Section 6: HW6 and Midterm

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(with material from Alex Mariakakis,
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Breadth-First Search (BFS)

Often used for discovering connectivity

Calculates the shortest path *if and only if* all edges have same positive or no weight

Depth-first search (DFS) is commonly mentioned with BFS

- BFS looks “wide”, DFS looks “deep”
- Can also be used for discovery, but not the shortest path

BFS Pseudocode

```
public boolean find(Node start, Node end) {
      put start node in a queue
      while (queue is not empty) {
            pop node N off queue
            if (N == end)
                  return true;
            else {
                  for each node O that is child of N
                        push O onto queue
            }
      }
      return false;
}
```

Breadth-First Search

START:

Q: <A>

Pop: A, Q: <>

Q: <B, C>

Pop: B, Q: <C>

Q: <C>

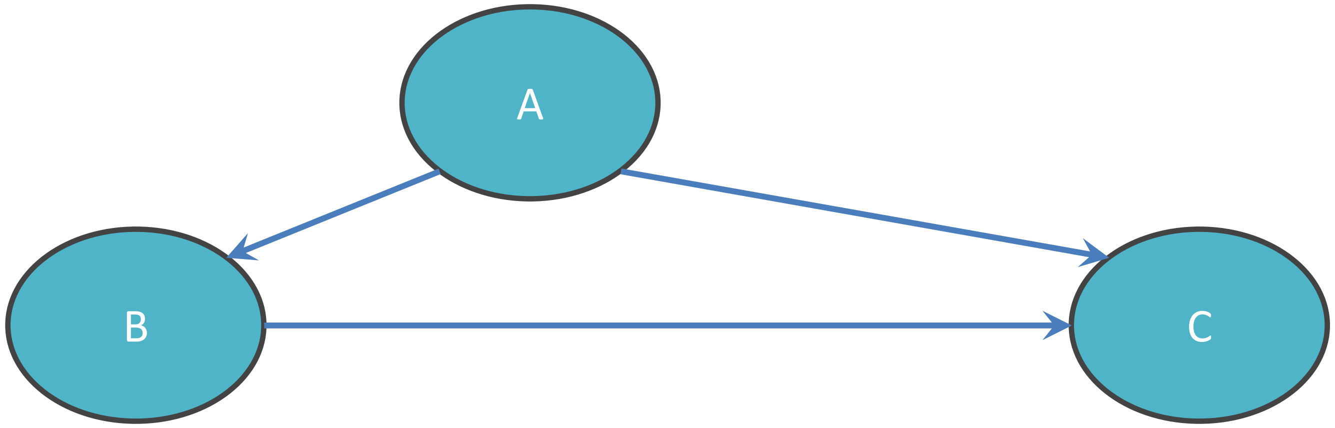
Pop: C, Q: <C>

Q: <>

DONE

Starting at A

Goal: Fully explore



Breadth-First Search with Cycle

START:

Q: <A>

Pop: A, Q: <>

Q: <B>

Pop: B, Q: <>

Q: <C>

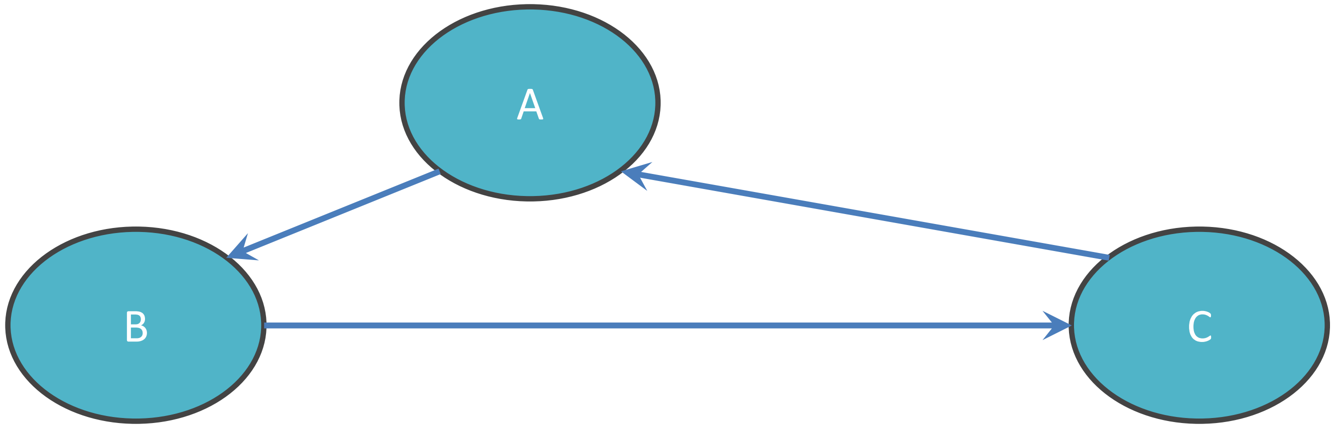
Pop: C, Q: <>

Q: <A>

NEVER DONE

Starting at A

Goal: Fully Explore



BFS Pseudocode

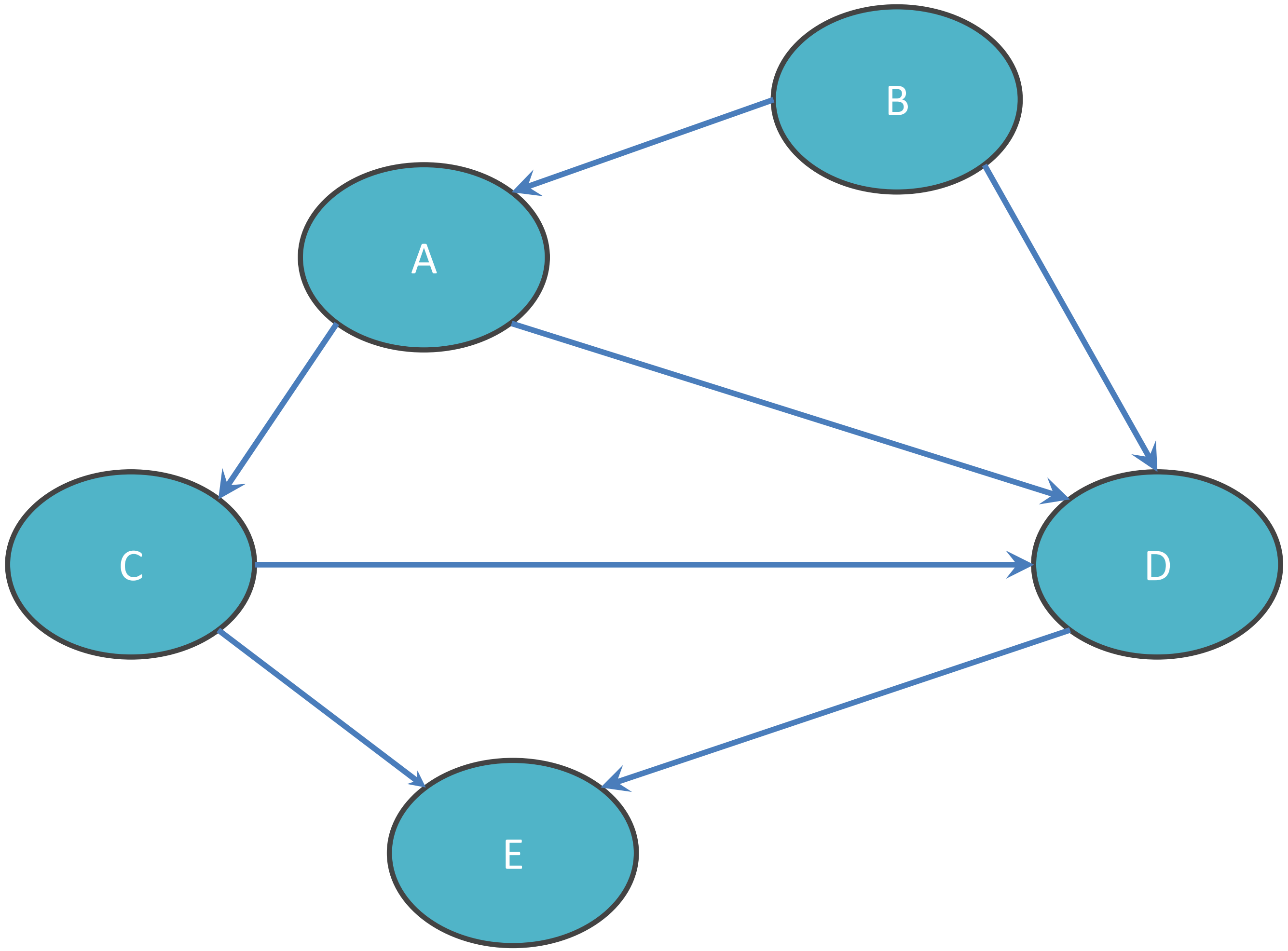
```
public boolean find(Node start, Node end) {
      put start node in a queue
      while (queue is not empty) {
            pop node N off queue
            mark node N as visited

            if (N is goal)
                  return true;
            else {
                  for each node O that is child of N
                        if O is not marked visited
                              push O onto queue
            }
      }
      return false;
}
```

Mark the node as visited!

