Section 1: Code Reasoning

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Introduction to code reasoning

• Determine what *facts* are true at each line of the program
• Accomplish two major goals:
  • Prove code is correct
  • Understand what your code is doing

• Essential for creating high quality and high complexity programs
Reasoning Techniques

• Forward Reasoning – determine what assertions follow from initial conditions

• Backward Reasoning – determine sufficient conditions which make a specific result true
Forward Reasoning Example 1

Suppose we know initially that: \( i \geq 2 \)
We want to ensure \( z > 1 \) by the end

\[
x = i \times 2;
\]
\[
// \quad x = i \times 2 \text{ and } i \geq 2
\]
\[
y = x;
\]
\[
// \quad y = x \text{ and } x = i \times 2 \text{ and } i \geq 2
\]
\[
z = (x + y) / 2;
\]
\[
// \quad z = (x + y) / 2 \text{ and } i \geq 2 \text{ and } x = i \times 2 \text{ and } y = x
\]
\[
\Rightarrow// \quad z = (i \times 2 + i \times 2) / 2 = i \times 2
\]
\[
\Rightarrow// \quad z \geq 2 \times 2 = 4
\]
Since we can imply that \( z \geq 4 \) by the end, it follows that \( z > 1 \) as well
Forward Reasoning Example 2

Suppose $x \geq 1$ and $y \geq 1$

We want to see what possible values $z$ evaluates to by the end

$x = x + y$

// $x_1 = x_0 + y$ and $x_0 \geq 1$ and $y \geq 1$

$y = 20$

// $y_1 = 20$ and $y_0 \geq 1$ and $x_1 = x_0 + y_0$ and $x_0 \geq 1$

$z = y - x$

// $z = y_1 - x_1$ and $y_1 = 20$ and $x_1 = x_0 + y_0$ and $y_0 \geq 1$ and $x_0 \geq 1$

$\Rightarrow$ // $z = 20 - x_1$ and $x_1 \geq 1 + 1 = 2$

$\Rightarrow$ // $z \leq 20 - 2 = 18$

The possible values of $z$ are all integers less than or equal to 18
Forward Reasoning Example 3

Suppose $x \neq 0$ and $w \geq 3$

We want to know what values $z$ is greater than or equal to by the end

$q = w / 2;$

// $q \geq 1$ and $x \neq 0$ and $w \geq 3$

$y = x \times x;$

// $q \geq 1$ and $y > 0$ and $x \neq 0$ and $w \geq 3$

$z = q \times y;$

// $z = q \times y$ and $q \geq 1$ and $y > 0$ and $x \neq 0$ and $w \geq 3$

$\rightarrow$ // $z \geq 1 \times 1 = 1$

$z$ is greater than or equal to 1
Backward Reasoning Example 1

Suppose we want to show that \( z \geq 6 \) by the end. What needs to be true about \( i \)?

```
// i * 4 >= 8 or equivalently i >= 2
x = i * 4;
// x + 4 >= 12 or equivalently x >= 8
i = x + 4;
// i / 2 >= 6 or equivalently i >= 12
z = i / 2;
// z >= 6
```
Suppose we want to show that $z \leq 10$ by the end.
What needs to be true about $x$ and $y$?

```plaintext
// x + y >= 10
x = x + y;
    // 20 - x <= 10 or equivalently x >= 10
y = 20;
    // y - x <= 10
z = y - x;
    // z <= 10
```
Backward Reasoning Example 3

Suppose we want to show that \( x > 0 \) by the end.
What needs to be true initially?

\[
\begin{align*}
// & \quad x + 3 \times c > 4 \\
& \quad a = x + c \\
// & \quad a + 2 \times c > 4 \\
& \quad b = 2 \times c - 4 \\
// & \quad a + b > 0 \\
& \quad x = a + b \\
// & \quad x > 0
\end{align*}
\]
Worksheet

• 20 Minutes – Get as far as you can and don’t worry if you get stuck!
• Feel free to collaborate with other students and ask me questions
• Go over solutions in slides after
Forward Reasoning Solutions
Worksheet – Problem 1

What is the value of \( z \) by the end in terms of \( x \) and \( y \)?

\[
\text{// } x > 0 \text{ and } y > 0
\]

\( w = x \times y; \)

\[
\text{\text{// } w = x \times y \text{ and } x > 0 \text{ and } y > 0}
\]

\( q = x \times x; \)

\[
\text{\text{// } q = x \times x \text{ and } w = x \times y \text{ and } x > 0 \text{ and } y > 0}
\]

\( z = w / q; \)

\[
\text{\text{// } z = y/x \text{ and } x > 0 \text{ and } y > 0}
\]
Worksheet – Problem 2

What are the possible values of $z$ by the end of the code?

```c
// x >= 0 and y >= 0
y = 25;
// y = 25 and x >= 0
x = x + y;
// x >= 25 and y = 25
x = sqrt(x);
// x >= 5 and y = 25
z = y - x;
// z <= 20
```
What are the possible values for $z$ by the end?

$// x \neq 0$ and $y < 0$

$z = x \times x$;

$// z > 0$ and $x \neq 0$ and $y < 0$

$z = z \times y$;

$// z < 0$ and $x \neq 0$ and $y < 0$

$z = z \times x$;

$// z > 0$ or $z < 0$

$\Rightarrow // z \neq 0$

$// z \neq 0$
Backward Reasoning Solutions
Worksheet – Problem 4

What are the sufficient conditions to ensure $z \neq -1$ by the end?

// $y \neq -2$ and $y \neq -3$

$x = y / 2;
// x \neq -1

z = x * 2;
// z \neq -2

z = z + 1;
// z \neq -1
Worksheet – Problem 5

What must be true initially for \( y > 20 \) by the end?

// x < 1

\[ x = 1 - x; \]

// x > 0

\[ x = x + 10; \]

// x > 10

\[ y = 2 * x; \]

// y > 20
Worksheet – Problem 6

What must be true initially for $x > y$ and $y > z$ by the end?

// $a < 4$ and $b < -a$

$b = -b;$

// $a < 4$ and $b > a$

$z = a * 2;$

// $a < 4$ and $a + b > z$

$x = b + 4$

// $x > a + b$ and $a + b > z$

$y = a + b;$

// $x > y$ and $y > z$
What are the possible values of $z$ after the if statement?

```java
// y = 5
if (x < 0) {
    // y = 5 and x < 0
    z = x - y + 1;
    // z <= -5
} else {
    // y = 5 and x >= 0
    z = x + y;
    // z >= 5
}
// z >= 5 or z <= -5
```
Conditional Reasoning (Backward)

What must be true initially to ensure \( z = 0 \) at the end?

\[
// (b\text{Condition} \text{ and } a + b = 0) \text{ or } (!b\text{Condition} \text{ and } x + y = 0)
\]

if (bCondition) {
  // a + b = 0
  z = a + b;
  // z = 0
}
else {
  // x + y = 0
  z = x + y;
  // z = 0
}

// z = 0
Conditional Reasoning Solution
Prove that the following code calculates the absolute value of $x$

```java
if (x > 0) {
    abs = x;
} else {
    abs = -x;
}
```

// $(x > 0$ and $abs = x)$ or $(x <= 0$ and $abs = -x)$

// $abs = |x|$