CSE 331
Software Design and Implementation

Lecture 7
Abstraction Functions

Hal Perkins / Winter 2016

Connecting implementations to specs

Representation Invariant: maps Object → boolean

- Indicates if an instance is *well-formed*
- Defines the set of valid concrete values
- Only values in the valid set make sense as implementations of an abstract value
- **For implementors/debuggers/maintainers of the abstraction: no object should *ever* violate the rep invariant**
  - Such an object has no useful meaning

Abstraction Function: maps Object → abstract value

- What the data structure *means* as an abstract value
- How the data structure is to be interpreted
- Only defined on objects meeting the rep invariant
- **For implementors/debuggers/maintainers of the abstraction:** Each procedure should meet its spec (abstract values) by "doing the right thing" with the concrete representation

Rep inv. constrains structure, not meaning

An implementation of **`insert`** that preserves the rep invariant:

```
public void insert(Character c) {
  Character cc = new Character(encrypt(c));
  if (!elts.contains(cc))
    elts.addElement(cc);
}
public boolean member(Char
  return elts.contains(c)
}
```

```
CharSet s = new CharSet();
s.insert('a');
if (s.member('a'))
    …
```

Program is still wrong

- Clients observe incorrect behavior
- What client code exposes the error?
- Where is the error?
- We must consider the *meaning*
- The *abstraction function* helps us

Abstraction function: rep→abstract value

The abstraction function maps the concrete representation to the abstract value it represents

AF: Object → abstract value

AF(CharSet this) = { c | c is contained in this.elts }

"set of Characters contained in this.elts"

Not executable because abstract values are "just" conceptual

The abstraction function lets us reason about what [concrete] methods do in terms of the clients' [abstract] view

Abstraction function and `insert`

Goal is to satisfy the specification of insert:

```
// modifies: this
// effects: this_post = this_pre U {c}
public void insert (Character c) {…}
```

The AF tells us what the rep means, which lets us place the blame

AF(CharSet this) = { c | c is contained in this.elts }

Consider a call to **`insert`**:

On *entry*, meaning is AF($this_{pre}$) = $elts_{pre}$

On *exit*, meaning is AF($this_{post}$) = AF($this_{pre}$) U {encrypt('a')}

What if we used this abstraction function instead?

AF(this) = { c | encrypt(c) is contained in this.elts }

= { decrypt(c) | c is contained in this.elts }

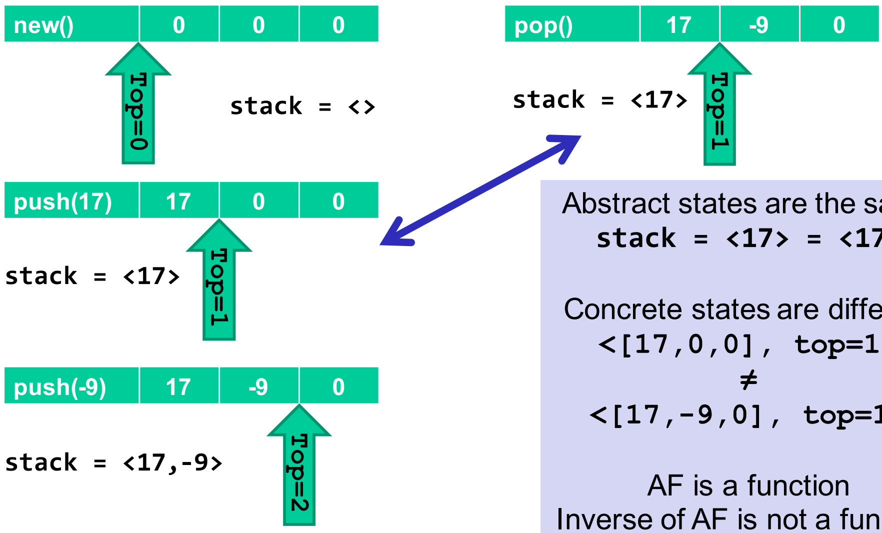
The abstraction function is a function

Why do we map concrete to abstract and not vice versa?

