Reasoning about code

CSE 331
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Reasoning about code

Determine what facts are true during execution

- `x > 0`
- for all nodes **`n`**: **`n.next.previous == n`**
- array **`a`** is sorted
- `x + y == z`
- if **`x != null`**, then **`x.a > x.b`**

Applications:

- Ensure code is correct (via reasoning or testing)
- Find defects
- Reproduce failures
- Understand why code is incorrect

Verify a representation invariant

Does this code maintain the rep invariant?

```
class NameList {

  // representation invariant: 0 ≤ index < names.length
  int index;
  String[] names;

  ...

  void addName(String name) {
    index++;
    if (index < names.length) {
      names[index] = name;
    }
  }

}
```

What must the caller do?

Incompletely documented:

```
// @param name a full name, last name first, like "Doe, John"
// @returns a two-element array of the first name and last name
String[] parseName(String name) {
  int commapos = name.indexOf(",");
  String lastName = name.substring(0, commapos);
  String firstName = name.substring(commapos + 2);
  return new String[] { firstName, lastName };
}
```

- What input produces [“John”, “Doe”]?
- What input produces [“ohn”, “Doe”]? [“ John”, “Doe”]?
- How can you improve the precondition?

Web server using SQL database

```
String userInput = …;
String query = "SELECT messages FROM users "
               + "WHERE name='" + userInput + "'";
statement.executeUpdate(query);  // execute DB query
```

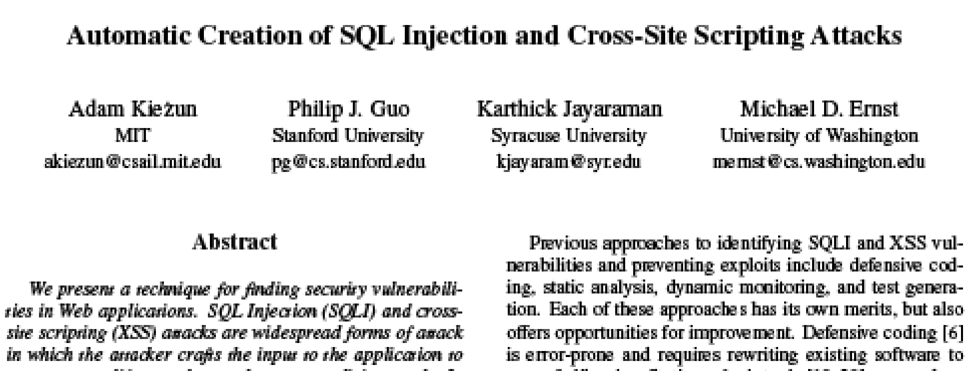
Is it possible to retrieve information for all users?

```
query = "SELECT messages FROM users
         WHERE name='a' or '1'='1'"
```

User inputs: **a' or '1'='1**

```
query  = "SELECT messages FROM users
         WHERE name='a' or '1'='1'"
```

http://xkcd.com/327/


Automatic Creation of SQL Injection and Cross-Site Scripting Attacks

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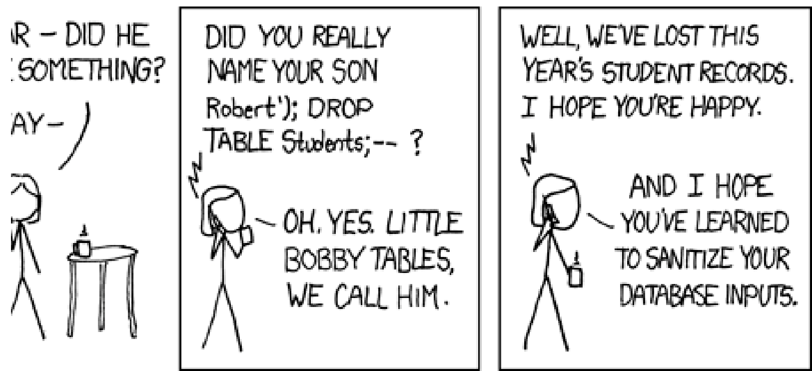
Abstract

