Warmup

A programmer's wife tells him, "Would you mind going to the store and picking up a loaf of bread. Also, if they have eggs, get a dozen."

The programmer returns with 12 loaves of bread.

Section 3:
HW4, ADTs, and more

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with material from Alex Mariakakis, Krysta Yousoufian, Mike Ernst, Kellen Donohue

Agenda

- Announcements
  - HW3: due tonight
  - HW4: due next Thursday
- Polynomial arithmetic
- Abstract data types (ADT)
- Representation invariants (RI)
- Abstraction Functions
- Further information found in **Calendar/info** & **docs/handouts** link on website

HW4: Polynomial Graphing Calculator

- **Problem 0:** Write pseudocode algorithms for polynomial operations
- **Problem 1:** Answer questions about RatNum
- **Problem 2:** Implement RatTerm
- **Problem 3:** Implement RatPoly
- **Problem 4:** Implement RatPolyStack
- **Problem 5:** Try out the calculator

RatThings

- RatNum
  - ADT for a Rational Number
  - Has NaN
- RatTerm
  - Single polynomial term
  - Coefficient (RatNum) & degree
- RatPoly
  - Sum of RatTerms
- RatPolyStack
  - Ordered collection of RatPolys

Polynomial Addition

$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$

Polynomial Addition

$$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrr} & 5x^4 & + 4x^3 & - x^2 & & + 5 \\ + & 3x^5 & - 2x^3 & & + x & - 5 \end{array}$$

Polynomial Addition

$$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrrr} & 5x^4 & +\ 4x^3 & -\ x^2 & 0x & & +\ 5 \\ + & 3x^5 & 0x^4 & -\ 2x^3 & 0x^2 & +\ x & -\ 5 \end{array}$$

Polynomial Addition

$$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrrr}
 & 5x^4 & +\ 4x^3 & -\ x^2 & 0x & & +\ 5 \\
+ & 3x^5 & 0x^4 & -\ 2x^3 & 0x^2 & +\ x & -\ 5 \\
\hline
 & 3x^5 & +\ 5x^4 & -\ 2x^3 & -\ x^2 & +\ x & +\ 0
\end{array}$$

Polynomial Subtraction

$$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrr} & 5x^4 & + 4x^3 & - x^2 & & + 5 \\ - & 3x^5 & - 2x^3 & & + x & - 5 \end{array}$$

Polynomial Subtraction

$$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrrr} & 5x^4 & +\ 4x^3 & -\ x^2 & 0x & +\ 5 \\ -\ 3x^5 & 0x^4 & -\ 2x^3 & 0x^2 & +\ x & -\ 5 \end{array}$$

Polynomial Subtraction

$$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrrr}
 & 5x^4 & +\ 4x^3 & -\ x^2 & 0x & +\ 5 \\
- & 3x^5 & 0x^4 & -\ 2x^3 & 0x^2 & +\ x & -\ 5 \\
\hline
 & -3x^5 & +\ 5x^4 & +\ 6x^3 & -\ x^2 & -\ x & +\ 10
\end{array}$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{r}
4x^3 - x^2 + 5 \\
* \qquad\qquad x - 5 \\
\hline
\end{array}
$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{rrrr}
 & 4x^3 & -\ x^2 & +\ 5 \\
* & & x & -\ 5 \\
\hline
 & -20x^3 & +\ 5x^2 & -\ 25
\end{array}
$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{rrrrr}
 & & 4x^3 & - x^2 & + 5 \\
* & & & x & - 5 \\
\hline
 & -20x^3 & + 5x^2 & & - 25 \\
4x^4 & -x^3 & & + 5x &
\end{array}
$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{rrrrrr}
 & & & 4x^3 & -\ x^2 & +\ 5 \\
* & & & & x & -\ 5 \\
\hline
 & & -20x^3 & +\ 5x^2 & & -\ 25 \\
+ & 4x^4 & -x^3 & & +\ 5x & \\
\hline
 & \mathbf{4x^4} & \mathbf{-21x^3} & \mathbf{+\ 5x^2} & \mathbf{+\ 5x} & \mathbf{-\ 25}
\end{array}
$$