Breadth-First Search

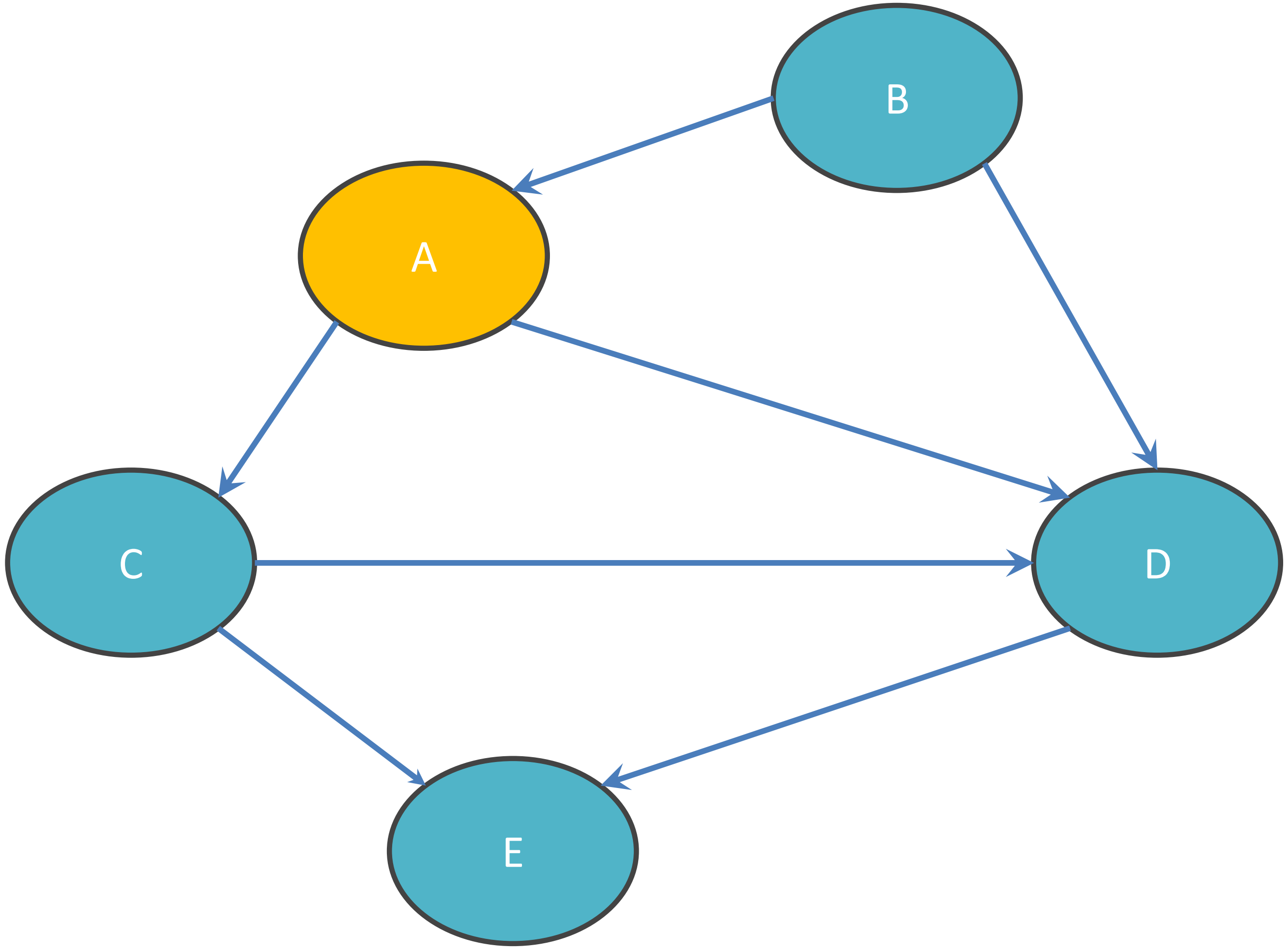
Q: <>



Breadth-First Search

Q: <>

Q: <A>

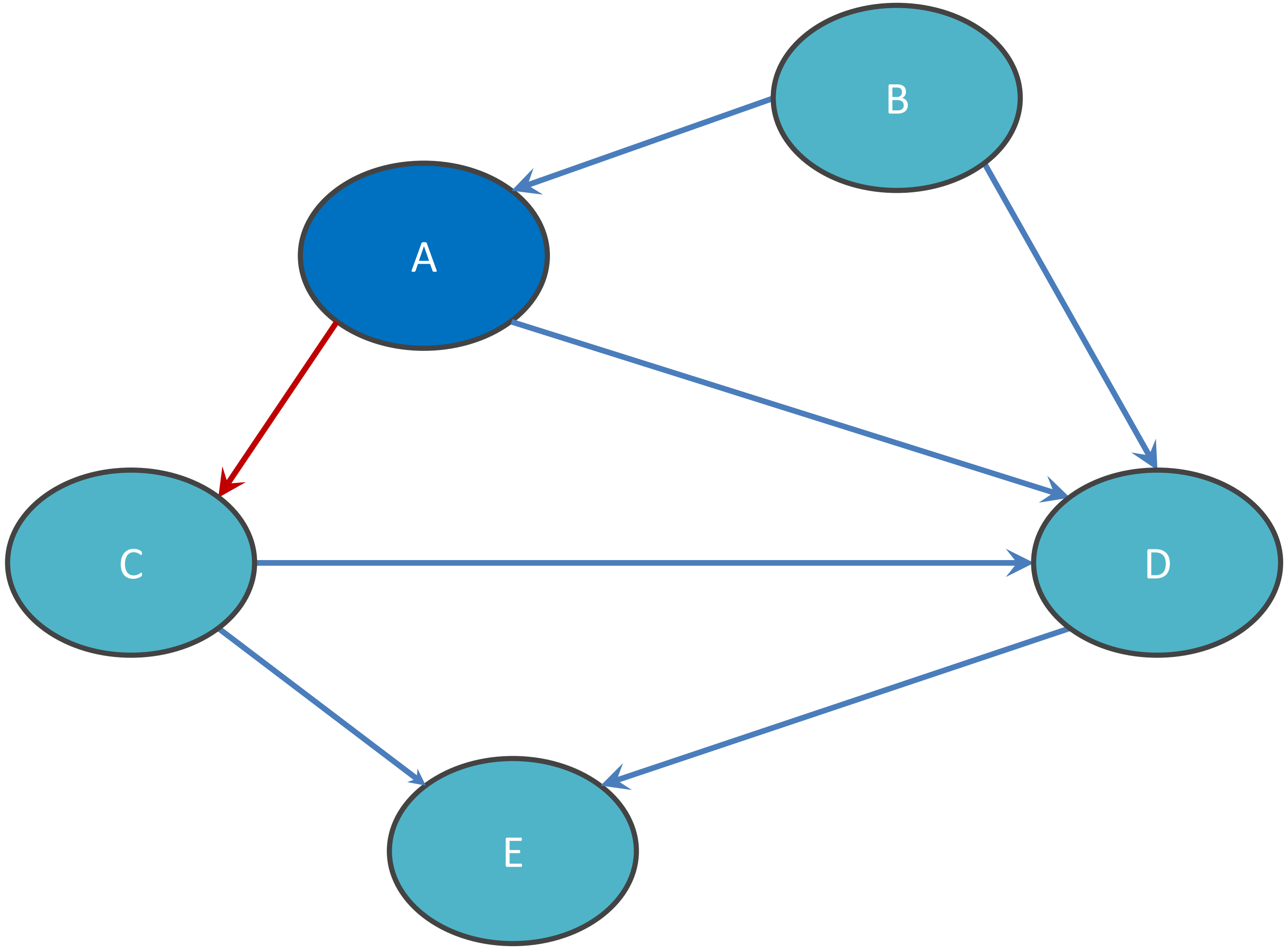


Breadth-First Search

Q: <>

Q: <A>

Q: <>



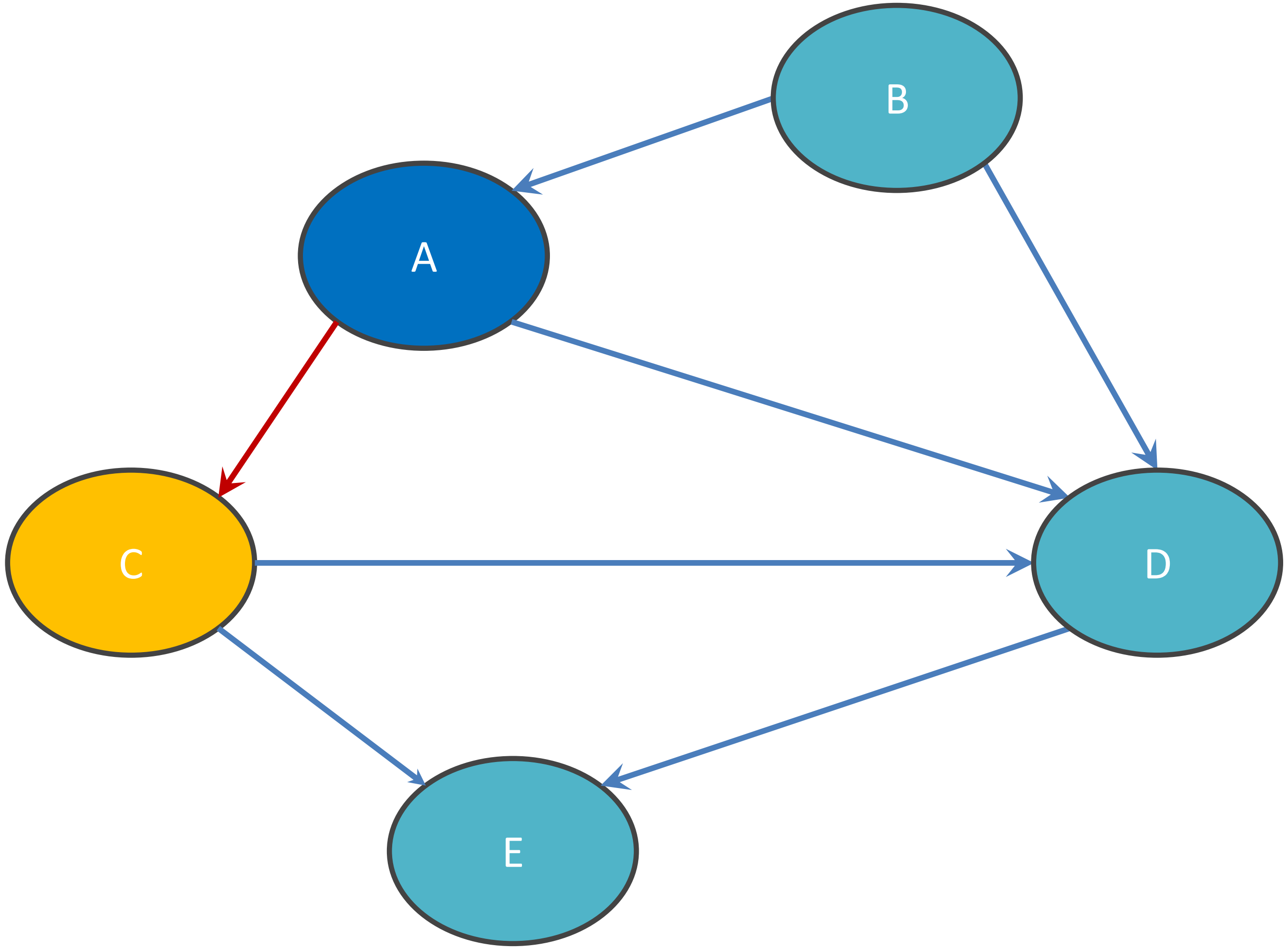
Breadth-First Search

Q: <>

Q: <A>

Q: <>