We present a technique for finding security vulnerabilities in Web applications. SQL Injection (SQLI) and cross-site scripting (XSS) attacks are widespread forms of attack in which the attacker crafts the input to the application to

Previous approaches to identifying SQLI and XSS vulnerabilities and preventing exploits include defensive coding, static analysis, dynamic monitoring, and test generation. Each of these approaches has its own merits, but also offers opportunities for improvement. Defensive coding [6] is error-prone and requires rewriting existing software to



Ways to get your *design* right

The hard way
- Start hacking
- When something doesn't work, hack some more
  - How do you know it doesn't work?
  - Need to reproduce the errors your users experience
- Apply caffeine liberally

The easier way
- Plan first (specs, system decomposition, tests, ...)
- Less apparent progress upfront
- Faster completion times
- Better delivered product
- Less frustration

Ways to verify your *code*

Goal: correct code

The hard way: hacking

- Make up some inputs
- If it doesn't crash, ship it
- When it fails in the field, attempt to debug

An easier way: systematic testing

- Reason about possible behaviors and desired outcomes
- Construct simple tests that exercise all behaviors

Another way that can be easy: reasoning

- Prove that the system does what you want
  - Rep invariants are preserved
  - Implementation satisfies specification
- Proof can be formal or informal (we will be informal)
- Complementary to testing

Forward reasoning

You know what is true before running the code

    What is true after running the code?

Given a precondition, what is the postcondition?

Example:

```
// precondition:  x is even
x = x + 3;
y = 2x;
x = 5;
// postcondition:  ??
```

Applications:

- Rep invariant holds before running the code
  - Does it still hold after running the code?
- Does a method satisfy its spec?
  - If precondition holds, does postcondition hold?

Backward reasoning

You know what you want to be true after running the code

    What must be true beforehand in order to ensure that?

Given a postcondition, what must the precondition be?

Example:

```
// precondition:  ??
x = x + 3;
y = 2x;
x = 5;
// postcondition:  y > x
```

What was your reasoning?

Application:

- (Re-)establish rep invariant at method exit: what requires?
- Reproduce a failure: what must the input have been?
- Exploit a defect

Forward vs. backward reasoning

Forward reasoning is more intuitive

- Simulates the code (“abstract interpretation”)
- Introduces facts that may be irrelevant to the goal
  - Set of current facts may get large

Backward reasoning is sometimes more helpful

- Helps you understand what should happen
- Given a specific goal, indicates how to achieve it
  - Given an error, gives a test case that exposes it

Does the postcondition hold?

Use forward reasoning

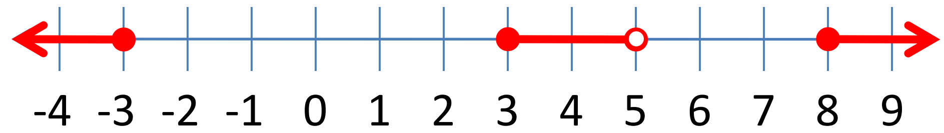
```
int x = …;
int z = …;
assert x >= 0;
                    // x ≥ 0
z = 0;
                    // x ≥ 0  &  z = 0
if (x != 0) {
                    // x > 0  &  z = 0
   z = x;
                    // x > 0  &  z = x
} else {
                    // x = 0  &  z = 0
   z = z + 1;
                    // x = 0  &  z = 1
}
                    // (x > 0  &  z = x)   OR   (x = 0  &  z = 1)
assert z > 0;
```

What input led to assertion failure?

Most common application of backward reasoning

```
                  // x < 0   OR   (x = 0  &  z ≤ -1)
                  // (x ≠ 0  &  x ≤ 0)   OR   (x = 0  &  z ≤ -1)
if (x != 0) {
                  // x ≤ 0
   z = x;
                  // z ≤ 0
} else {
                  // z ≤ -1
   z = z + 1;
                  // z ≤ 0
}
                  // z ≤ 0
assert z > 0;
```

Another example of backward reasoning



// precondition: ??