Polynomial Division

$(5x^6 + 4x^4 - x^3 + 5) / (x^3 - 2x - 5)$

Polynomial Division

$(5x^6 + 4x^4 - x^3 + 5) \; / \; (x^3 - 2x - 5)$

$$x^3 - 2x - 5 \;\overline{\big)\; 5x^6 + 4x^4 - x^3 + 5}$$

Polynomial Division

1	0	-2	-5	5	0	4	-1	0	0	5	

Polynomial Division

$$
\begin{array}{r|rrrrrrr}
 & & & & & 5 & & \\
\hline
1 \quad 0 \quad -2 \quad -5 & 5 & 0 & 4 & -1 & 0 & 0 & 5
\end{array}
$$

Polynomial Division

```
                          5
                 ___________________________________
1  0 -2 -5 |  5  0  4  -1  0  0  5
            |
            |  5  0-10 -25
```

Polynomial Division

							5				
1	0	-2	-5	5	0	4	-1	0	0	5	
				5	0	-10	-25				
				0	0	14	24				

Polynomial Division

```
                                5
                 ------------------------------------
1   0 -2 -5 |  5   0   4   -1   0   0   5
                5   0-10 -25
               ------------------------
                0   0 14   24
                      14   24 0
```

Polynomial Division

```
                          5  0
                 ------------------------------
1  0 -2 -5 |  5  0  4  -1  0  0  5
            |  5  0-10 -25
            ----------------------
            |  0  0 14  24
            |       14  24 0
```

Polynomial Division

```
                          5   0
                 ------------------------------------
1   0 -2 -5 |  5   0   4   -1   0   0   5
                 5   0-10 -25
                 ------------------------
                 0   0 14   24
                       14   24 0
                       14   24 0   0
```

Polynomial Division

```
                                5   0 14
                 ------------------------------------
1   0 -2 -5 |  5   0   4   -1   0   0   5
                5   0-10 -25
                ------------------------
                0   0 14   24
                      14   24 0
                      14   24 0   0
```

Polynomial Division

```
                                5   0 14
                 ------------------------------------
1   0 -2 -5 |  5   0   4   -1   0   0   5
                5   0-10 -25
               ------------------------
                0   0 14   24
                      14   24 0
                      14   24 0   0
                      14   0 -28 -70
```

Polynomial Division

```
                                5   0 14
                 ------------------------------------
1   0 -2 -5 |  5   0   4   -1   0   0   5
                5   0-10 -25
               ------------------------
                0   0 14   24
                      14   24 0
                      14   24 0   0
                      14   0 -28 -70
                    ------------------------------
                      0   24   28   70
```

Polynomial Division

```
                                5   0 14
                 ------------------------------------
1   0 -2 -5 |  5   0   4   -1   0   0   5
                5   0-10 -25
               ------------------------
                0   0 14   24
                      14   24 0
                      14   24 0   0
                      14   0 -28 -70
                    ------------------------------
                      0   24   28   70
                          24   28   70 5
```

Polynomial Division

```
                                  5   0 14 24
                 ------------------------------------------
1   0 -2 -5 |  5   0   4   -1   0   0   5
                5   0-10 -25
               ------------------------
                0   0 14   24
                      14   24 0
                      14   24 0   0
                      14   0 -28 -70
                    ------------------------
                      0   24   28   70
                          24   28   70 5
                          24   0   -48 -120
```

Polynomial Division

```
                                  5   0 14 24
                 ------------------------------------------
1   0 -2 -5 |  5   0   4   -1   0   0   5
                5   0-10 -25
               ------------------------
                0   0 14   24
                      14   24 0
                      14   24 0   0
                      14   0 -28 -70
                     ------------------------
                      0   24   28   70
                          24   28   70 5
                          24   0   -48 -120
                         ------------------
                          0    28 118 125
```

