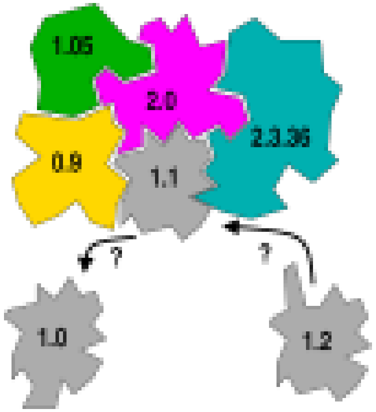

Comparing procedure specifications

CSE 331
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Outline

- Satisfying a specification; substitutability
- Stronger and weaker specifications
  - Comparing by hand
  - Comparing via logical formulas
  - Comparing via transition relations
    - Full transition relations
    - Abbreviated transition relations
- Specification style; checking preconditions

Satisfaction of a specification

- Let P be an implementation and S a specification
- *P satisfies S* iff
  - Every behavior of P is permitted by S
  - “The behavior of P is a subset of S”
- The statement “P is correct” is meaningless
  - Though often made!
- If P does not satisfy S, either (or both!) could be “wrong”
  - *“One person’s feature is another person’s bug.”*
  - It’s usually better to change the program than the spec

Why compare specifications?

We wish to compare procedures to specifications

- Does the procedure satisfy the specification?
- Has the implementer succeeded?

We wish to compare specifications to one another

- Which specification (if either) is stronger?
- Substitutability:
  A procedure satisfying a stronger specification
  can be used anywhere
  that a weaker specification is required

A specification denotes a set of procedures

Some set of procedures satisfies a specification

Suppose a procedure takes an integer as an argument

- Spec 1: “returns an integer ≥ its argument”
- Spec 2: “returns a non-negative integer ≥ its argument”
- Spec 3: “returns argument + 1”
- Spec 4: “returns argument^2 ”
- Spec 5: “returns Integer.MAX_VALUE”

Consider these implementations:

- Code 1: `return arg * 2;`
- Code 2: `return abs(arg);`
- Code 3: `return arg + 5;`
- Code 4: `return arg * arg;`
- Code 5: `return Integer.MAX_VALUE;`

Spec1	Spec2	Spec3	Spec4	Spec5
No				
Yes				

Ignore overflow

How does overflow affect answers?

Review:
Specification strength and substitutability

- A stronger specification promises more
  - It constrains the implementation more
  - The client can make more assumptions
  - *Weaker* preconditions ("contravariance")
  - Stronger postconditions
- Substitutability
  - A stronger specification can always be substituted for a weaker one

Procedure specifications

Example of a procedure specification:

```
// requires i > 0
// modifies nothing
// returns true iff i is a prime number
public static boolean isPrime(int i)
```

General form of a procedure specification:

// requires	a logical formula (a Boolean expression)
// modifies	a list of (Java) expressions
// throws	a list of exceptions, each with a condition
// effects	a logical formula (a Boolean expression)
// returns	(a condition on) the return value; like “// effects result = …”

How to compare specifications

Three ways to compare

1. By hand; examine each clause
   Advantage: can be checked manually
2. Logical formulas representing the specification
   Advantage: mechanizable in tools
3. Transition relations
   Advantage: captures intuition of “stronger = smaller”
   a. Full transition relations
   b. Abbreviated transition relations

Use whichever is most convenient

Technique 1: Comparing by hand

Idea: compare the specification field-by-field

S_2 is stronger than S_1 if

- S_2 requires is easier to satisfy (weaker requires)
  - Preconditions are *contravariant* (other clauses are *covariant*)
- S_2 modifies is smaller (stronger modifies)
- S_2 effects is harder to satisfy (stronger effects)
- S_2 throws guarantees more (stronger throws)
- S_2 returns guarantees more (stronger returns)

Trivia:

The strongest (most constraining) spec has the following:

requires clause: true	(equivalently, "requires nothing")
modifies clause: Ø	(equivalently, "modifies nothing")
effects clause: false	
throws clause: nothing	
returns clause:	*(there is no strongest returns clause)*

(This particular spec is so strong as to be useless.)

Technique 2: Comparing logical formulas

Essentially the same as technique 1 (comparing by hand).

Technique 1:

- 5 small comparisons
- Combine them to determine whether S_2 is stronger than S_1

Technique 2:

- One big comparison

Why do we care? Why should we learn another technique?

- Good for automated tools (you are unlikely to use it manually)
- Gives another perspective
- Helps to explicate rules (explains contravariance)

Technique 2: Comparing logical formulas

Specification S2 is stronger than S1 iff:

$\forall$ implementation P, (P satisfies S2) $\Rightarrow$ (P satisfies S1)

If each specification is a logical formula, this is equivalent to:

S2 $\Rightarrow$ S1

So, convert each spec to a formula (in 2 steps, see following slides)

This specification:

// requires *R*
// modifies *M*
// effects *E*

is equivalent to this single logical formula:

$R \Rightarrow (E \wedge$ (nothing but M is modified))

What about throws and returns? Absorb them into effects.