Q: <C>



Breadth-First Search

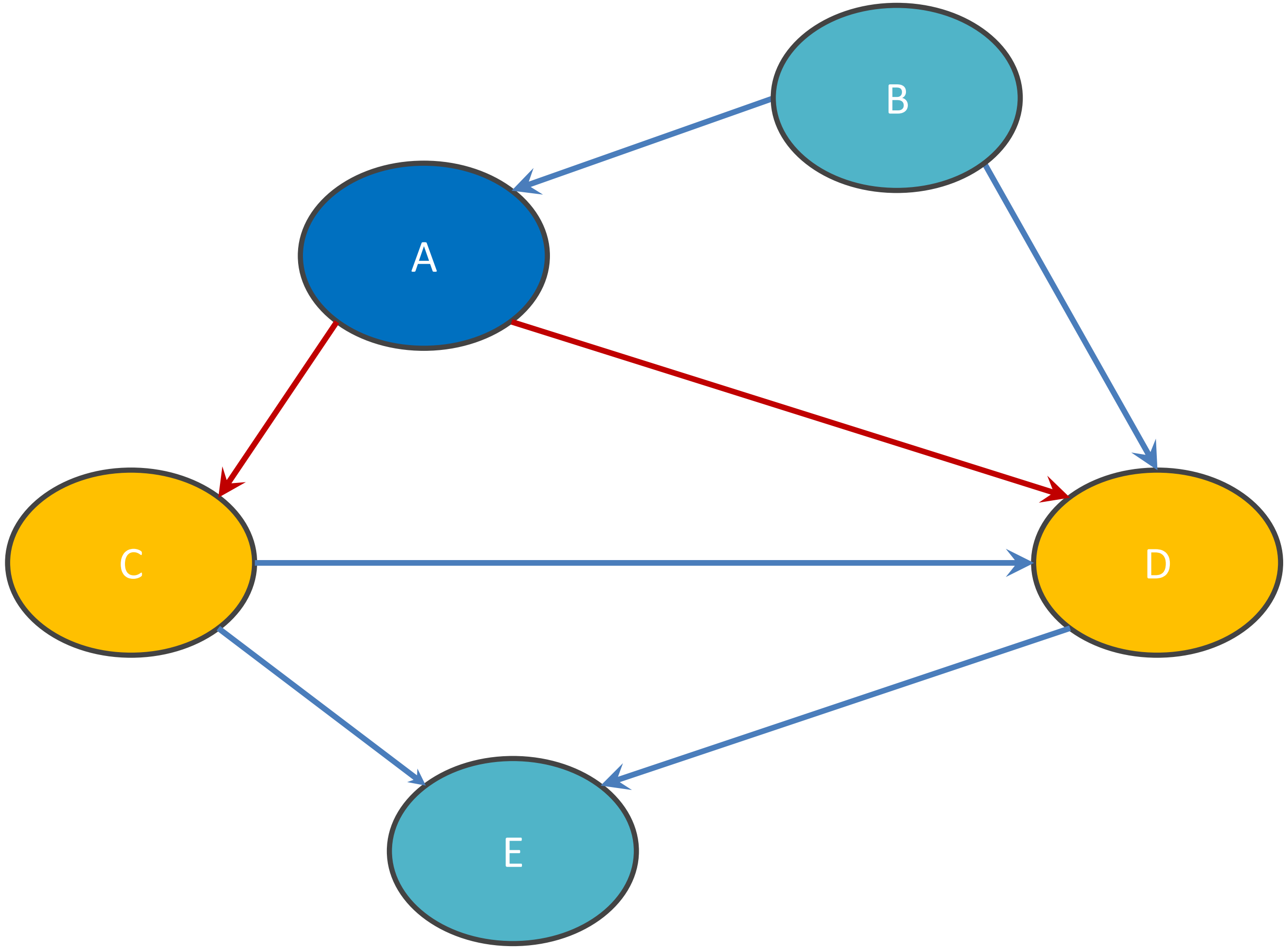
Q: <>

Q: <A>

Q: <>

Q: <C>

Q: <C ,D>



Breadth-First Search

Q: <>

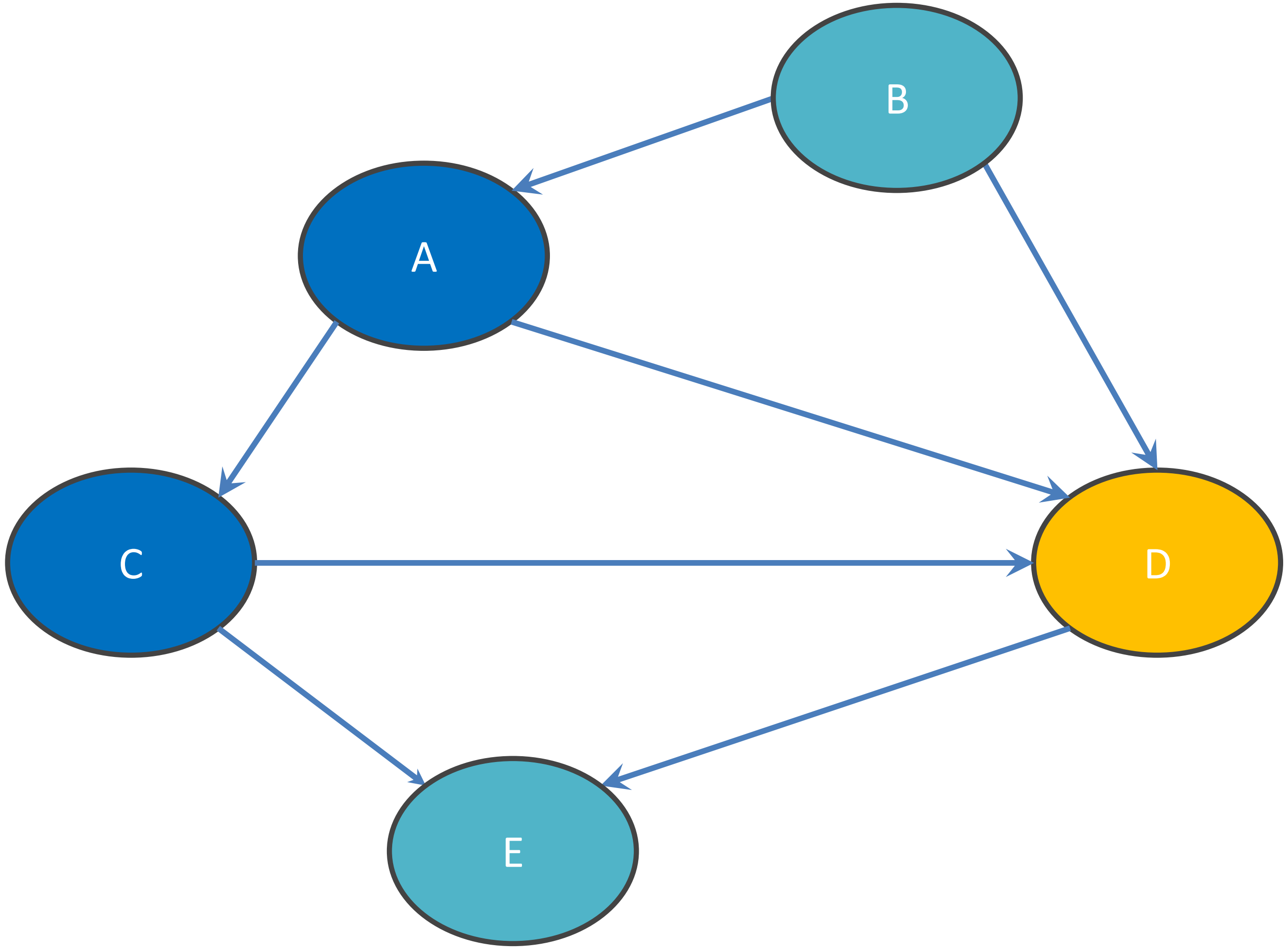
Q: <A>

Q: <>

Q: <C>

Q: <C ,D>

Q: <D>



Breadth-First Search

Q: <>

Q: <A>

