Warmup

A programmer's roommate tells him, "Would you mind going to the store and picking up a loaf of bread. Also, if they have eggs, get a dozen."

The programmer returns with 12 loaves of bread.

Section 3:
HW4, ADTs, and more

WITH MATERIAL FROM VINOD RATHNAM, ALEX MARIAKAKIS, KRYSTA YOUSOUFIAN, MIKE ERNST, KELLEN DONOHUE

Agenda

Polynomial arithmetic

Abstract data types (ADT)

Representation invariants (RI)

Abstraction Functions

HW4: Polynomial Graphing Calculator

Problem 0: Write pseudocode algorithms for polynomial operations

Problem 1: Answer questions about RatNum

Problem 2: Implement RatTerm

Problem 3: Implement RatPoly

Problem 4: Implement RatPolyStack

Problem 5: Try out the calculator

RatThings

RatNum
- ADT for a Rational Number
- Has NaN

RatTerm
- Single polynomial term
- Coefficient (RatNum) & degree

RatPoly
- Sum of RatTerms

RatPolyStack
- Ordered collection of RatPolys

Polynomial Addition

$$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$$

Polynomial Addition

$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$

$$\begin{array}{rcccccc} & & 5x^4 & + 4x^3 & - x^2 & & + 5 \\ + & 3x^5 & & - 2x^3 & & + x & - 5 \end{array}$$

Polynomial Addition

$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$

$$\begin{array}{rcccccc} & & 5x^4 & + 4x^3 & - x^2 & 0x & + 5 \\ + & 3x^5 & 0x^4 & - 2x^3 & 0x^2 & + x & - 5 \end{array}$$

Polynomial Addition

$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$

$$
\begin{array}{rrrrrrr}
 & & 5x^4 & + 4x^3 & - x^2 & 0x & + 5 \\
+ & 3x^5 & 0x^4 & - 2x^3 & 0x^2 & + x & - 5 \\
\hline
 & 3x^5 & + 5x^4 & + 2x^3 & - x^2 & + x & + 0
\end{array}
$$

Polynomial Subtraction

$$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrrr} & & 5x^4 & + 4x^3 & - x^2 & & + 5 \\ - & 3x^5 & & - 2x^3 & & + x & - 5 \end{array}$$

Polynomial Subtraction

$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$

$$
\begin{array}{rrrrrrr}
 & & 5x^4 & + 4x^3 & - x^2 & 0x & + 5 \\
- & 3x^5 & 0x^4 & - 2x^3 & 0x^2 & + x & - 5
\end{array}
$$

Polynomial Subtraction

$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$

$$\begin{array}{rrrrrrr} & & 5x^4 & + 4x^3 & - x^2 & 0x & + 5 \\ - & 3x^5 & 0x^4 & - 2x^3 & 0x^2 & + x & - 5 \\ \hline & -3x^5 & + 5x^4 & + 6x^3 & - x^2 & - x & + 10 \end{array}$$

Polynomial Multiplication

$$(4x^3 - x^2 + 5) * (x - 5)$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{r}
4x^3 - x^2 + 5 \\
* \qquad\qquad x - 5 \\
\hline
\end{array}
$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{lr}
 & 4x^3 - x^2 + 5 \\
* & x - 5 \\
\hline
 & -20x^3 + 5x^2 \qquad - 25
\end{array}
$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{r}
4x^3 - x^2 + 5 \\
* \qquad\qquad x - 5 \\
\hline
-20x^3 + 5x^2 \qquad - 25 \\
4x^4 \quad -x^3 \qquad + 5x \qquad
\end{array}
$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{rrrrrr}
 & & 4x^3 & - x^2 & & + 5 \\
* & & & & x & - 5 \\
\hline
 & & -20x^3 & + 5x^2 & & - 25 \\
+ & 4x^4 & -x^3 & & + 5x & \\
\hline
 & 4x^4 & -21x^3 & + 5x^2 & + 5x & - 25
\end{array}
$$

Poly Division

$(5x^6 + 4x^4 - x^3 + 5)$ / $(x^3 - 2x - 5)$

Poly Division

$(5x^6 + 4x^4 - x^3 + 5)$ / $(x^3 - 2x - 5)$

$x^3 - 2x - 5 \overline{\big)\, 5x^6 + 4x^4 - x^3 + 5}$

Poly Division

```
1  0 -2 -5 | 5  0  4  -1  0  0  5
```

Poly Division

```
                                   5
1  0 -2 -5 |  5   0   4   -1   0   0   5
```

Poly Division

```
                        5
1  0 -2 -5 |  5   0   4   -1   0   0   5
              5   0-10 -25
```