Final result: S2 is stronger than S1 iff

$(R_2 \Rightarrow (E_2 \wedge \text{only-modifies-}M_2)) \Rightarrow (R_1 \Rightarrow (E_1 \wedge \text{only-modifies-}M_1))$

Convert spec to formula, step 1: absorb throws and returns into effects

CSE 331 style:
requires (unchanged)
modifies (unchanged)
throws
effects } correspond to resulting "effects"
returns

Example (from `java.util.ArrayList<T>`):

```
// requires: true
// modifies: this[index]
// throws: IndexOutOfBoundsException if index < 0 || index ≥ size()
// effects: this_post[index] = element
// returns: this_pre[index]
T set(int index, T element)
```

Equivalent spec, after absorbing throws and returns into effects:

```
// requires: true
// modifies: this[index]
// effects: if index < 0 || index ≥ size() then throws IndexOutOfBoundsException
//          else this_post[index] = element && returns this_pre[index]
T set(int index, T element)
```

Convert spec to formula, step 2: eliminate requires, modifies

Single logical formula

requires $\Rightarrow$ (effects $\wedge$ (*not-modified*))

"not-modified" preserves every field *not* in the modifies clause

Logical fact: If precondition is false, formula is true

Recall: For any x and y: $x \Rightarrow \text{true}$; $\text{false} \Rightarrow x$; $(x \Rightarrow y) \equiv (\neg x \vee y)$

Example:

// requires: true

// modifies: this[index]

// effects: *E*

```
T set(int index, T element)
```

Result:

$\text{true} \Rightarrow (E \wedge (\forall i{\neq}\text{index}.\ \text{this}_{\text{pre}}[i] = \text{this}_{\text{post}}[i]))$

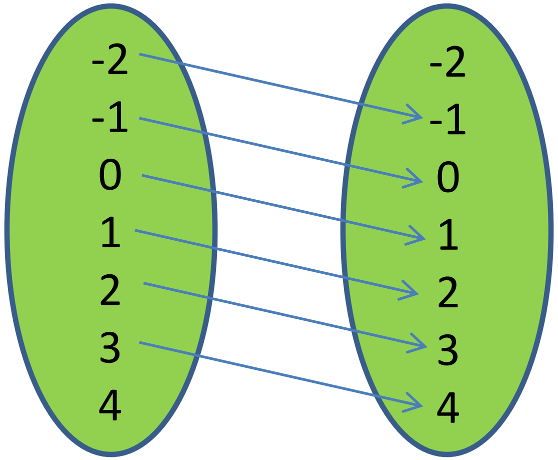
Technique 3: Comparing transition relations

Transition relation relates prestates to poststates

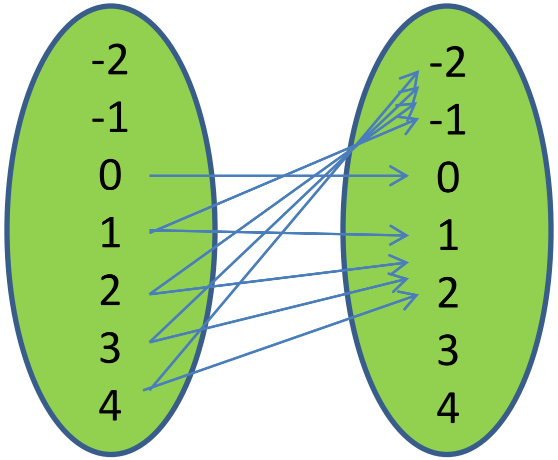
- Includes all possible behaviors

Transition relation maps procedure arguments to results

```
int increment(int i) {
  return i+1;
}
```



```
// requires: a ≥ 0
double mySqrt(double a) {
  if (Random.nextBoolean())
    return Math.sqrt(a);
  else
    return - Math.sqrt(a);
}
```



A specification has a transition relation, too

- Contains just as much information as other forms of specification

Satisfaction via transition relations

A stronger specification has a smaller transition relation

Rule: P satisfies S iff P is a subset of S

(when both are viewed as transition relations)

sqrt specification (S_{sqrt})

```
// requires x is a perfect square
// returns positive or negative square root
int sqrt(int x)
```

Transition relation: ⟨0,0⟩, ⟨1,1⟩, ⟨1,-1⟩, ⟨4,2⟩, ⟨4,-2⟩, …

Expressed as ⟨input,output⟩ pairs

sqrt code (P_{sqrt})

```
int sqrt(int x) {
    // … always returns positive square root
}
```

Transition relation: ⟨0,0⟩, ⟨1,1⟩, ⟨4,2⟩, …

P_{sqrt} satisfies S_{sqrt} because P_{sqrt} is a subset of S_{sqrt}

Beware transition relations in abbreviated form

"P satisfies S iff P is a subset of S" is a good rule

- But it gives the wrong answer for transition relations in abbreviated form
- (The transition relations we have seen so far are in abbreviated form!)

