Understanding an ADT implementation: Abstraction functions

CSE 331
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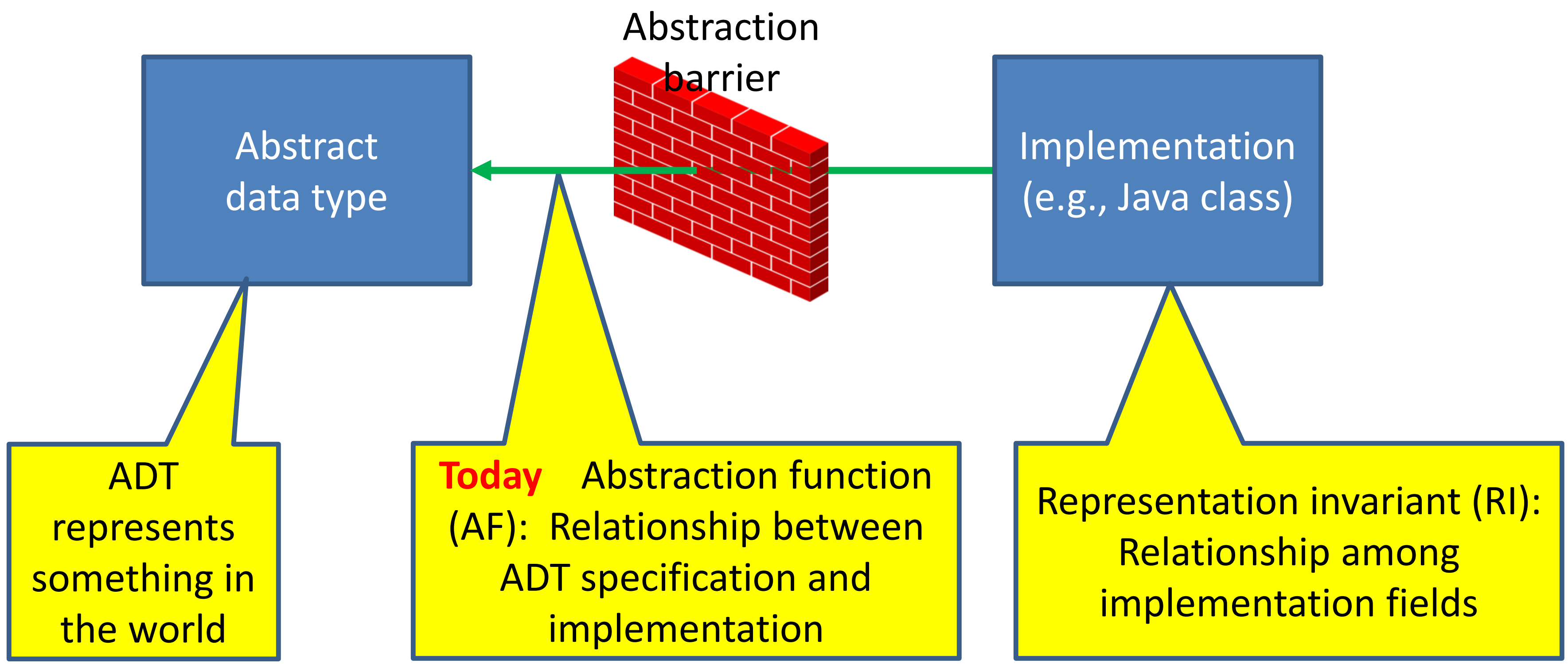
Outline of data abstraction lectures

ADT specification

ADT implementation



Review: Connecting specifications and implementations

Representation invariant: Object → boolean

- Indicates whether rep/instance is well-formed
- Defines the set of valid values of the data structure
- Only well-formed representations are meaningful

Abstraction function: Object → abstract value

- What the rep/instance means as an abstract value
- How the rep/instance is to be interpreted

Used by implementors/maintainers of the abstraction

Abstraction function: rep → abstract value

The abstraction function maps the concrete representation to the abstract value it represents

AF: Object → abstract value

AF(CharSet this) = { c | c is contained in this.elts }

"set of Characters contained in this.elts"

Typically *not* executable (Why?)