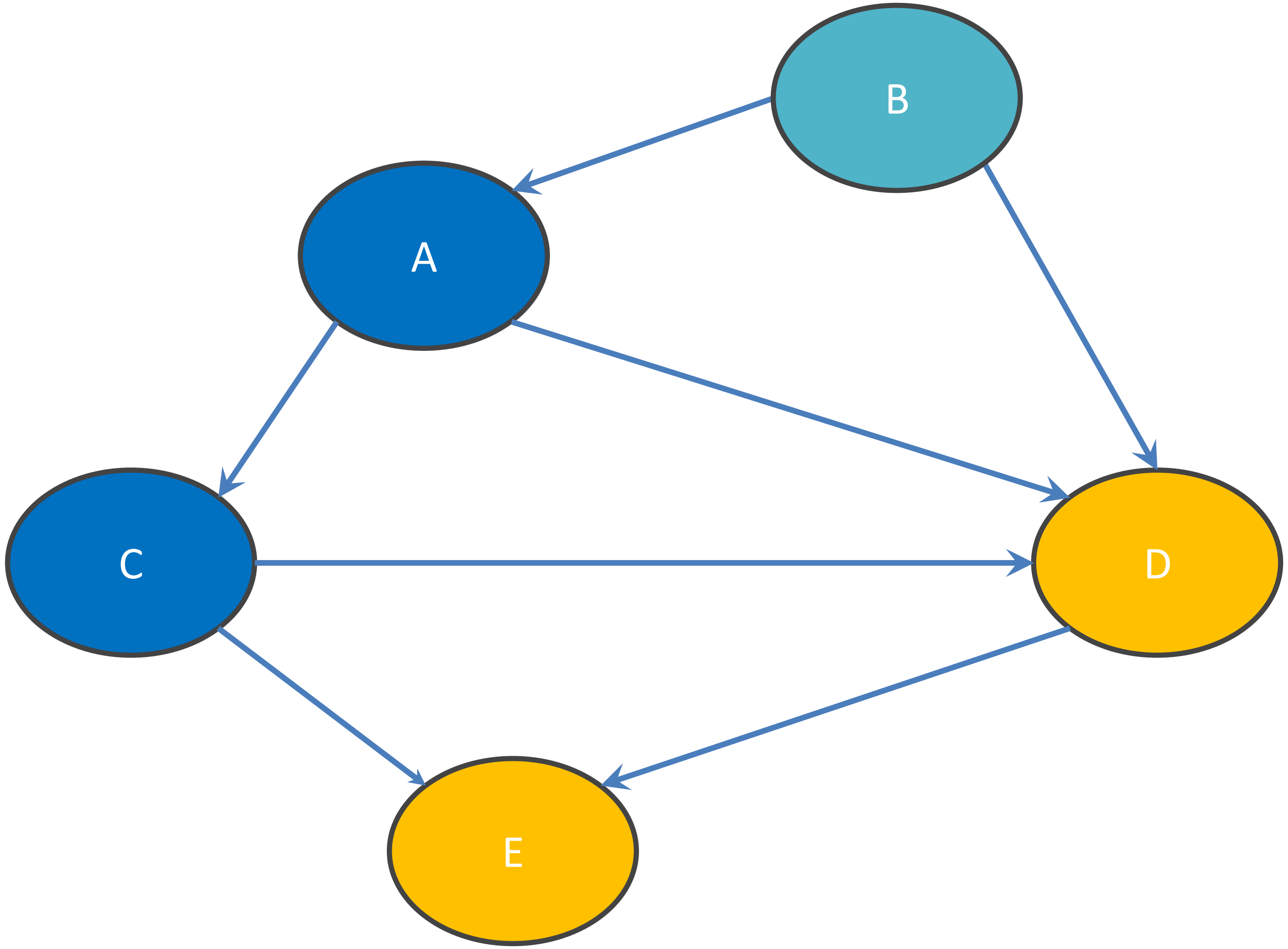
Q: <>

Q: <C>

Q: <C ,D>

Q: <D>

Q: <D, E>



Breadth-First Search

Q: <>

Q: <A>

Q: <>

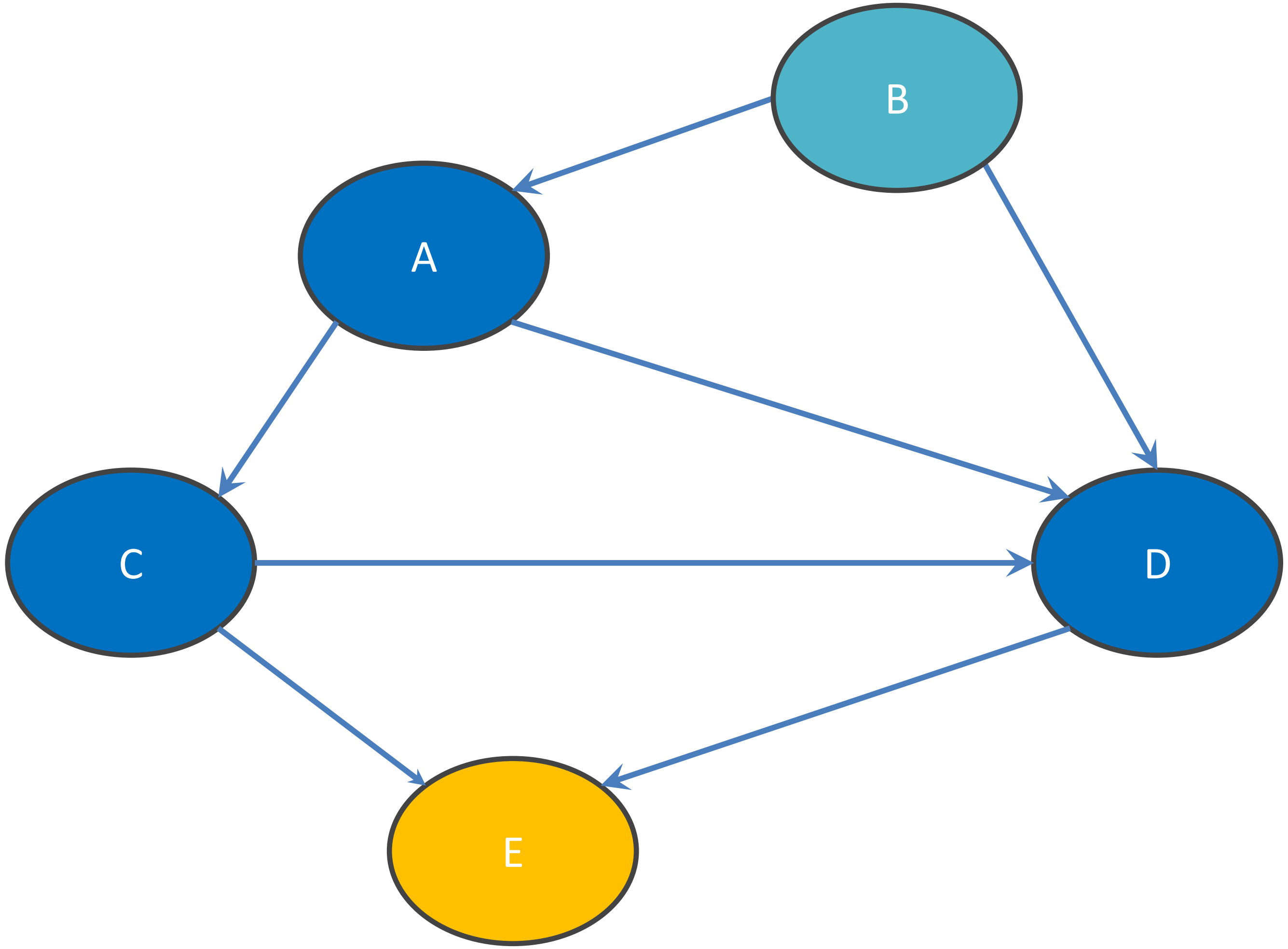
Q: <C>

Q: <C ,D>

Q: <D>

Q: <D, E>

Q: <E>



Breadth-First Search

Q: <>

Q: <A>

Q: <>

Q: <C>

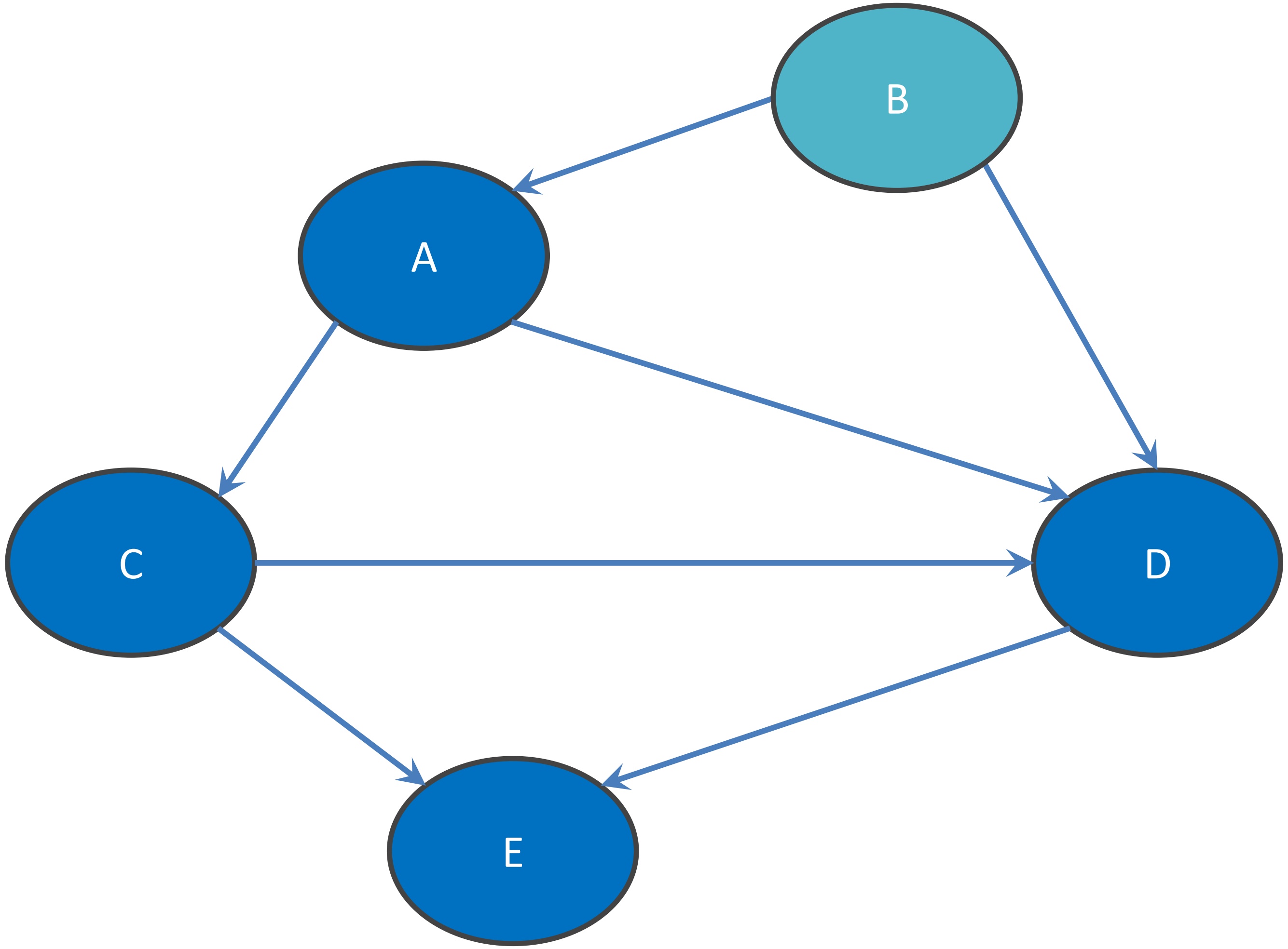