Polynomial Division

$(5x^6 + 4x^4 - x^3 + 5) / (x^3 - 2x - 5)$

$5x^3 + 14x + 24$

Polynomial Division

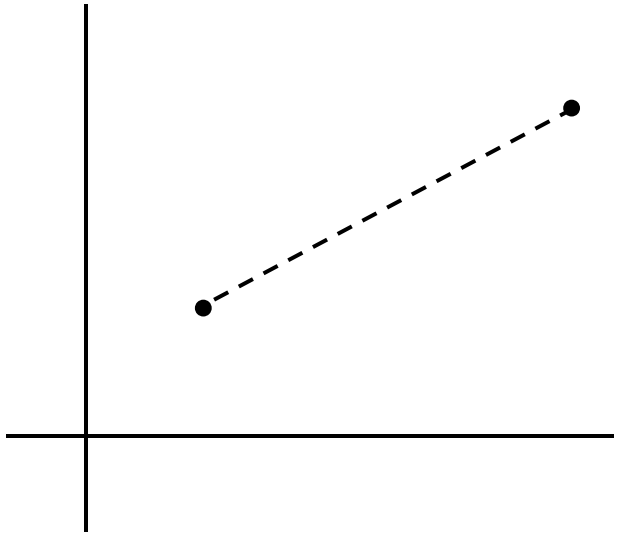
$(5x^6 + 4x^4 - x^3 + 5) \; / \; (x^3 - 2x - 5)$

$$5x^3 + 14x + 24 + \frac{28x^2 + 118x + 125}{x^3 - 2x - 5}$$

CalculatorFrame Demo

ADT Example: Line

Suppose we want to make a `Line` class that represents lines on the Cartesian plane



See http://courses.cs.washington.edu/courses/cse331/13au/conceptual-info/specifications.html for more

Specification vs Derived Fields

- **Specification Fields**: describes components of the abstract state of a class

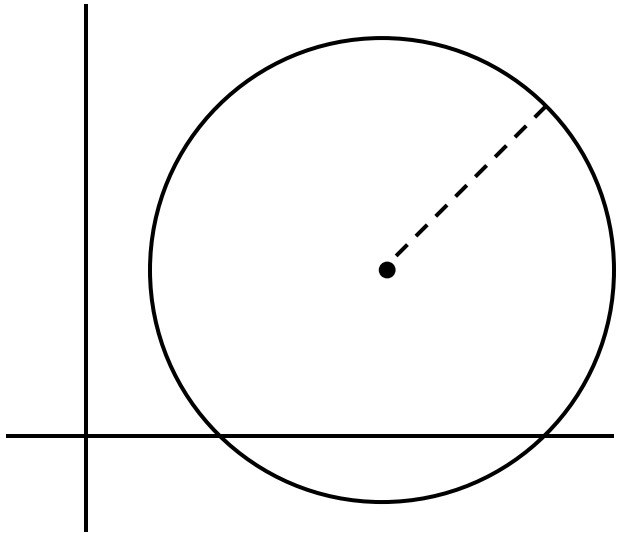
- **Derived Specification Fields**: information that can be derived from specification fields but useful to have

ADT Example: Line

```
/**
 * This class represents the mathematical concept of a line segment.
 *
 * Specification fields:
 *  @specfield start-point : point  // The starting point of the line.
 *  @specfield end-point   : point  // The ending point of the line.
 *
 * Derived specification fields:
 *  @derivedfield length : real     // The length of the line.
 *
 *
 */
public class Line {
…
}
```

ADT Example: Circle

- Circle on the Cartesian coordinate plane



Representation Invariants

- Constrains an object's internal state
- Maps concrete representation of object to a boolean
- If representation invariant is false/violated, the object is "broken" – doesn't map to any abstract value

Circle: Class Specification

What are the abstract/specification fields (what the client sees)?

