
CSE 331

Software Design & Implementation

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Representation Invariants & Abstraction Functions

(Based on slides by Mike Ernst, Dan Grossman, David Notkin, Hal Perkins, Zach Tatlock)

Reminders

- HW3 due tomorrow night

Implementing a Data Abstraction (ADT)

Specify an ADT by defining operations on **abstract states**

To implement an ADT:

- select the representation of instances
- implement operations in terms of that representation

Choose a representation so that:

- it is possible to implement required operations
- the most frequently used operations are efficient / simple / ...
 - abstraction allows the rep to change later
 - almost always better to start simple

Next, need to use **reasoning** to verify the operations are correct

- two intellectual tools are helpful for this...

Connecting implementations to specs

For implementers / debuggers / maintainers of the implementation:

Representation Invariant: maps Object \rightarrow boolean

- defines the set of valid concrete values
- must hold at all times (outside of mutators)
- **no object should ever violate the rep invariant**
 - such an object has no useful meaning

Abstraction Function: maps Object \rightarrow abstract state

- says what the data structure *means* in vocabulary of the ADT
- **only defined** on objects meeting the rep invariant
- connects the concrete representation back to the specification
 - can check that the abstract value after each method meets the postcondition described in the specification

Example: Circle

```
/** Represents a mutable circle in the plane. For example,  
 * it can be a circle with center (0,0) and radius 1. */  
public class Circle {  
  
    // Rep invariant: center != null and rad > 0  
    private Point center;  
    private double rad;  
  
    // Abstraction function:  
    // AF(this) = a circle with center at this.center  
    //   and radius this.rad  
  
    // ...  
}
```

Example: Circle 2

```
/** Represents a mutable circle in the plane. For example,  
 * it can be a circle with center (0,0) and radius 1. */  
public class Circle {  
  
    // Rep invariant: center != null and edge != null  
    //   and !center.equals(edge)  
    private Point center, edge;  
  
    // Abstraction function:  
    // AF(this) = a circle with center at this.center  
    //   and radius this.center.distanceTo(this.edge)  
  
    // ...  
}
```

Example: Polynomial

```
/** An immutable polynomial with integer coefficients.
 * Examples include 0, 2x, and x + 3x^2 + 5x. */
public class IntPoly {

    // Rep invariant: coeffs != null
    private final int[] coeffs;

    // Abstraction function:
    // AF(this) = sum of this.coeffs[i] * x^i
    // for i = 0 .. this.coeffs.length

    /** Returns the highest exponent with nonzero coefficient
     * or zero if none exists. */
    public int degree() { ... }

    // ...
}
```

Example: Polynomial 2

```
/** An immutable polynomial with integer coefficients.
 * Examples include 0, 2x, and x + 3x^2 + 5x. */
public class IntPoly {

    // Rep invariant: terms != null
    private final List<IntTerm> terms;

    // Abstraction function:
    // AF(this) = sum of monomials in this.terms

    /** Returns the highest exponent with nonzero coefficient
     * or zero if none exists. */
    public int degree() { ... }

    // ...
}
```


Another example

```
class Account {
    private int balance;
    // history of all transactions
    private List<Transaction> transactions;
    ...
}
```

Real-world constraints:

- Balance ≥ 0
- Balance = $\sum_i \text{transactions.get}(i).\text{amount}$

Implementation-related constraints:

- Transactions \neq null
- No nulls in transactions

Example: Text File

Abstract state is a list of lines, each a sequence of (char, color) pairs.

How would we actually represent this?

Probably okay to store lines in an array:

- most files have < 10k lines, so copying is not too expensive
- most characters are inserted into existing lines not creating new
- (always better to start simple... can change this later)

Example: Text File

Probably not okay to make each (char, color) pair an object

- object overhead is at least 10 bytes per object
- 16 bytes * 10k lines * 50 characters = 8 Mb
- 125 files = 1 Gb = unhappy users

Instead store characters and colors in arrays

- problem: inefficient to insert in the middle of an array
... **except** at the end of the array (assuming there is space)

Old trick: have two arrays

- one is the beginning of the line
- one is the end of the line in reverse order
- easy to add new characters at the split between parts

Example: TextLine

```
// Overview: Represents one line of the text file...
public class TextLine {

    // Representation invariant: all arrays are non-null and
    // and prefixCharsLen + suffixCharsLen equals
    //     prefixColorsLen + suffixColorsLen
    // and  $0 \leq \text{prefixCharsLen} \leq \text{prefixChars.length}$  and ...
    private char[] prefixChars, suffixChars;
    private int[] prefixColors, suffixColors;
    private int prefixCharsLen, suffixCharsLen;
    private int prefixColorsLen, suffixColorsLen;

    // Abstraction function: AF(this) = zip sum of
    //   prefixChars[0..prefixCharsLen-1] + reverse(suffixChars[0..suffixCharsLen-1]) and
    //   prefixColors[0..prefixColorsLen-1] + reverse(suffixColors[0..suffixColorsLen-1])
    // (I.e., the list of pairs (char, color), where the i-th
    // char is paired with the i-th color.)

    // ...
}
```