Q: <C ,D>

Q: <D>

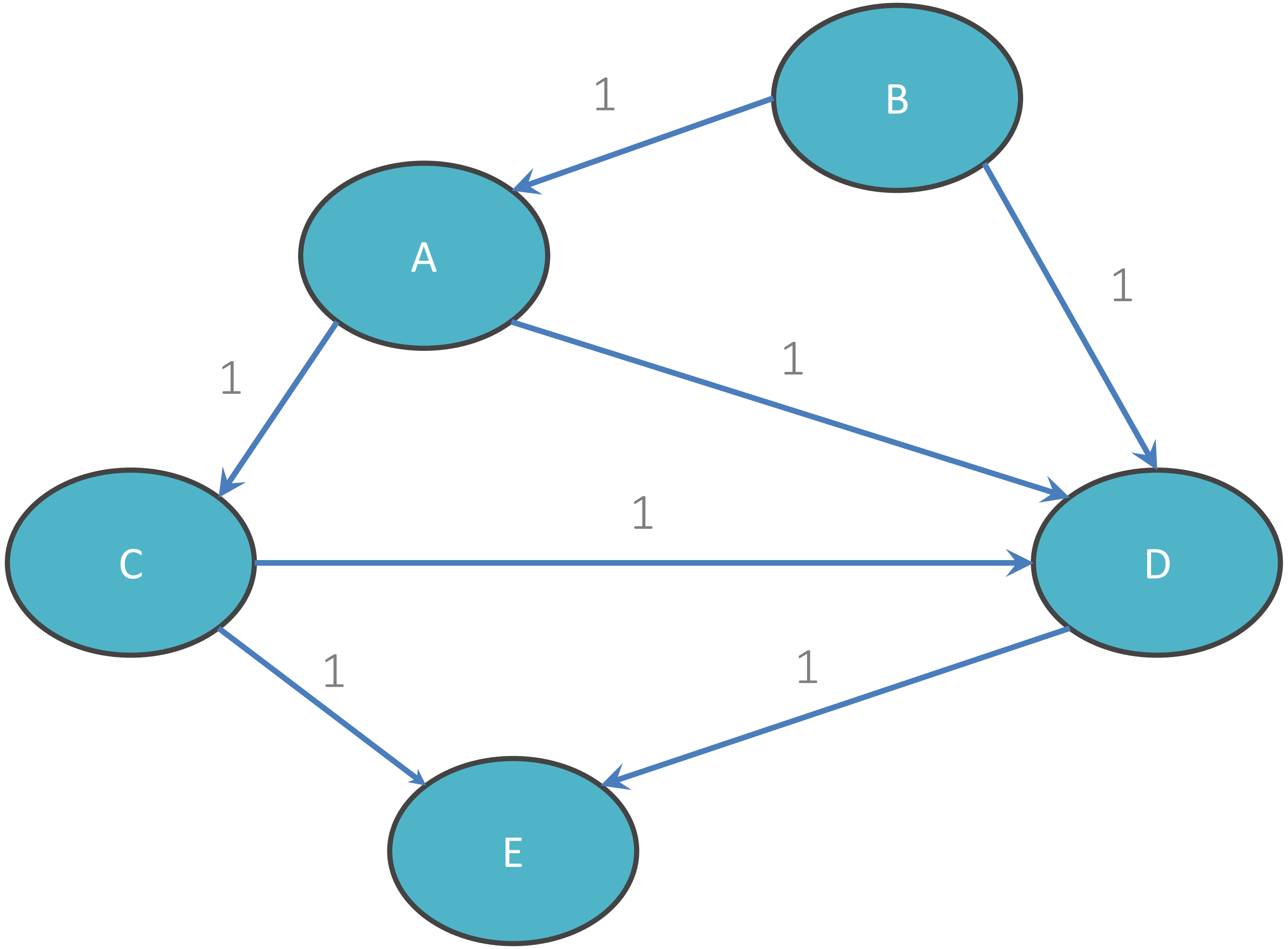
Q: <D, E>

Q: <E>

DONE



Shortest Paths with BFS

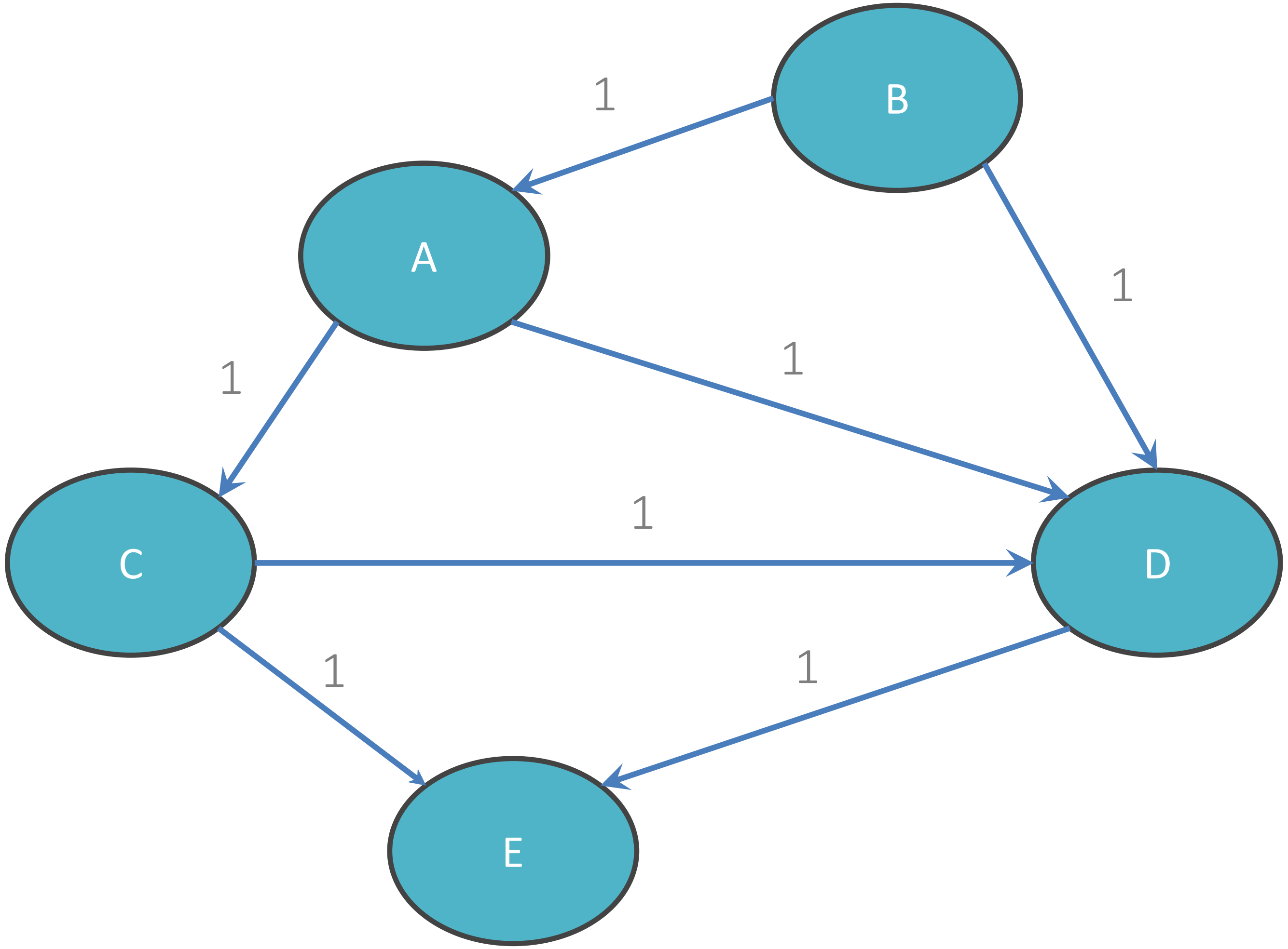


From Node B

Destination	Path	Cost
A	<B,A>	1
B	<B>	0
C	<B,A,C>	2
D		
E		

Shortest path to D? to E?
What are the costs?