Poly Division

```
                                        5
1  0 -2 -5 |  5   0   4   -1   0   0   5
              5   0-10 -25
              ----------------
              0   0 14   24
```

Poly Division

```
                                        5
1  0 -2 -5 |  5   0   4   -1   0   0   5
              5   0-10  -25
              ---------------
              0   0 14   24
                    14   24 0
```

Poly Division

```
                          5  0
1  0 -2 -5 |  5  0  4  -1  0  0  5
              5  0-10 -25
              0  0 14  24
                   14  24 0
```

Poly Division

```
                                5   0
                    ________________________________________
1   0  -2  -5      |  5   0   4   -1   0   0   5
                      5   0-10  -25
                      ____________________
                      0   0  14   24
                             14   24  0
                             14   24  0   0
```

Poly Division

```
                        5   0 14
1   0 -2 -5 |  5   0   4   -1   0   0   5
               5   0-10  -25
               0   0 14   24
                     14   24 0
                     14   24 0   0
```

Poly Division

```
                              5   0 14
1  0 -2 -5 |  5   0   4   -1   0   0   5
              5   0-10 -25
              0   0 14   24
                    14   24 0
                    14   24 0   0
                    14   0 -28 -70
```

Poly Division

5 0 14

1 0 -2 -5 | 5 0 4 -1 0 0 5

5 0-10 -25

0 0 14 24

14 24 0

14 24 0 0

14 0 -28 -70

0 24 28 70

Poly Division

```
                           5   0 14
1  0 -2 -5 | 5   0   4   -1   0   0   5
             5   0-10 -25
             ------------------
             0   0 14   24
                   14   24 0
                   14   24 0   0
                   14   0 -28 -70
                   -------------------
                   0   24   28   70
                       24   28   70 5
```

Poly Division

```
                              5   0 14  24
 1   0 -2 -5   |  5   0   4   -1   0   0   5
                  5   0-10  -25
                  0   0 14   24
                         14   24 0
                         14   24 0   0
                         14   0 -28 -70
                         0   24   28   70
                             24   28   70 5
                             24   0   -48 -120
```

Poly Division

```
                        5   0 14 24
1  0 -2 -5  |  5   0   4  -1   0   0   5
               5   0-10 -25
               0   0 14   24
                     14   24 0
                     14   24 0   0
                     14   0 -28 -70
                     0   24   28   70
                         24   28   70 5
                          24   0   -48 -120
                          0    28 118 125
```

Poly Division

$(5x^6 + 4x^4 - x^3 + 5)$ / $(x^3 - 2x - 5)$

$5x^3 + 14x + 24$

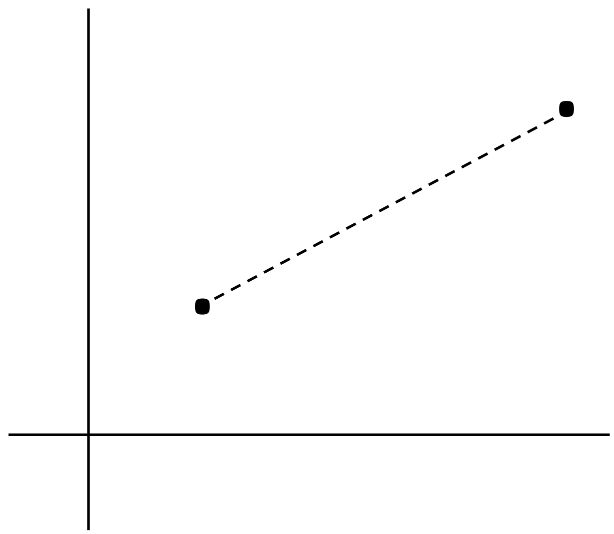
Poly Division

$(5x^6 + 4x^4 - x^3 + 5) / (x^3 - 2x - 5)$

$$5x^3 + 14x + 24 + \frac{28x^2 + 118x + 125}{x^3 - 2x - 5}$$

ADT Example: Line

Suppose we want to make a `Line` class that represents lines on the Cartesian plane