The abstraction function

- Allows us to check correctness
 - use reasoning to show that the method leaves the abstract state such that it satisfies the postcondition
- Can assume the abstract state initially satisfies precondition
 - but it can be in any concrete representation corresponding to an abstract state that satisfies the precondition
 - (many possible abstract states, potentially many concrete representations for each abstract state)

TextLine.java

(note: just chars, no colors)

IntDeque.java

Example: CharSet ADT

```
// Overview: A CharSet is a finite mutable set of Characters
// @effects: creates a fresh, empty CharSet
public CharSet() {...}

// @modifies: this
// @effects: thispost = thispre + {c}
public void insert(Character c) {...}

// @modifies: this
// @effects: thispost = thispre - {c}
public void delete(Character c) {...}

// @return: (c in this)
public boolean member(Character c) {...}

// @return: cardinality of this
public int size() {...}
```


An implementation: Is it right?

```
class CharSet {
    private List<Character> elts =
        new ArrayList<Character>();
    public void insert(Character c) {
        elts.add(c);
    }
    public void delete(Character c) {
        elts.remove(c);
    }
    public boolean member(Character c) {
        return elts.contains(c);
    }
    public int size() {
        return elts.size();
    }
}
```

An implementation

```
class CharSet {
    private List<Character>
        elts = new ArrayList<Character>();
    public void insert(Character c) {
        elts.add(c);
    }
    public void delete(Character c) {
        elts.remove(c);
    }
    public boolean member(Character c) {
        return elts.contains(c);
    }
    public int size() {
        return elts.size();
    }
}
```

```
CharSet s = new CharSet();
Character a = new Character('a');
s.insert(a);
s.insert(a);
s.delete(a);
if (s.member(a))
    System.out.print("wrong");
else
    System.out.print("right");
```

An implementation

```
class CharSet {
    private List<Character>
        elts = new ArrayList<Character>();
    public void insert(Character c) {
        elts.add(c);
    }
    public void delete(Character c) {
        elts.remove(c);
    }
    public boolean member(Character c) {
        return elts.contains(c);
    }
    public int size() {
        return elts.size();
    }
}
```

```
CharSet s = new CharSet();
Character a = new Character('a');
s.insert(a);
s.insert(a);
s.delete(a);
if (s.member(a))
    System.out.print("wrong");
else
    System.out.print("right");
```

Where is the error?

Where Is the Error?

- Answer this and you know what to fix
- *Perhaps* `delete` is wrong
 - should remove all occurrences?
- *Perhaps* `insert` is wrong
 - should not insert a character that is already there?
- The `representation invariant` tells us which is correct
 - this is how we document our choice for “the right answer”

The representation invariant

- Defines data structure well-formedness
- Must hold before and after every `CharSet` operation
- Operations (methods) may depend on it
- Write it like this:

```
class CharSet {  
    // Rep invariant:  
    //   elts has no nulls and no duplicates  
    private List<Character> elts = ...  
    ...  
}
```

Or, more formally (if you prefer):

for all indices i of `elts`, we have `elts.elementAt(i) ≠ null`

for all indices i, j of `elts` with $i \neq j$,

we have `! elts.elementAt(i).equals(elts.elementAt(j))`

Now we can locate the error

```
// Rep invariant:  
//   elts has no nulls and no duplicates  
  
public void insert(Character c) {  
    elts.add(c);  
}  
  
public void delete(Character c) {  
    elts.remove(c);  
}
```

Checking rep invariants

Should you write code to check that the rep invariant holds?

- Yes, if it's inexpensive [depends on the invariant]
- Yes, for debugging [even when it's expensive]
- Often hard to justify turning the checking off
 - better argument is removing clutter (improve understandability)
- Some private methods need not check (Why?)