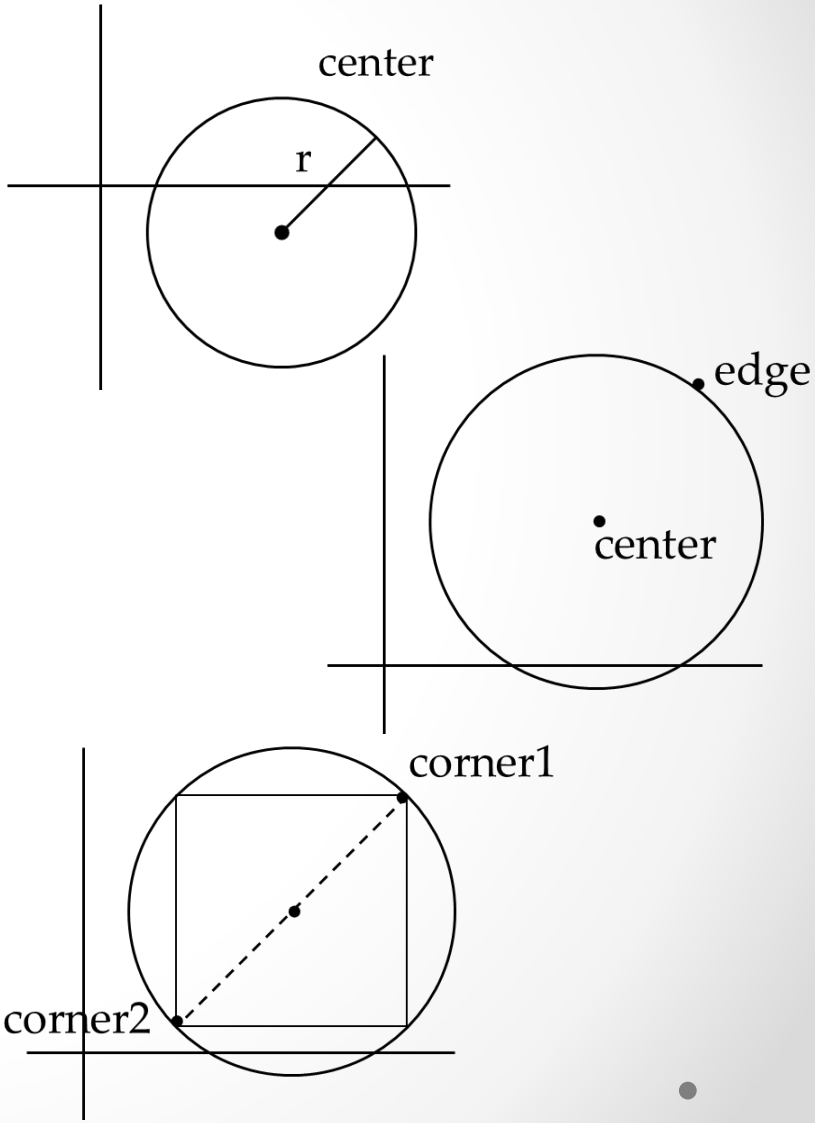
- Center
- Radius

What are some derived fields?

- Circumference
- Area

How can we implement this?

- #1: Center, radius
- #2: Center, edge
- #3: Corner of diameter



Circle Implementation 1

```
public class Circle1 {
      private Point center;
      private double rad;

      // Rep invariant:
      //

      //  ...
}
```

Circle Implementation 1

```
public class Circle1 {
      private Point center;
      private double rad;

      // Rep invariant:
      // center != null && rad > 0

      //  ...
}
```

Circle Implementation 2

```
public class Circle2 {
      private Point center;
      private Point edge;

      // Rep invariant:
      //

      //     ...
}
```

Circle Implementation 2

```
public class Circle2 {
      private Point center;
      private Point edge;

      // Rep invariant:
      // center != null && edge ! null &&
        !center.equals(edge)

      //     ...
}
```