$(x \le -3)$ OR $(x \ge 3$ & $x < 5)$ OR $(x \ge 8)$

$(x < 5$ & $x*x \ge 9)$ OR $(x \ge 5$ & $x+1 \ge 9)$

```
if (x < 5) {
```

$x*x \ge 9$

```
    x = x*x;
```

$x \ge 9$

```
} else {
```

$x+1 \ge 9$

```
    x = x+1;
```

$x \ge 9$

```
}
```

// postcondition: $x \ge 9$

Called the “**weakest precondition**” or “wp”

If statements review

Forward reasoning

{P}
```
if B
```
  {P ∧ B}
```
  S1
```
  {Q1}
```
else
```
  {P ∧ !B}
```
  S2
```
  {Q2}

{Q1 ∨ Q2}

Backward reasoning

{ (B ∧ wp(S1, Q)) ∨ (¬B ∧ wp(S2, Q)) }
```
if B
```
  {wp(S1, Q)}
```
  S1
```
  {Q}
```
else
```
  {wp(S2, Q)}
```
  S2
```
  {Q}

{Q}

Forward reasoning with a loop

```
assert x >= 0;
```
// $x \geq 0$
```
i = x;
```
// $x \geq 0$ & $i = x$

($x \geq 0$ & $i = x$ & $z = 0$)
OR LOOP-END
```
z = 0;
```
// $x \geq 0$ & $i = x$ & $z = 0$
```
while (i != 0) {
```
// LOOP-BEGIN $i \neq 0$ & (($x \geq 0$ & $i = x$ & $z = 0$)
OR LOOP-END)
```
  z = z + 1;
  i = i - 1;
```
// LOOP-END
```
}
```
// $x \geq 0$ & $i = 0$ & $z = x$
```
assert x == z;
```

Infinite number of paths through this code
How do you know that the overall conclusion is correct?
Induction on the length of the computation

Reasoning about loops

A loop represents an unknown number of paths

- Case analysis is problematic
- Recursion presents the same problem as loops

Cannot enumerate all paths

- This is what makes testing and reasoning hard

Things to prove about a loop:

1. It computes the correct value
2. It terminates (no infinite loop)

Reasoning about loops: values and termination

```
// assert x ≥ 0 & y = 0
while (x != y) {
    y = y + 1;
}
// assert x = y
```

Does "x=y" hold after this loop?
Does this loop terminate?
1) Pre-assertion guarantees that $x \geq y$
2) Every time through loop
    $x \geq y$ holds before at the test
    If the body is entered, $x > y$ -- this is LOOP-BEGIN
    y is incremented by 1
    x is unchanged
    Therefore, y is closer to x (but $x \geq y$ still holds) – this is LOOP-END
3) Since there are only a finite number of integers between x and y, y will eventually equal x
4) Execution exits the loop as soon as $x = y$ (but $x \geq y$ still holds)

Understanding loops by induction

We just made an inductive argument
- Inducting over the *number of iterations*

Computation induction
- Show that conjecture holds if zero iterations
- Show that it holds after n+1 iterations
  - (assuming that it holds after n iterations)

Two things to prove
1. Some property is preserved (known as "partial correctness"), *if* the code terminates
   - Loop invariant is preserved by each iteration, if the iteration completes
2. The loop completes (known as "termination")
   - The "decrementing function" is reduced by each iteration and cannot be reduced forever

Example: Factorial

{ arg≥0 & n=arg } // n is a temporary variable

```
r=1;
```

arg≥0 ∧ n=arg ∧ r=1

```
while (n≠0) {
```

r=arg!/n! ∧ arg≥n>0

“Loop invariant”. Where did this come from?

```
  r=r*n;
```

r=arg!/(n-1)! ∧ arg≥n>0

```
  n=n-1;
```

r=arg!/n! ∧ arg≥n≥0

```
}
```

(n = 0 ∧ arg≥0 ∧ n=arg ∧ r=1) OR (n = 0 ∧ r=arg!/n! ∧ arg≥n≥0)

{ r=arg! } (n = 0 ∧ arg=0 ∧ r=1) OR (n = 0 ∧ arg≥0 ∧ r=arg!)