- It's not a function in the other direction
  - Example: lists **`[a,b]`** and **`[b,a]`** might each represent the set **`{a, b}`**
- It's not as useful in the other direction
  - Purpose is to reason about whether our methods are manipulating concrete representations correctly in terms of the abstract specifications

Stack AF example

Abstract stack with array and "top" index implementation



Abstract states are the same

`stack = <17> = <17>`

Concrete states are different

`<[17,0,0], top=1>`

≠

`<[17,-9,0], top=1>`

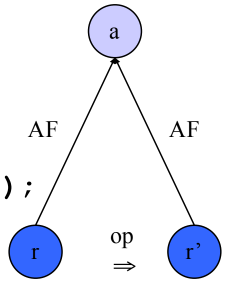
AF is a function

Inverse of AF is not a function

Benevolent side effects

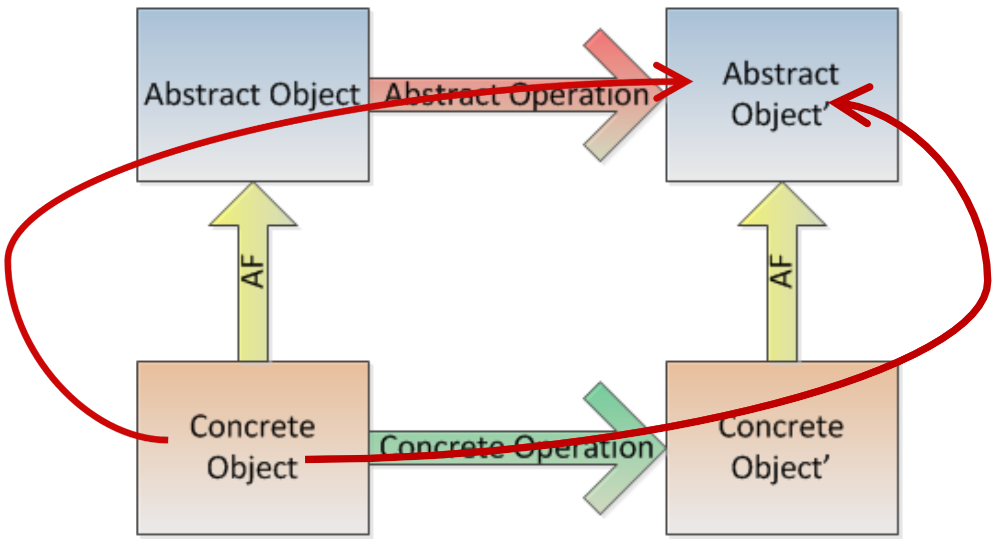
Different implementation of **`member`**:

```
boolean member(Character c1) {
  int i = elts.indexOf(c1);
  if (i == -1)
    return false;
  // move-to-front optimization
  Character c2 = elts.elementAt(0);
  elts.set(0, c1);
  elts.set(i, c2);
  return true;
}
```



- Move-to-front speeds up repeated membership tests
- Mutates rep, but does not change *abstract* value
  - *AF maps both reps to the same abstract value*
    - Precise reasoning/explanation for "clients can't tell"

For any correct operation…



Data Abstraction: Summary

Rep invariant
- Which concrete values represent abstract values

Abstraction function
- For each concrete value, which abstract value it represents

Together, they modularize the implementation
- Neither one is part of the ADT's specification
- Both are needed to reason an implementation satisfies the specification

In practice, representation invariants are documented more often and more carefully than abstraction functions
- A more widely understood and appreciated concept

Writing an abstraction function

Domain: all representations that satisfy the rep invariant

Range: can be tricky to denote

For mathematical entities like sets: easy

For more complex abstractions: give names to specification
- AF defines the value of each "specification field"

Overview section of the specification should provide a notation of writing abstract values
- Could implement a method for printing in this notation
  - Useful for debugging
  - Often a good choice for **`toString`**