anyOdd specification (S_{anyOdd})

```
// requires x = 0
// returns any odd integer
int anyOdd(int x)
```

- **Abbreviated** transition relation: ⟨0,1⟩, ⟨0,3⟩, ⟨0,5⟩, ⟨0,7⟩, …

anyOdd code (P_{anyOdd})

```
int anyOdd(int x) {
    return 3;
}
```

- Transition relation: ⟨0,3⟩, ⟨1,3⟩, ⟨2,3⟩, ⟨3,3⟩, …

The code satisfies the specification, but the rule says it does not

- P_{anyOdd} is not a subset of S_{anyOdd}
- because ⟨1,3⟩ is not in the specification's transition relation

We will see two solutions to this problem: full or abbreviated transition relations

Satisfaction via *full* transition relations (option 1)

The transition relation should make explicit everything an implementation may do.

Problem: Abbreviated transition relation for S does not indicate all possibilities.

anyOdd specification (S_{anyOdd}): // same as before

```
// requires x = 0
// returns any odd integer
int anyOdd(int x)
```

Full transition relation: ⟨0,1⟩, ⟨0,3⟩, ⟨0,5⟩, ⟨0,7⟩, … // on previous slide

⟨1, 0⟩, ⟨1, 1⟩, ⟨1, 2⟩, …, ⟨1, exception⟩, ⟨1, infinite loop⟩, … // new

⟨2, 0⟩, ⟨2, 1⟩, ⟨2, 2⟩, …, ⟨2, exception⟩, ⟨2, infinite loop⟩, … // new

anyOdd code (P_{anyOdd}): // same as before

```
int anyOdd(int x) {
   return 3;
}
```

Transition relation: ⟨0,3⟩, ⟨1,3⟩, ⟨2,3⟩, ⟨3,3⟩, … // same as before

The rule "P satisfies S iff P is a subset of S" gives the right answer for full relations.

Downside: Writing the full transition relation is bulky and inconvenient.

It's more convenient to make the implicit notational assumption:

For elements not in the domain of S, any behavior is permitted.

(Recall that a relation maps a *domain* to a *range*.)

Satisfaction via *abbreviated* transition relations (option 2)

New rule: P satisfies S iff P | (Domain of S) is a subset of S

where “P | D” = “P restricted to the domain D”

i.e., remove from P all pairs whose first member is not in D

(Recall that a relation maps a *domain* to a *range*.)

anyOdd specification (S_{anyOdd})

```
// requires x = 0
// returns any odd integer
int anyOdd(int x)
```

Abbreviated transition relation: ⟨0,1⟩, ⟨0,3⟩, ⟨0,5⟩, ⟨0,7⟩, …

anyOdd code (P_{anyOdd})

```
int anyOdd(int x) {
    return 3;
}
```

Transition relation: ⟨0,3⟩, ⟨1,3⟩, ⟨2,3⟩, ⟨3,3⟩, …

Domain of S = { 0 }

P | (domain of S) = ⟨0,3⟩, which is a subset of S, so P satisfies S.

The new rule gives the right answer even for abbreviated transition relations.

We’ll use this version of the notation in CSE 331.

Abbreviated transition relations, summary

True transition relation:

- Contains all the pairs, all comparisons work
- Bulky to read and write

Abbreviated transition relation

- Shorter and more convenient
- Naively doing comparisons leads to wrong result

How to do comparisons:

- – Use the expanded transition relation, or
- – Restrict the domain when comparing

Either approach makes the “smaller is stronger” intuition work

Review: ways to compare specifications

A stronger specification is satisfied by fewer implementations

A stronger specification has

- *weaker* preconditions (note contravariance)
- stronger postcondition
- fewer modifications

  Advantage of this view: can be checked by hand

A stronger specification has a (logically) stronger formula

  Advantage of this view: mechanizable in tools

A stronger specification has a smaller transition relation

  Advantage of this view: captures intuition of “stronger = smaller” (fewer choices)

Specification style

The point of a specification is to be helpful

  Formalism helps, overformalism doesn't

A specification should be

- coherent: not too many cases
- informative: a bad example is **`HashMap.get`**
- strong enough: to do something useful, to make guarantees
- weak enough: to permit (efficient) implementation

A procedure has a side effect *or* is called for its value

  Bad style to have *both* effects and returns

  Exception: return old value, as for **`HashMap.put`**

Should preconditions be checked?

Checking preconditions
  - makes an implementation more robust
  - provides better feedback to the client (fail fast)
  - avoids silent failures, avoids delayed failures

Preconditions are common in “helper” methods/classes
  - In public APIs, no precondition ⇒ handle all possible input
  - Why does binarySearch impose a precondition?

Rule of thumb: Check if it is cheap to do so
  - Example: list must be non-null ⇒ check
  - Example: list must be sorted ⇒ don’t check

A quality implementation checks preconditions whenever it is *inexpensive* and *convenient* to do so