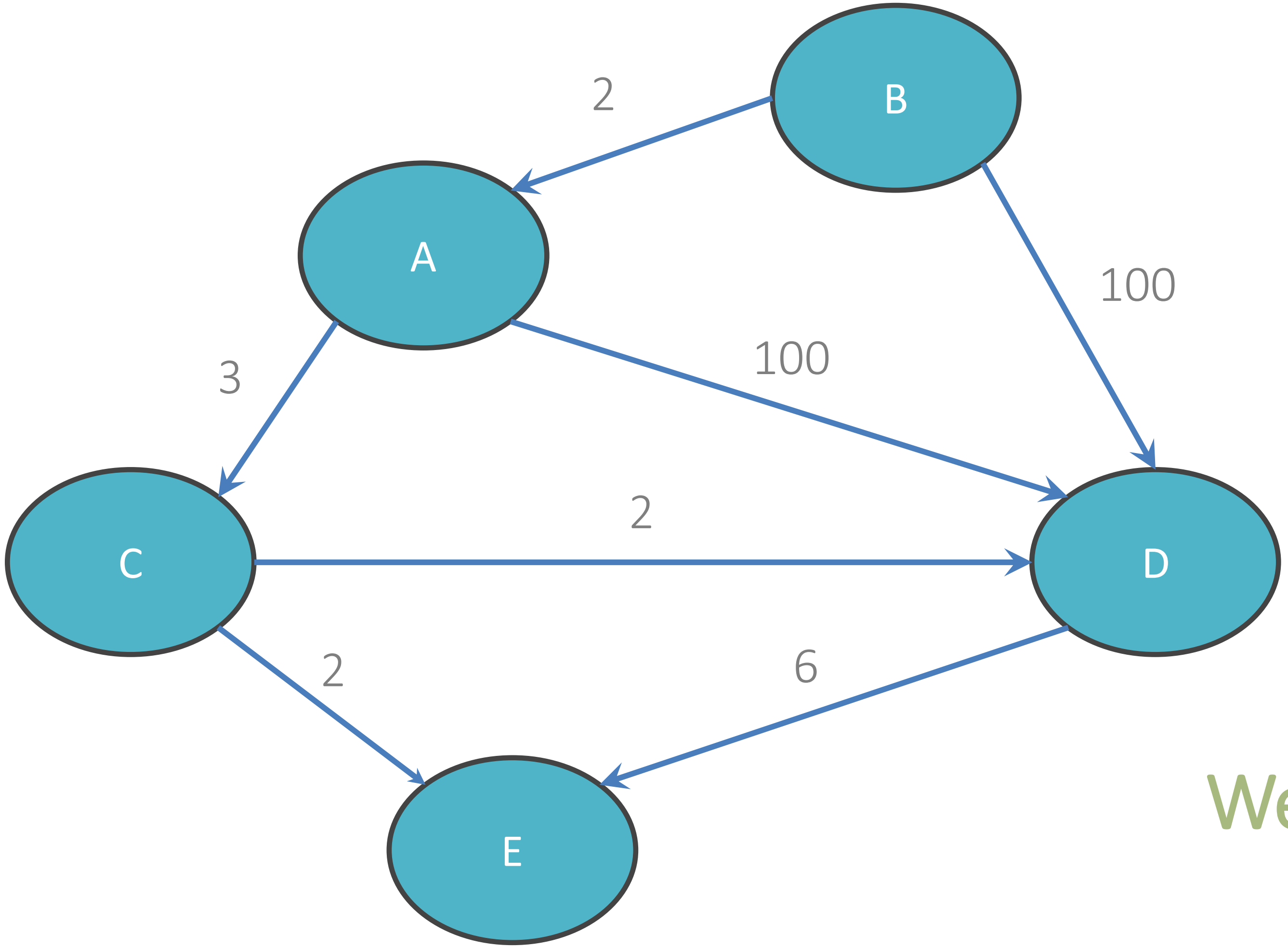
Shortest Paths with BFS



From Node B

Destination	Path	Cost
A	<B,A>	1
B	<B>	0
C	<B,A,C>	2
D	<B,D>	1
E	<B,D,E>	2

Shortest Paths with Weights



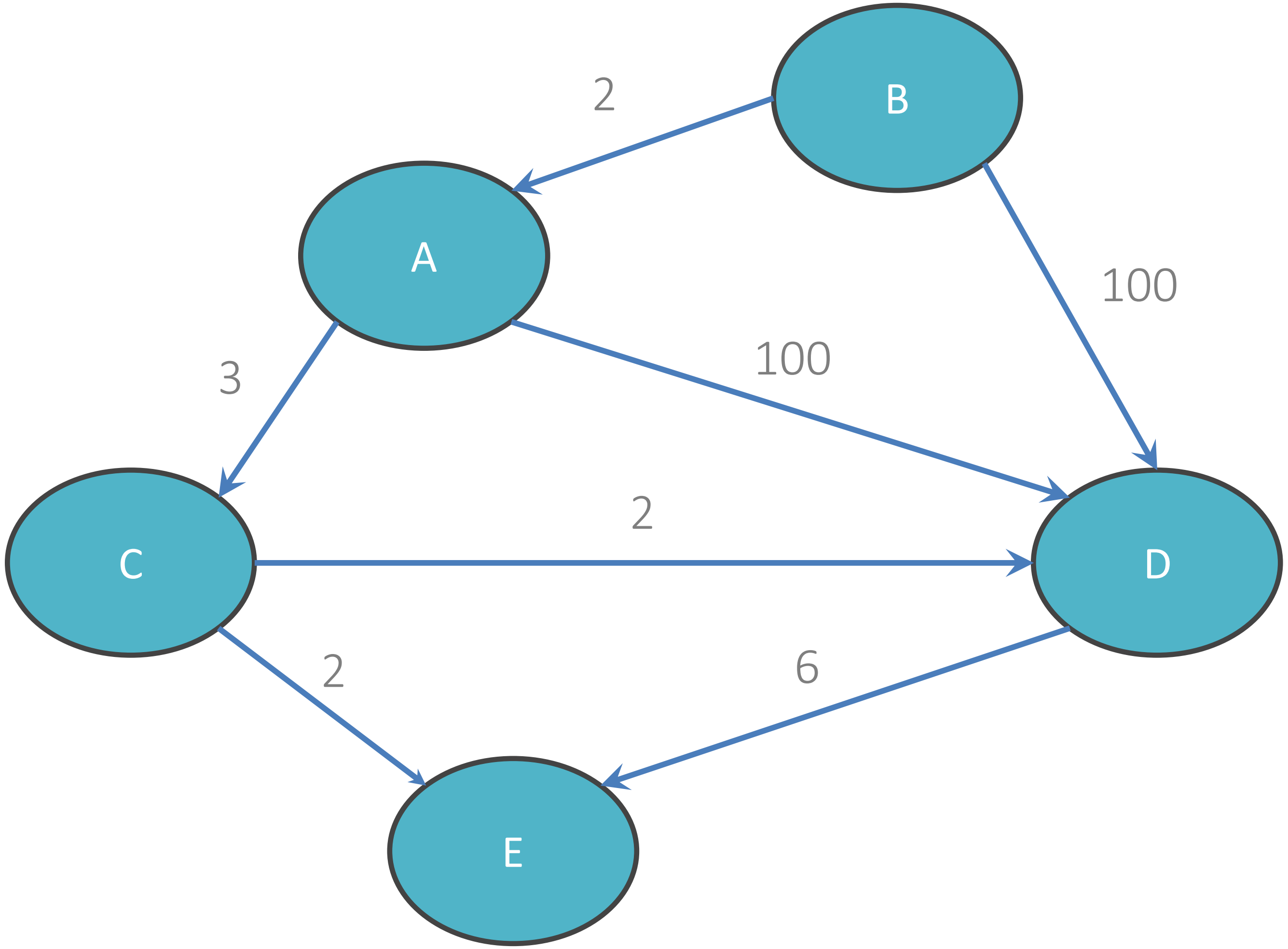
From Node B

Destination	Path	Cost
A	<B,A>	2
B	<B>	0
C	<B,A,C>	5
D		
E		

Weights are not the same!

Are the paths?

Shortest Paths with Weights



From Node B

Destination	Path	Cost
A	<B,A>	2
B	<B>	0
C	<B,A,C>	5
D	<B,A,C,D>	7
E	<B,A,C,E>	7

Midterm review

Midterm topics

Reasoning about code

Specification vs. Implementation

Abstract Data Types (ADTs)

Identity & equality

Testing

Reasoning about code 1

Using backwards reasoning, find the weakest precondition for each sequence of statements and postcondition below. Insert appropriate assertions in each blank line. You should simplify your answers if possible.

```
{_______________}
z = x + y;
{_______________}
y = z – 3;
{x > y}
```

Reasoning about code 1

Using backwards reasoning, find the weakest precondition for each sequence of statements and postcondition below. Insert appropriate assertions in each blank line. You should simplify your answers if possible.

```
{_______________}
z = x + y;
{x > z - 3}
y = z - 3;
{x > y}
```

Reasoning about code 1

Using backwards reasoning, find the weakest precondition for each sequence of statements and postcondition below. Insert appropriate assertions in each blank line. You should simplify your answers if possible.

```
{x > x + y - 3 => y < 3}
z = x + y;
{x > z - 3}
y = z - 3;
{x > y}
```

Reasoning about code 1

Using backwards reasoning, find the weakest precondition for each sequence of statements and postcondition below. Insert appropriate assertions in each blank line. You should simplify your answers if possible.

```
{_______________}
p = a + b;
{_______________}
q = a - b;
{p + q = 42}
```

Reasoning about code 1

Using backwards reasoning, find the weakest precondition for each sequence of statements and postcondition below. Insert appropriate assertions in each blank line. You should simplify your answers if possible.

```
{________________}
p = a + b;
{p + a - b = 42}
q = a - b;
{p + q = 42}
```

Reasoning about code 1

Using backwards reasoning, find the weakest precondition for each sequence of statements and postcondition below. Insert appropriate assertions in each blank line. You should simplify your answers if possible.

```
{a + b + a – b = 42 ⇒ a = 21}
p = a + b;
{p + a - b = 42}
q = a - b;
{p + q = 42}
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

```
A.  @effects decreases balance by amount

B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount

C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

Which specifications does this implementation meet?

```
I.   void withdraw(int amount) {
        balance -= amount;
     }
```

Another way to ask the question:

If the client does not know the implementation, will the method do what the client expects it to do based on the specification?

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

```
A.  @effects decreases balance by amount
```

✓ does exactly what the spec says

```
B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount

C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

Which specifications does this implementation meet?

```
I.   void withdraw(int amount) {
        balance -= amount;
     }
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

```
A.  @effects decreases balance by amount
```

✓ does exactly what the spec says

```
B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount
```

✓ If the client follows the `@requires` precondition, the code will execute as expected

```
C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

Which specifications does this implementation meet?

```
I.   void withdraw(int amount) {
         balance -= amount;
     }
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

A. `@effects decreases balance by amount` ✓ does exactly what the spec says

B. `@requires amount >= 0 and amount <= balance`
   `@effects decreases balance by amount` ✓ If the client follows the `@requires` precondition, the code will execute as expected

C. `@throws InsufficientFundsException`
   `    if balance < amount`
   `@effects decreases balance by amount` ✗ Method never throws an exception

Which specifications does this implementation meet?

```
I.  void withdraw(int amount) {
        balance -= amount;
    }
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

```
A.  @effects decreases balance by amount

B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount

C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

Which specifications does this implementation meet?

```
II. void withdraw(int amount) {
        if (balance >= amount) balance -= amount;
    }
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

```
A.  @effects decreases balance by amount
```

✗ balance does not always decrease

```
B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount

C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

Which specifications does this implementation meet?

```
II. void withdraw(int amount) {
        if (balance >= amount) balance -= amount;
    }
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

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B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount
```

✓ If the client follows the `@requires` precondition, the code will execute as expected

```
C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

Which specifications does this implementation meet?

```
II. void withdraw(int amount) {
        if (balance >= amount) balance -= amount;
    }
```

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Suppose we have a `BankAccount` *class with instance variable* `balance`. *Consider the following specifications:*

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✗ balance does not always decrease

```
B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount
```

✓ If the client follows the `@requires` precondition, the code will execute as expected

```
C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

✗ Method never throws an exception

Which specifications does this implementation meet?

```
II. void withdraw(int amount) {
        if (balance >= amount) balance -= amount;
    }
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

```
A.  @effects decreases balance by amount

B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount

C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

Which specifications does this implementation meet?

```
III.void withdraw(int amount) {
        if (amount < 0) throw new IllegalArgumentException();
        balance -= amount;
    }
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

```
A.  @effects decreases balance by amount
```

✗ balance does not always decrease

```
B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount

C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

Which specifications does this implementation meet?

```
III.void withdraw(int amount) {
        if (amount < 0) throw new IllegalArgumentException();
        balance -= amount;
    }
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

```
A.  @effects decreases balance by amount
```

✗ balance does not always decrease

```
B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount
```

✓ If the client follows the `@requires` precondition, the code will execute as expected

```
C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

Which specifications does this implementation meet?

```
III.void withdraw(int amount) {
        if (amount < 0) throw new IllegalArgumentException();
        balance -= amount;
    }
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`. *Consider the following specifications:*

```
A.  @effects decreases balance by amount
```

✗ balance does not always decrease

```
B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount
```

✓ If the client follows the `@requires` precondition, the code will execute as expected

```
C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

✗ Method throws wrong exception for wrong reason

Which specifications does this implementation meet?

```
III.void withdraw(int amount) {
        if (amount < 0) throw new IllegalArgumentException();
        balance -= amount;
    }
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

```
A.  @effects decreases balance by amount

B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount

C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

Which specifications does this implementation meet?

```
IV. void withdraw(int amount) throws InsufficientFundsException {
        if (balance < amount) throw new InsufficientFundsException();
        balance -= amount;
    }
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