A great debugging technique:

Design your code to catch bugs by implementing and using a function to check the rep-invariant

Checking the rep invariant

Rule of thumb: check on entry *and* on exit (why?)

```
public void delete(Character c) {
    checkRep();
    elts.remove(c);

    // Is this guaranteed to get called?
    // (could guarantee it with a finally block)
    checkRep();
}
...
/** Verify that elts contains no duplicates. */
private void checkRep() {
    for (int i = 0; i < elts.size(); i++) {
        assert elts.indexOf(elts.elementAt(i)) == i;
    }
}
```


Practice *defensive programming*

- You **will** make mistakes
 - if you haven't made many yet, you haven't written enough code
 - “No physician is really good before he's killed a few patients” – Hindu Proverb
- Question is not: will you make mistakes? You will.
- Question is: will you **catch** those mistakes before customers do?
- Write and incorporate code designed to catch the errors you make
 - check rep invariant on entry and exit (of mutators)
 - check preconditions (don't trust other programmers)
 - check postconditions (don't trust yourself either)
- Checking the rep invariant helps *discover* errors while testing
- Reasoning about the rep invariant helps *discover* errors while coding

Practice *defensive programming*

- Checking pre- and post-conditions and rep invariants is one tip
- More of these in Effective Java
 - Reading Quiz #2 focused on these
- In particular, focus on defensive programming against subtle bugs
 - obvious bugs (e.g. crashing every time) will be caught in testing
 - subtle bugs that only occasionally cause problems can sneak out
 - be especially defensive against these
 - tips in Reading Quiz #2 mainly combat these

Listing the elements of a CharSet

Consider adding the following method to `CharSet`

```
// returns: a List containing the members of this
public List<Character> getElts();
```

Consider this implementation:

```
// Rep invariant: elts has no nulls and no dups
private List<Character> elts;
public List<Character> getElts() { return elts; }
```

Does the implementation of `getElts` preserve the rep invariant?

Can't say!

Representation exposure

Consider this client code (outside the `CharSet` implementation):

```
CharSet s = new CharSet();
Character a = new Character('a');
s.insert(a);
s.getElts().add(a);
s.delete(a);
if (s.member(a)) ...
```

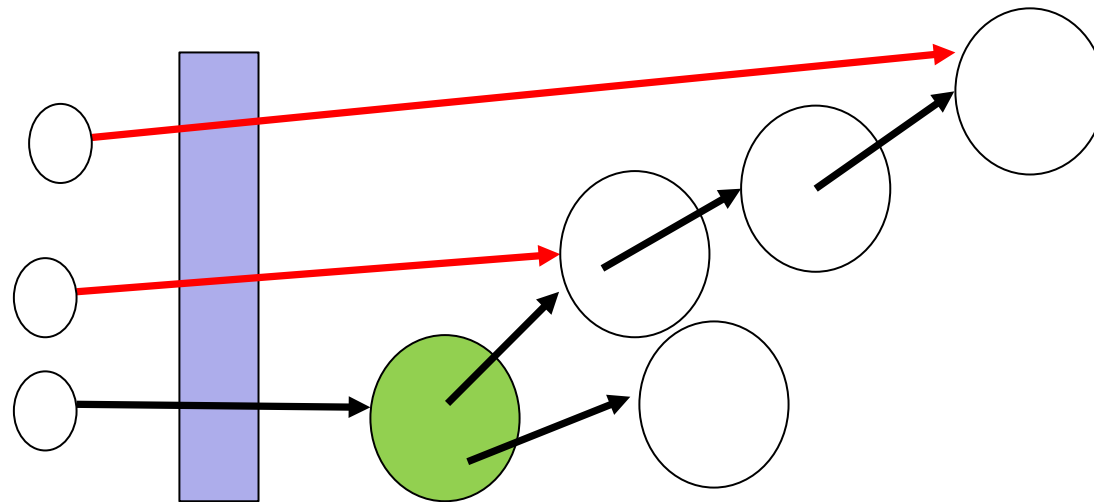
- [Representation exposure](#) is external access to the rep
- Representation exposure is almost always **EVIL**
 - can cause bugs that will be **very hard to detect**
- Rule #1: Don't do it!
- Rule #2: If you do it, document it clearly and then feel guilty about it!

Avoiding representation exposure

- *Understand* what representation exposure is
- *Design* ADT implementations to make sure it doesn't happen
- Treat rep exposure as a bug: *fix* your bugs
 - absolutely must avoid in libraries with many clients
 - can allow (but feel guilty) for code with few clients
- *Test* for it with *adversarial clients*:
 - pass values to methods and then mutate them
 - mutate values returned from methods

private is not enough

- Making fields `private` does *not* suffice to prevent rep exposure
 - see our example
 - issue is *aliasing of mutable data outside the abstraction*



- So `private` is a hint to you: no aliases outside abstraction to references to mutable data reachable from `private` fields
- Three general ways to avoid representation exposure...