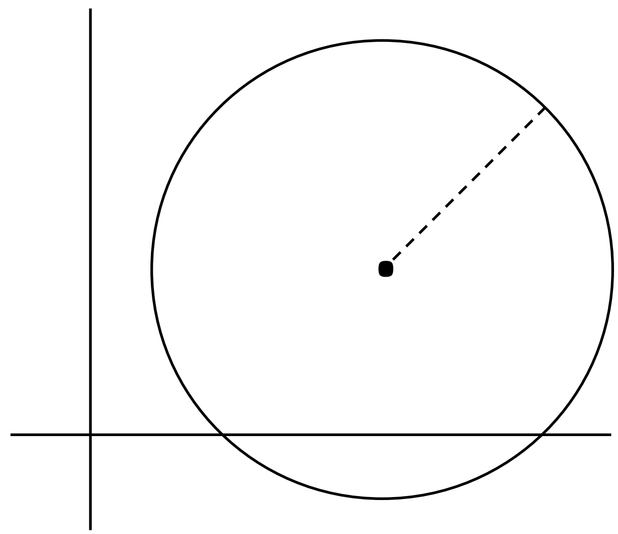
See
http://courses.cs.washington.edu/courses/cse331/15sp/concepts/specifications.html
for more

ADT Example: Line

```
/**
 * This class represents the mathematical concept of a line segment.
 *
 * A line is an immutable line segment on the 2D plane that has
 * endpoints p1 and p2
 */
public class Line {
…
}
```

ADT Example: Circle

Circle on the Cartesian coordinate plane



Circle: Class Specification

What represents the abstract state of a Circle?

What are some properties of a circle we can determine?

How can we implement this?

What are some ways to “break” a circle?

Representation Invariants

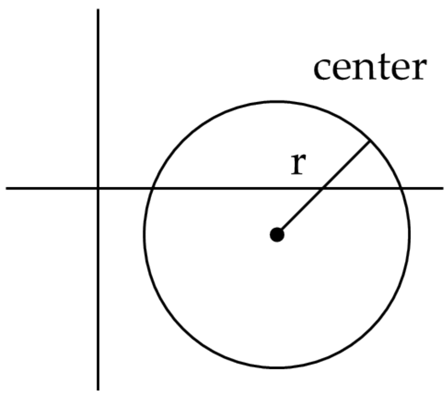
Constrains an object's internal state

Maps concrete representation of object to a boolean

If representation invariant is false/violated, the object is "broken" – doesn't map to any abstract value

Circle Implementation 1

```
public class Circle1 {
      private Point center;
      private double rad;

      // Rep invariant:
      //

      //  ...
}
```

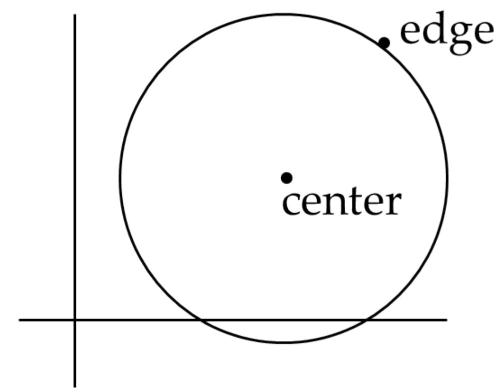


Circle Implementation 1

```
public class Circle1 {
	private Point center;
	private double rad;

	// Rep invariant:
	// center != null && rad > 0

	//  ...
}
```

Circle Implementation 2

```
public class Circle2 {
        private Point center;
        private Point edge;

        // Rep invariant:
        //
        //      ...
}
```

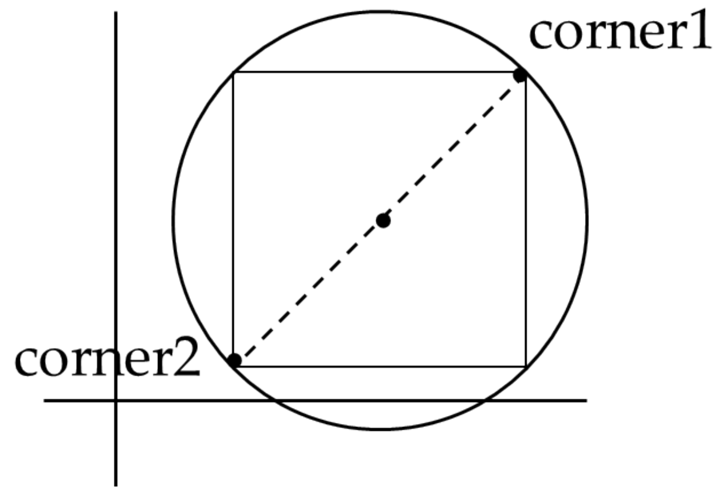


Circle Implementation 2

```
public class Circle2 {
        private Point center;
        private Point edge;

        // Rep invariant:
        // center != null &&
        // edge != null &&
        // !center.equals(edge)
        //     ...
}
```