The abstraction function lets us reason about concrete method behavior from the client (abstract) perspective

Rep invariant constrains structure, not meaning

An implementation of **`insert`** that preserves the rep invariant (no nulls or duplicates in **`elts`**):

```
public void insert(Character c) {
  Character cc = new Character(encrypt(c));
  if (!elts.contains(cc))
    elts.addElement(cc);
}
public boolean member(Character c) {
  return elts.contains(c);
}
```

Implementation is still wrong; this client code observes incorrect behavior:

```
CharSet s = new CharSet();
s.insert('a');
if (s.member('a'))
    …
```

Abstraction function and insert impl.

Our real goal is to satisfy the specification of insert:

```
// modifies: this
// effects: this_post = this_pre U {c}
public void insert(Character c);
```

The AF tells us what the rep means (and lets us place the blame)

AF(CharSet this) = { c | c is contained in this.elts }

Consider a call **`insert('a')`**:

On entry, the meaning is AF($this_{pre}$) ≈ $elts_{pre}$

On exit, the meaning is AF($this_{post}$) = AF($this_{pre}$) U {encrypt('a')}

What if we used this abstraction function instead?

AF(this) = { c | encrypt(c) is contained in this.elts }

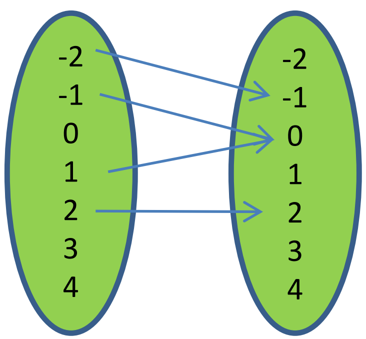
= { decrypt(c) | c is contained in this.elts }

The abstraction function: concrete → abstract

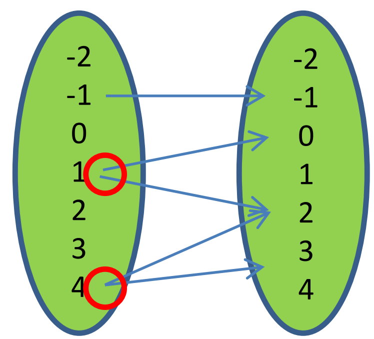
Q: Why don’t we use the inverse of the AF? What function maps abstract to concrete?

1. It’s not a function in the other direction.
   E.g., lists [a,b] and [b,a] each represent the set {a, b}
2. To go from abstract to concrete, just construct and modify objects via the provided operators
3. Not helpful in reasoning about impl cerrectness