Avoiding rep exposure (way #1)

- One way to avoid rep exposure is to make **copies** of all data that cross the abstraction barrier
 - Copy in [parameters that become part of the implementation]
 - Copy out [results that are part of the implementation]
- Examples of copying (assume `Point` is a mutable ADT):

```
class Line {
    private Point s, e;
    public Line(Point s, Point e) {
        this.s = new Point(s.x,s.y);
        this.e = new Point(e.x,e.y);
    }
    public Point getStart() {
        return new Point(this.s.x,this.s.y);
    }
}
```

...

Need deep copying

- “Shallow” copying is not enough
 - prevent any aliasing to mutable data inside/outside abstraction

- What’s the bug (assuming `Point` is a mutable ADT)?

```
class PointSet {  
    private List<Point> points = ...  
    public List<Point> getElts() {  
        return new ArrayList<Point>(points);  
    }  
}
```

- Not in example: Also need deep copying on “copy in”

Avoiding rep exposure (way #2)

- One way to avoid rep exposure is to exploit the **immutability** of (other) ADTs the implementation uses
 - aliasing is no problem if nobody can change data
 - have to mutate the rep to break the rep invariant
- Examples (assuming `Point` is an *immutable* ADT):

```
class Line {
    private Point s, e;
    public Line(Point s, Point e) {
        this.s = s;
        this.e = e;
    }
    public Point getStart() {
        return this.s;
    }
}
```

...

Why [not] immutability?

- Several advantages of immutability
 - aliasing does not matter
 - no need to make copies with identical contents
 - rep invariants cannot be broken
 - see CSE341 for more!
- Does require different code (e.g., if `Point` immutable)

```
void raiseLine(double deltaY) {  
    this.s = new Point(s.x, s.y+deltaY);  
    this.e = new Point(e.x, e.y+deltaY);  
}
```
- Immutable classes in Java libraries include `String`, `Character`, `Integer`, ...

Deepness, redux

- An immutable ADT must be immutable “all the way down”
 - No references *reachable* to data that may be mutated
- So combining our two ways to avoid rep exposure:
 - Must copy-in, copy-out “all the way down” to immutable parts

Back to getElts

Recall our initial rep-exposure example:

```
class CharSet {
    // Rep invariant: elts has no nulls and no dups
    private List<Character> elts = ...;

    // returns: elts currently in the set
    public List<Character> getElts() {
        return new ArrayList<Character>(elts); //copy out!
    }
    ...
}
```

Alternative #3

```
// returns: elts currently in the set
public List<Character> getElts() { // version 1
    return new ArrayList<Character>(elts); //copy out!
}

public List<Character> getElts() { // version 2
    return Collections.unmodifiableList(elts);
}
```

From the JavaDoc for `Collections.unmodifiableList`:

*Returns an unmodifiable view of the specified list. This method allows modules to provide users with "read-only" access to internal lists. Query operations on the returned list "read through" to the specified list, and attempts to modify the returned list... result in an **UnsupportedOperationException**.*

The good news

```
public List<Character> getElts() { // version 2
    return Collections.unmodifiableList(elts);
}
```

- Clients cannot *modify (mutate)* the rep
 - cannot break the rep invariant
- (For long lists,) more efficient than copy out
- Uses standard libraries

The bad news

```
public List<Character> getElts() { // version 1
    return new ArrayList<Character>(elts); //copy out!
}
```

```
public List<Character> getElts() { // version 2
    return Collections.unmodifiableList(elts);
}
```

The two implementations do not do the same thing!

- both avoid allowing clients to break the rep invariant
- both return a list containing the elements

But consider:

```
xs = s.getElts();
s.insert('a');
xs.contains('a');
```

Version 2 is *observing* an exposed rep, leading to different behavior

Different specifications

Ambiguity of “returns a list containing the current set elements”

“returns a fresh mutable list containing the elements in the set
at the time of the call”

versus

“returns read-only access to a list that the ADT
continues to update to hold the current elements in the set”

A third spec weaker than both [but less simple and useful!]

“returns a list containing the current set elements. *Behavior is unspecified (!) if client attempts to mutate the list or to access the list after the set’s elements are changed*”

Also note: Version 2’s spec also makes changing the rep later harder
– only “simple” to implement with rep as a **List**

Suggestions

Best options for implementing `getElts()`

- if $O(n)$ time is acceptable for relevant use cases, copy the list
 - safest option
 - best option for changeability
- if $O(1)$ time is required, then return an unmodifiable list
 - prevents breaking rep invariant
 - clearly document that behavior is unspecified after mutation
 - ideally, write a your own unmodifiable view of the list that throws an exception on all operations after mutation
- if $O(1)$ time is required and there is no unmodifiable version and you don't have time to write one, expose rep and feel guilty