```
A.  @effects decreases balance by amount
```

✗ balance does not always decrease

```
B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount

C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

Which specifications does this implementation meet?

```
IV. void withdraw(int amount) throws InsufficientFundsException {
        if (balance < amount) throw new InsufficientFundsException();
        balance -= amount;
    }
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

```
A.  @effects decreases balance by amount
```

✗ balance does not always decrease

```
B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount
```

✓ If the client follows the `@requires` precondition, the code will execute as expected

```
C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

Which specifications does this implementation meet?

```
IV. void withdraw(int amount) throws InsufficientFundsException {
        if (balance < amount) throw new InsufficientFundsException();
        balance -= amount;
    }
```

Specification vs. Implementation

Suppose we have a `BankAccount` *class with instance variable* `balance`*. Consider the following specifications:*

```
A.  @effects decreases balance by amount
```

✗ balance does not always decrease

```
B.  @requires amount >= 0 and amount <= balance
    @effects decreases balance by amount
```

✓ If the client follows the `@requires` precondition, the code will execute as expected

```
C.  @throws InsufficientFundsException
        if balance < amount
    @effects decreases balance by amount
```

✓ Method does what the spec says

Which specifications does this implementation meet?

```
IV. void withdraw(int amount) throws InsufficientFundsException {
        if (balance < amount) throw new InsufficientFundsException();
        balance -= amount;
    }
```

Writing Code Given Invariant

Given two strings *a* and *b* where a.length > 0 and b.length > 0 that are only comprised of alphabetic characters a-z, fill in the implementation on the following slides for the method **`arePermutations`** which returns true if *a* and *b* are permutations of each other and false otherwise.

In general we ask that you ***do not use additional loops in your answer****, but specifically for the following two implementations, you* ***may*** *use additional loops.*

Examples:

```
arePermutations("abcd", "dbca") -> true
arePermutations("efgh", "efgi") -> false
arePermutations("", "abcd") -> false
```

Writing Code Given Invariant 1

```
public boolean arePermutations(String a, String b) {




{inv: sortedA = sorted(a[0] ... a[k-1]) && sortedB = sorted(b[0] ... b[k-1]) && a.length == b.length}
    while (                ) {














    }

}
```

Writing Code Given Invariant 1

```
public boolean arePermutations(String a, String b) {
    if (a.length() != b.length()) return false;
    String sortedA = “”;
    String sortedB = “”;
    int k = 0;
{inv: sortedA = sorted(a[0] ... a[k-1]) && sortedB = sorted(b[0] ... b[k-1]) && a.length == b.length}
    while (                ) {



    }
    return sortedA.equals(sortedB);
}
```

Writing Code Given Invariant 1

```
public boolean arePermutations(String a, String b) {
    if (a.length() != b.length()) return false;
    String sortedA = “”;
    String sortedB = “”;
    int k = 0;
{inv: sortedA = sorted(a[0] ... a[k-1]) && sortedB = sorted(b[0] ... b[k-1]) && a.length == b.length}
    while (               ) {
        char letterA = a[k];
        char letterB = b[k];
        int i = 0;
        while (i != sortedA.length() && sortedA[i] < letterA) {
            i++;
        }
        sortedA = sortedA.substring(0, i) + letterA + sortedA.substring(i, sortedA.length());







    }
    return sortedA.equals(sortedB);
}
```

Writing Code Given Invariant 1

```
public boolean arePermutations(String a, String b) {
    if (a.length() != b.length()) return false;
    String sortedA = “”;
    String sortedB = “”;
    int k = 0;
{inv: sortedA = sorted(a[0] ... a[k-1]) && sortedB = sorted(b[0] ... b[k-1]) && a.length == b.length}
    while (               ) {
        char letterA = a[k];
        char letterB = b[k];
        int i = 0;
        while (i != sortedA.length() && sortedA[i] < letterA) {
            i++;
        }
        sortedA = sortedA.substring(0, i) + letterA + sortedA.substring(i, sortedA.length());
        i = 0;
        while (i != sortedB.length() && sortedB[i] < letterB) {
            i++;
        }
        sortedB = sortedB.substring(0, i) + letterB + sortedB.substring(i, sortedB.length());

    }
    return sortedA.equals(sortedB);
}
```

Writing Code Given Invariant 1

```
public boolean arePermutations(String a, String b) {
    if (a.length() != b.length()) return false;
    String sortedA = “”;
    String sortedB = “”;
    int k = 0;
{inv: sortedA = sorted(a[0] ... a[k-1]) && sortedB = sorted(b[0] ... b[k-1]) && a.length == b.length}
    while (k != a.length()) {
        char letterA = a[k];
        char letterB = b[k];
        int i = 0;
        while (i != sortedA.length() && sortedA[i] < letterA) {
            i++;
        }
        sortedA = sortedA.substring(0, i) + letterA + sortedA.substring(i, sortedA.length());
        i = 0;
        while (i != sortedB.length() && sortedB[i] < letterB) {
            i++;
        }
        sortedB = sortedB.substring(0, i) + letterB + sortedB.substring(i, sortedB.length());
        k++;
    }
    return sortedA.equals(sortedB);
}
```

Writing Code Given Invariant 2

```
public boolean arePermutations(String a, String b) {



    {inv: counts[0] = # of a’s in a[0], ..., a[i-1], ..., counts[25] = # of z’s in a[0], ..., a[i-1]
     && a.length == b.length}
    while (                ) {



    }

    {inv: counts[0] >= 0, ... , counts[25] >= 0 && a.length == b.length}
    while (                ) {




    }

}
```

Writing Code Given Invariant 2

```
public boolean arePermutations(String a, String b) {
    if (a.length() != b.length()) return false;
    int[] counts = new int[26];
    int i = 0;
    {inv: counts[0] = # of a’s in a[0], ..., a[i-1], ..., counts[25] = # of z’s in a[0], ..., a[i-1]
     && a.length == b.length}
    while (               ) {



    }

    {inv: counts[0] >= 0, ... , counts[25] >= 0 && a.length == b.length}
    while (               ) {




    }
}
```

Writing Code Given Invariant 2

```
public boolean arePermutations(String a, String b) {
    if (a.length() != b.length()) return false;
    int[] counts = new int[26];
    int i = 0;
    {inv: counts[0] = # of a’s in a[0], ..., a[i-1], ..., counts[25] = # of z’s in a[0], ..., a[i-1]
     && a.length == b.length}
    while (i != a.length()) {
        char letter = a.charAt(i);
        counts[letter – ‘a’]++;
        i++;
    }

    {inv: counts[0] >= 0, ... , counts[25] >= 0 && a.length == b.length}
    while (               ) {



    }

}
```

Writing Code Given Invariant 2

```
public boolean arePermutations(String a, String b) {
    if (a.length() != b.length()) return false;
    int[] counts = new int[26];
    int i = 0;
    {inv: counts[0] = # of a’s in a[0], ..., a[i-1], ..., counts[25] = # of z’s in a[0], ..., a[i-1]
     && a.length == b.length}
    while (i != a.length()) {
        char letter = a.charAt(i);
        counts[letter – ‘a’]++;
        i++;
    }
    i = 0;
    {inv: counts[0] >= 0, ... , counts[25] >= 0 && a.length == b.length}
    while (i != a.length()) {
        char letter = b.charAt(i);
        counts[letter – ‘a’]--;
        if (counts[letter – ‘a’] < 0) return false;
        i++;
    }
    return true;
}
```

Testing `arePermutations` Implementation

For the previous implementations of `arePermutations`, write two test cases where the inputs result in expected/actual behavior that is fundamentally different from each other. Write a brief explanation convincing someone else why your test cases test different behavior. You can define behavior in terms of expected (black box) or actual (clear box) execution equivalence.

Test Cases 1

`Input: a = “abcd” and b = “abc”`

`Returns: false`

This test case tests the behavior where two Strings that are not of equal length cannot be permutations of each other by definition. In terms of the specific implementation of `arePermutations`, this tests the case where the loop is never entered because the Strings do not have the same length to begin with.

Test Cases 2

`Input: a = “abcabc” and b = “baccba”`

`Returns: true`

This test case tests the behavior where two Strings are permutations of each other but contain repeated characters that appear in different orders in each String. In terms of the specific implementation of `arePermutations`, this tests the case where the frequency of letters from String *a,* when compared against String *b,* never becomes negative despite the letters not appearing in the same order relative to one another (assuming implementation 2).

Specifications 2

```
/**
 * An IntPoly is an immutable, integer-valued polynomial
 * with integer coefficients. A typical IntPoly value
 * is a_0 + a_1*x + a_2*x^2 + ... + a_n*x_n. An IntPoly
 * with degree n has coefficent a_n != 0, except that the
 * zero polynomial is represented as a polynomial of
 * degree 0 and a_0 = 0 in that case.
 */

public class IntPoly {
    int a[];
    // AF(this) = a has n+1 entries, and for each entry,
    // a[i] = coefficient a_i of the polynomial.
}
```

Specifications 2

```
/**
 * Return a new IntPoly that is the sum of this and other
 * @requires
 * @modifies
 * @effects
 * @return
 * @throws
 */
public IntPoly add(IntPoly other)
```

Specifications 2

```
/**
 * Return a new IntPoly that is the sum of this and other
 * @requires other != null
 * @modifies none
 * @effects none
 * @return a new IntPoly representing the sum of this and other
 * @throws none
 */
public IntPoly add(IntPoly other)
```

Representation invariants

One of your colleagues is worried that this creates a potential representation exposure problem. Another colleague says there's no problem since an `IntPoly` *is immutable. Is there a problem? Give a brief justification for your answer.*

```
public class IntPoly {
    int a[];
    // AF(this) = a has n+1 entries, and for each entry,
    // a[i] = coefficient a_i of the polynomial.

    // Return the coefficients of this IntPoly
    public int[] getCoeffs() {
        return a;
    }
}
```

Representation invariants

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```
public class IntPoly {
    int a[];
    // AF(this) = a has n+1 entries, and for each entry,
    // a[i] = coefficient a_i of the polynomial.

    // Return the coefficients of this IntPoly
    public int[] getCoeffs() {
        return a;
    }
}
```

The return value is a reference to the same coefficient array stored in the **`IntPoly`** and the client code could alter those coefficients.

Representation invariants

If there is a representation exposure problem, give a new or repaired implementation of `getCoeffs` *that fixes the problem but still returns the coefficients of the* `IntPoly` *to the client. If it saves time you can give a precise description of the changes needed instead of writing the detailed Java code.*

```
public class IntPoly {
    int a[];
    // AF(this) = a has n+1 entries, and for each entry,
    // a[i] = coefficient a_i of the polynomial.