A function maps each argument to at most one value



Function



Not a function

Multiple reps for the same abstract value

Stack rep:
`int[] elements;`
`int top;` // first unused index

`new Stack()`

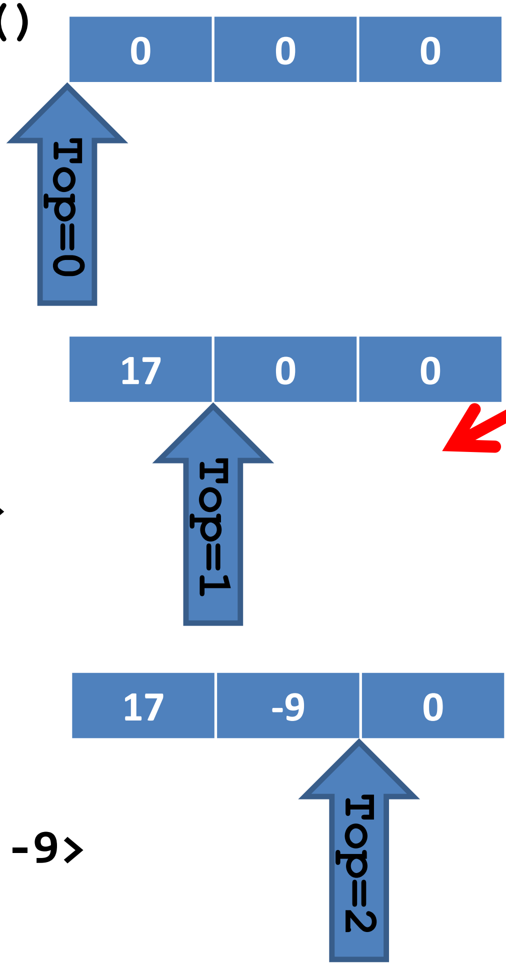
`stack = <>`

`push(17)`

`stack = <17>`

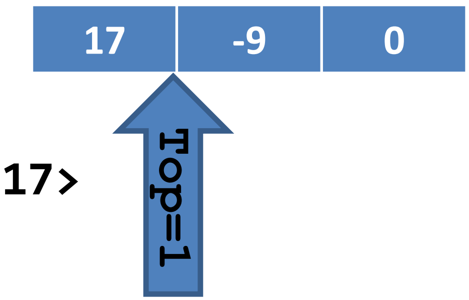
`push(-9)`

`stack = <17,-9>`



`pop()`

`stack = <17>`



Abstract states are the same
`stack = <17> = <17>`

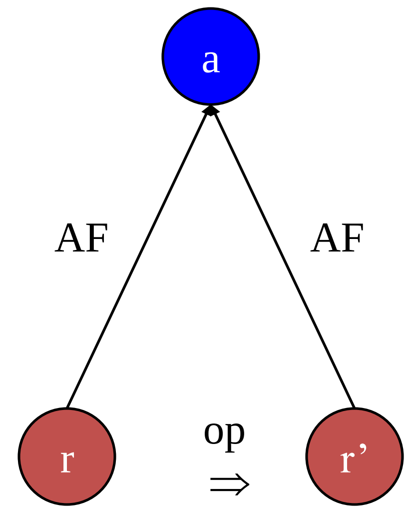
Concrete states are different
`<[17,0,0], top=1>`
≠
`<[17,-9,0], top=1>`

AF is a function
AF^{-1} is not a function

Benevolent side effects

Different implementation of member:

```
boolean member(Character c) {
   int i = elts.indexOf(c);
   if (i == -1)
     return false;
   // move-to-front optimization
   Character tmp = elts.elementAt(0);
   elts.set(0, c);
   elts.set(i, tmp);
   return true;
}
```