Loop invariant

1. When reverse engineering: *guess* it
2. When designing: choose it *before* writing code

To design loops or recursion:

- Decompose large problems into smaller ones
  - "Divide and conquer" or "*Wishful thinking*" design methodology
- "I don't know how to compute $n!$, but I could compute it if you told me $(n-1)!$."
  - What about 0! ?

Loop design methodology

1. Decompose problem into
   - Assumption that most of the problem is solved
   - A small increment of remaining work
2. Write an invariant that expresses the milestone of each iteration
3. Write a loop body to perform the increment while maintaining the invariant
4. Write the loop test so false-implies-postcondition
5. Write initialization code to establish invariant

Loop design example

Set **`max`** to hold the largest value in array **`items`**

`max` = *amax*(**`items[0..len]`**)

1. Decomposition: Given *amax*(**`items[0..len-1]`**), can determine *amax*(**`items[0..len]`**)

   amax(**`items[0..len]`**)
   = max(*amax*(**`items[0..len-1]`**), **`items[len]`**)

2. Invariant: **`max`** holds largest value in range **`items[0..k-1]`**

Loop design example

Set **`max`** to hold the largest value in array **`items`**

3. Write a loop body to perform the increment and maintain the invariant

```
// inv: max holds largest value in items[0..k-1]
  while (...) {
    // inv holds
    if (items[k] > max) {
      max = items[k]; // breaks inv temporarily
    } else {
      // nothing to do
    }
    // max holds largest value in items[0..k]
    k = k+1; // invariant holds again
  }
```

Loop design example

Set **`max`** to hold the largest value in array **`items`**

4. Write the loop test so false-implies-postcondition

```
// inv: max holds largest value in items[0..k-1]
  while (k != items.length) {
    // inv holds
    if (items[k] > max) {
      max = items[k]; // breaks inv temporarily
    } else {
      // nothing to do
    }
    // max holds largest value in items[0..k]
    k = k+1; // invariant holds again
  }
```

Loop design example

Set **`max`** to hold the largest value in array **`items`**

5. Write initialization code to establish invariant

```
k = 1;
max = items[0];
// inv: max holds largest value in items[0..k-1]
while (k != items.length) {
   …
}
```

Loop design edge case

Our initialization code has a precondition: `items.size > 0`

```
// items.length > 0
k = 1;
max = items[0];
// inv: max holds largest value in items[0..k-1]}
while (k != items.size) {
   …
}
```

Such a (specified!) precondition may be appropriate

Else need a different postcondition ("if size is 0, …") and a conditional that checks for the empty case

Or the `Integer.MIN_VALUE` "trick" and logical reasoning

Neat: Precise preconditions should expose all this to you!

Example: Quotient and remainder

Compute quotient and remainder for num/denom

num = 0 $\times$ denom + num

```
q := 0;
```

num = 0 $\times$ denom + r

```
r := num;
```

num = r + q $\times$ denom

```
while (denom <= r) {
  r := r - denom;
  q := 1 + q;
}
```

// num = q $\times$ denom +r & r < denom

Example: Greatest common divisor

$\{ x1 > 0 \ \wedge \ x2 > 0 \}$

```
y1:=x1;
y2:=x2;
while (y1≠y2) do
   if  y1>y2
      then y1 := y1-y2
      else y2 := y2-y1
```

$\{ y1 = gcd(x1,x2) \}$

Recall: if y1,y2 are both positive integers, then:

- If y1>y2 then gcd(y1,y2)=gcd(y1-y2,y2)
- If y2>y1 then gcd(y1,y2)=gcd(y1,y2-y1)
- If y1-y2 then gcd(y1,y2)=y1=y2