Circle Implementation 3

```
public class Circle3 {
       private Point corner1, corner2;

       // Rep invariant:
       //
       //    ...
}
```



Circle Implementation 3

```
public class Circle3 {
       private Point corner1, corner2;

       // Rep invariant:
       // corner1 != null &&
       // corner2 != null &&
       // !corner1.equals(corner2)
       //    ...
}
```

Checking Rep Invariants

- Representation invariant should hold before and after every public method

Write and use `checkRep()`
- Call before and after methods that can modify the state
- Can make use of Java's assert syntax (pluses and minuses)
- OK that it adds extra code
  - Code is usually a small part of download size
  - Important for finding bugs

checkRep() Example with Asserts

```
public class Circle1 {
        private Point center;
        private double rad;

        private void checkRep() {
          assert center != null : “This does not have a center”;
          assert radius > 0 : “This circle has a negative radius”;
        }
}
```

A lot neater!

Using Asserts

To enable asserts: Go to Run->Run Configurations…->Arguments tab-> input

–ea in VM arguments section

- Do this for every test file
- Demo!

Abstraction Function

Abstraction function: a **mapping** from **internal state** to **abstract value**

Abstract fields may not map directly to representation fields

- Circle has **radius** but not necessarily

```
private int radius;
```

Internal representation can be anything as long as it somehow encodes the abstract value

Representation Invariant excludes values for which the abstraction function has no meaning

Circle Implementation 1

```
public class Circle1 {
        private Point center;
        private double rad;

        // Abstraction function:
        // AF(this) = a circle c such that
        //     c.center =
        //     c.radius =

        // Rep invariant:
        // center != null && rad > 0

        //  ...
}
```

Circle Implementation 1

```
public class Circle1 {
        private Point center;
        private double rad;

        // Abstraction function:
        // AF(this) = a circle c such that
        //      c.center = this.center
        //      c.radius = this.rad

        // Rep invariant:
        // center != null && rad > 0

        //  ...
}
```

Circle Implementation 2

```
public class Circle2 {
      private Point center;
      private Point edge;

      // Abstraction function:
      // AF(this) = a circle c such that
      //      c.center =
      //      c.radius =

      // Rep invariant:
      // center != null && edge != null &&!center.equals(edge)

      //     ...
}
```

Circle Implementation 2

```
public class Circle2 {
       private Point center;
       private Point edge;

       // Abstraction function:
       // AF(this) = a circle c such that
       //     c.center = this.center
       //     c.radius = sqrt((center.x-edge.x)^2 +
       //          (center.y-edge.y)^2)

       // Rep invariant:
       // center != null && edge != null && !center.equals(edge)
       //    ...
}
```

Circle Implementation 3

```
public class Circle3 {
	private Point corner1, corner2;

	// Abstraction function:
	// AF(this) = a circle c such that
	//     c.center =
	//     c.radius =

	// Rep invariant:
	// corner1 != null && corner2 != null &&
	// !corner1.equals(corner2)

	//    ...
}
```

Circle Implementation 3

```
public class Circle3 {
        private Point corner1, corner2;

        // Abstraction function:
        // AF(this) = a circle c such that
        //  c.center = <(corner1.x + corner2.x) / 2,
        //      (corner.y + corner2.y) / 2>

        //  c.radius = (1/2)*sqrt((corner1.x-corner2.x)^2 +
        //      (corner1.y-corner2.y)^2)

        // Rep invariant:
        // corner1 != null && corner2 != null &&
        // !corner1.equals(corner2)

        //    ...
}
```

ADT Example: NonNullStringList

```
public class NonNullStringList  {
       // Abstraction function:
       // ??

       // Rep invariant:
       // ??

       public void add(String s) { ... }
       public boolean remove(String s) { ... }
       public String get(int i) { ... }
}
```

NonNullStringList Implementation 1

```
public class NonNullStringList  {
      // Abstraction function:
      // Index i in arr contains the ith element in the list

      // Rep invariant:
      // RI = [0,count-1] != null &&
      // Count >=0 && arr != null

      private String[] arr;
      private int count;

      public void add(String s) { ... }
      public boolean remove(String s) { ... }
      public String get(int i) { ... }
}
```

Problems?

NonNullStringList Implementation 2

```
public class NonNullStringList  {
        // Abstraction function:
        // Value in the nth node after head contains the
        // nth item in the list

        // Rep invariant:
        // RI = Head has size nodes after it, each of whose
        // value is non-null, no cycle in ListNodes

        public int size;
        public ListNode head;

        public void add(String s) { ... }
        public boolean remove(String s) { ... }
        public String get(int i) { ... }
}
```