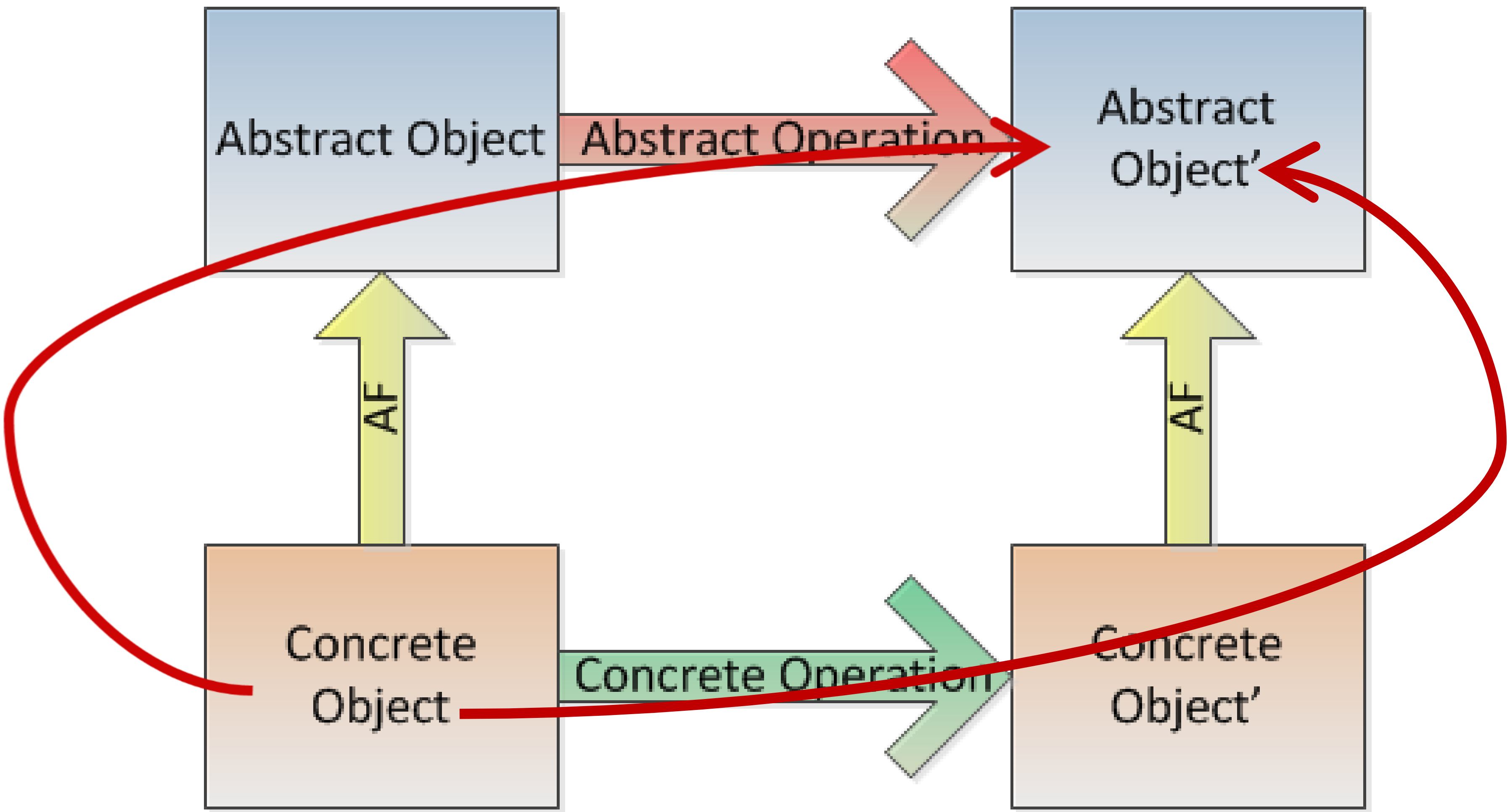


Move-to-front speeds up repeated membership tests

Mutates rep, but does not change *abstract* value

- AF maps both reps to the same abstract value
- Example: { a, c, i, n, o, t, u } = AF([a|u|c|t|i|o|n]) = AF([c|a|u|t|i|o|n])

For any correct operation



Writing an abstraction function

- The domain: all representations that satisfy the rep invariant
- The range: can be tricky to denote
  - For mathematical entities like sets: easy
  - For more complex abstractions: give them fields
    - AF defines the value of each “specification field”
    - For “derived specification fields”, see the handouts
- The overview section of the specification should provide a way of writing abstract values
  - This printed representation is valuable for debugging (`toString`)

ADTs and Java language features

- Java classes
  - Make operations in the ADT public
  - Make other operations and fields of the class private
  - Clients can only access ADT operations
- Java interfaces
  - Clients only see the ADT, not the implementation
  - Multiple implementations have no code in common
  - Cannot include creators (constructors) or fields
- Both classes and interfaces are sometimes appropriate
  - Write and rely upon careful specifications
  - Prefer interface types instead of specific classes in declarations (e.g., **`List`** instead of **`ArrayList`** for variables and parameters)

Connecting ADTs to implementations: Summary

Rep invariant

- Which concrete values represent abstract values

Abstraction function

- For each concrete value, which abstract value it represents

Neither one is part of the abstraction (the ADT)

Use both to reason that an implementation satisfies the specification

- They modularize the implementation
- Can examine operators one at a time

When you program:

- Always write a rep invariant (standard industry best practice)
- Write an abstraction function when you need it
  - Write an informal one for most non-trivial classes
  - A formal one is harder to write and often less useful
    - Helps with reasoning and debugging

Invariants simplify reasoning

- Why focus so much on invariants (properties of code that do not change)?
- Why focus so much on immutability (a specific kind of invariant)?

- Software is complex – invariants/immutability reduce the intellectual complexity
- If we can assume some property remains unchanged, we don't have to worry about it
- Reducing what we need to think about can be a huge benefit