    // Return the coefficients of this IntPoly
    public int[] getCoeffs() {
        return a;
    }
}
```

Representation invariants

If there is a representation exposure problem, give a new or repaired implementation of `getCoeffs` *that fixes the problem but still returns the coefficients of the* `IntPoly` *to the client. If it saves time you can give a precise description of the changes needed instead of writing the detailed Java code.*

```
public int[] getCoeffs() {
    int[] copyA = new int[a.length];
    for (int i = 0; i < copyA.length; i++) {
        copyA[i] = a[i]
    }
    return copyA
}
```

Representation invariants

If there is a representation exposure problem, give a new or repaired implementation of `getCoeffs` *that fixes the problem but still returns the coefficients of the* `IntPoly` *to the client. If it saves time you can give a precise description of the changes needed instead of writing the detailed Java code.*

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    }
    return copyA
}
```

1. Make a copy
2. Return the copy

Representation invariants

If there is a representation exposure problem, give a new or repaired implementation of `getCoeffs` *that fixes the problem but still returns the coefficients of the* `IntPoly` *to the client. If it saves time you can give a precise description of the changes needed instead of writing the detailed Java code.*

```
public int[] getCoeffs() {
    int[] copyA = new int[a.length];
    for (int i = 0; i < copyA.length; i++) {
        copyA[i] = a[i]
    }
    return copyA
}
```

1. Make a copy
2. Return the copy

Alternatively, we can just use…

```
Arrays.copyOf(a, a.length)
```

Reasoning about code 2

We would like to add a method to this class that evaluates the `IntPoly` *at a particular value x. In other words, given a value x, the method* `valueAt(x)` *should return* $a_0 + a_1x + a_2x^2 + ... + a_nx^n$*, where* a_0 *through an are the coefficients of this* `IntPoly`*.*

For this problem, develop an implementation of this method and prove that your implementation is correct.

(see starter code on next slide)

Reasoning about code 2

```
/** Return the value of this IntPoly at point x */
public int valueAt(int x) {
    int val = a[0];
    int xk = 1;
    int k = 0;
    int n = a.length-1; // degree of this, n >=0
    {_____}
    while (k != n) {
        {_____}
        xk = xk * x;
        {_____}
        val = val + a[k+1]*xk;
        {_____}
        k = k + 1;
        {_____}
    }
    {_____}
    return val;
}
```

Reasoning about code 2

```
/** Return the value of this IntPoly at point x */
public int valueAt(int x) {
    int val = a[0];
    int xk = 1;
    int k = 0;
    int n = a.length-1; // degree of this, n >=0
    {inv: xk = x^k && val = a[0] + a[1]*x + ... + a[k]*x^k}
    while (k != n) {
        {_____}
        xk = xk * x;
        {_____}
        val = val + a[k+1]*xk;
        {_____}
        k = k + 1;
        {_____}
    }
    {_____}
    return val;
}
```

This should come with the code...

Reasoning about code 2

```
/** Return the value of this IntPoly at point x */
public int valueAt(int x) {
    int val = a[0];
    int xk = 1;
    int k = 0;
    int n = a.length-1; // degree of this, n >=0
    {inv: xk = x^k && val = a[0] + a[1]*x + ... + a[k]*x^k}
    while (k != n) {
        {inv && k != n}
        xk = xk * x;
        {_____}
        val = val + a[k+1]*xk;
        {_____}
        k = k + 1;
        {_____}
    }
    {_____}
    return val;
}
```

Reasoning about code 2

```
/** Return the value of this IntPoly at point x */
public int valueAt(int x) {
    int val = a[0];
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    int k = 0;
    int n = a.length-1; // degree of this, n >=0
    {inv: xk = x^k && val = a[0] + a[1]*x + ... + a[k]*x^k}
    while (k != n) {
        {inv && k != n}
        xk = xk * x;
        {xk = x^(k+1) && val = a[0] + a[1]*x + ... + a[k]*x^k}
        val = val + a[k+1]*xk;
        {_____}
        k = k + 1;
        {_____}
    }
    {_____}
    return val;
}
```

Reasoning about code 2

```
/** Return the value of this IntPoly at point x */
public int valueAt(int x) {
    int val = a[0];
    int xk = 1;
    int k = 0;
    int n = a.length-1; // degree of this, n >=0
    {inv: xk = x^k && val = a[0] + a[1]*x + ... + a[k]*x^k}
    while (k != n) {
        {inv && k != n}
        xk = xk * x;
        {xk = x^(k+1) && val = a[0] + a[1]*x + ... + a[k]*x^k}
        val = val + a[k+1]*xk;
        {xk = x^(k+1) && val = a[0] + a[1]*x + ... + a[k+1]*x^(k+1)}
        k = k + 1;
        {_____}
    }
    {_____}
    return val;
}
```

Reasoning about code 2

```
/** Return the value of this IntPoly at point x */
public int valueAt(int x) {
    int val = a[0];
    int xk = 1;
    int k = 0;
    int n = a.length-1; // degree of this, n >=0
    {inv: xk = x^k && val = a[0] + a[1]*x + ... + a[k]*x^k}
    while (k != n) {
        {inv && k != n}
        xk = xk * x;
        {xk = x^(k+1) && val = a[0] + a[1]*x + ... + a[k]*x^k}
        val = val + a[k+1]*xk;
        {xk = x^(k+1) && val = a[0] + a[1]*x + ... + a[k+1]*x^(k+1)}
        k = k + 1;
        {inv}
    }
    {_____}
    return val;
}
```

Reasoning about code 2

```
/** Return the value of this IntPoly at point x */
public int valueAt(int x) {
    int val = a[0];
    int xk = 1;
    int k = 0;
    int n = a.length-1; // degree of this, n >=0
    {inv: xk = x^k && val = a[0] + a[1]*x + ... + a[k]*x^k}
    while (k != n) {
        {inv && k != n}
        xk = xk * x;
        {xk = x^(k+1) && val = a[0] + a[1]*x + ... + a[k]*x^k}
        val = val + a[k+1]*xk;
        {xk = x^(k+1) && val = a[0] + a[1]*x + ... + a[k+1]*x^(k+1)}
        k = k + 1;
        {inv}
    }
    {inv && k = n ⇒ val = a[0] + a[1]*x + ... + a[n]*x^n}
    return val;
}
```

Equality

Suppose we are defining a class **`StockItem`** *to represent items stocked by an online grocery store. Here is the start of the class definition, including the class name and instance variables:*

```
public class StockItem {
    String name;
    String size;
    String description;
    int quantity;

    /* Construct a new StockItem */
    public StockItem(...);
}
```

Equality

A summer intern was asked to implement an `equals` *function for this class that treats two* `StockItem` *objects as equal if their* `name` *and* `size` *fields match. Here's the result:*

```
/** return true if the name and size fields match */
public boolean equals(StockItem other) {
    return name.equals(other.name) && size.equals(other.size);
}
```

This `equals` *method seems to work sometimes but not always. Give an example showing a situation when it fails.*

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}
```

This `equals` *method seems to work sometimes but not always. Give an example showing a situation when it fails.*

```
Object s1 = new StockItem("thing", 1, "stuff", 1);
Object s2 = new StockItem("thing", 1, "stuff", 1);
System.out.println(s1.equals(s2));
```

Equality

A summer intern was asked to implement an `equals` *function for this class that treats two* `StockItem` *objects as equal if their* `name` *and* `size` *fields match. Here's the result:*

```
/** return true if the name and size fields match */
public boolean equals(StockItem other) {  // equals is overloaded, not overridden
    return name.equals(other.name) && size.equals(other.size);
}
```

This `equals` *method seems to work sometimes but not always. Give an example showing a situation when it fails.*

```
Object s1 = new StockItem("thing", 1, "stuff", 1);
Object s2 = new StockItem("thing", 1, "stuff", 1);
System.out.println(s1.equals(s2));
```

Equality

Show how you would fix the `equals` *method so it works properly (*`StockItems` *are equal if their* `names` *and* `sizes` *are equal)*

```
/** return true if the name and size fields match */
```

Equality

Show how you would fix the `equals` *method so it works properly (*`StockItems` *are equal if their* `names` *and* `sizes` *are equal)*

```
/** return true if the name and size fields match */
@Override
public boolean equals(Object o) {
    if (!(o instanceof StockItem)) {
        return false;
    }
    StockItem other = (StockItem) o;
    return name.equals(other.name) && size.equals(other.size);
}
```

hashCode

Which of the following implementations of `hashCode()` *for the* ***`StockItem`*** *class are legal:*

```
1. return name.hashCode();
2. return name.hashCode() * 17 + size.hashCode();
3. return name.hashCode() * 17 + quantity;
4. return quantity;
```

hashCode

Which of the following implementations of `hashCode()` *for the* ***`StockItem`*** *class are legal:*

1. `return name.hashCode();` ✓ legal
2. `return name.hashCode() * 17 + size.hashCode();`
3. `return name.hashCode() * 17 + quantity;`
4. `return quantity;`

hashCode

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hashCode

Which of the following implementations of `hashCode()` *for the* ***`StockItem`*** *class are legal:*

1. `return name.hashCode();` ✓ legal
2. `return name.hashCode() * 17 + size.hashCode();` ✓ legal
3. `return name.hashCode() * 17 + quantity;` ✗ illegal!
4. `return quantity;`

hashCode

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1. `return name.hashCode();` ✓ legal
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3. `return name.hashCode() * 17 + quantity;` ✗ illegal!
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hashCode

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1. `return name.hashCode();` ✓ legal
2. `return name.hashCode() * 17 + size.hashCode();` ✓ legal
3. `return name.hashCode() * 17 + quantity;` ✗ illegal!
4. `return quantity;` ✗ illegal!

The **`equals`** method does not care about **`quantity`**

hashCode

Which implementation do you prefer?

```
public int hashCode() {
    return name.hashCode();
}
```

```
public int hashCode() {
    return name.hashCode()*17 + size.hashCode();
}
```

hashCode

Which implementation do you prefer?

```
public int hashCode() {
    return name.hashCode();
}
```

(ii) will likely do the best job since it takes into account both the size and name fields. (i) is also legal but it gives the same **`hashCode`** for **`StockItems`** that have different sizes as long as they have the same name, so it doesn't differentiate between different **`StockItems`** as well as (ii).

```
public int hashCode() {
    return name.hashCode()*17 + size.hashCode();
}
```