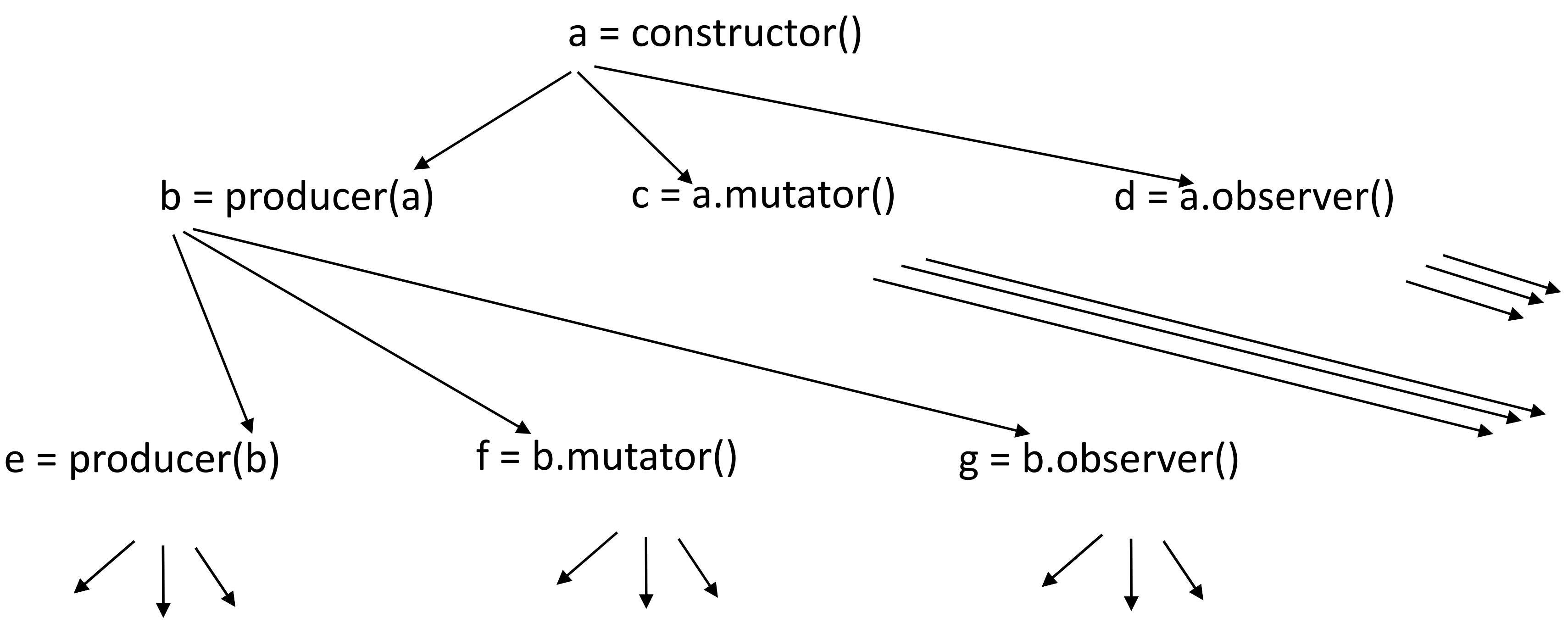
Goal: Demonstrate that rep invariant is satisfied

- Exhaustive testing
  - Create every possible object of the type
  - Check rep invariant for each object
  - Problem: impractical
- Limited testing
  - Choose representative objects of the type
  - Check rep invariant for each object
  - Problem: did you choose well?
- Reasoning
  - Prove that all objects of the type satisfy the rep invariant
  - Sometimes easier than testing, sometimes harder
  - Every good programmer uses it as appropriate

All possible objects (and values) of a type

- Make a new object
  - constructors
  - producers
- Modify an existing object
  - mutators
  - observers, producers (why?)
- Limited number of operations, but infinitely many objects
  - Maybe infinitely many values as well

Examples of making objects



Infinitely many possibilities

We cannot perform a proof that considers each possibility case-by-case

Solution: induction

Induction: technique for proving *infinitely* many facts using *finitely* many proof steps

For constructors ("basis step")

- Prove the property holds on exit

For all other methods ("inductive step")

- Prove that:
  - if the property holds on entry, then it holds on exit

If the basis and inductive steps are true:

- There is no way to make an object for which the property does not hold
- Therefore, the property holds for all objects