Circle Implementation 3

```
public class Circle3 {
      private Point corner1, corner2;

      // Rep invariant:
      //



      //     ...
}
```

Circle Implementation 3

```
public class Circle3 {
      private Point corner1, corner2;

      // Rep invariant:
      // corner1 != null && corner2 != null &&
      !corner1.equals(corner2)

      //    ...
}
```

Checking RIs

- Representation invariant should hold before and after every public method
- Write and use `checkRep()`
  - Call before and after public methods
  - Make use of Java's assert syntax!
  - OK that it adds extra code
    - Asserts won't be included on release builds
    - Important for finding bugs

checkRep() Example With Exceptions

```
public class Circle1 {
        private Point center;
        private double rad;

        private void checkRep() throws RuntimeException {
                if (center == null) {
                        throw new RuntimeException(“This does
                                not have a center");
                }

                if (radius <= 0) {
                        throw new RuntimeException(“This
                        circle has a negative radius”);
                }
        }
}
```

checkRep() Example with Asserts

```
public class Circle1 {
        private Point center;
        private double rad;

        private void checkRep() throws RuntimeException {
                assert center != null : “This does not have a
                                        center”;
                assert radius > 0 : “This circle has a negative
                                    radius”;
        }
}
```

A lot neater!

Using Asserts

- To enable asserts: Go to Run->Run Configurations…->Arguments tab-> input **–ea** in VM arguments section
  - Do this for every test file
  - Demo!

Abstraction Function

- Abstraction function: a **mapping** from **internal state** to **abstract value**
- Abstract fields may not map directly to representation fields
  - Circle has **radius** but not necessarily

    ```
    private int radius;
    ```
- Internal representation can be anything as long as it somehow encodes the abstract value
- Representation Invariant excludes values for which the abstraction function has no meaning

Circle Implementation 1

```
public class Circle1 {
        private Point center;
        private double rad;

        // Abstraction function:
        // AF(this) = a circle c such that
        //      c.center =
        //      c.radius =

        // Rep invariant:
        // center != null && rad > 0

        //  ...
}
```

Circle Implementation 1

```
public class Circle1 {
        private Point center;
        private double rad;

        // Abstraction function:
        // AF(this) = a circle c such that
        //      c.center = this.center
        //      c.radius = this.rad

        // Rep invariant:
        // center != null && rad > 0

        //  ...
}
```

Circle Implementation 2

```
public class Circle2 {
        private Point center;
        private Point edge;

        // Abstraction function:
        // AF(this) = a circle c such that
        //     c.center =
        //     c.radius =

        // Rep invariant:
        // center != null && edge ! null &&
           !center.equals(edge)

        //    ...
}
```

Circle Implementation 2

```
public class Circle2 {
        private Point center;
        private Point edge;

        // Abstraction function:
        // AF(this) = a circle c such that
        //      c.center = this.center
        //      c.radius = sqrt((center.x-edge.x)^2 +
                           (center.y-edge.y)^2)

        // Rep invariant:
        // center != null && edge ! null &&
           !center.equals(edge)

        //    ...
}
```

Circle Implementation 3

```
public class Circle3 {
        private Point corner1, corner2;

        // Abstraction function:
        // AF(this) = a circle c such that
        //     c.center =
        //     c.radius =

        // Rep invariant:
        // corner1 != null && corner2 != null &&
           !corner1.equals(corner2)

        //    ...
}
```

Circle Implementation 3

```
public class Circle3 {
        private Point corner1, corner2;

        // Abstraction function:
        // AF(this) = a circle c such that
        //      c.center = <(corner1.x + corner2.x) / 2,
                            (corner.y + corner2.y) / 2>

        //      c.radius = (1/2)*sqrt((corner1.x-
                            corner2.x)^2 + (corner1.y-
                            corner2.y)^2)

        // Rep invariant:
        // corner1 != null && corner2 != null &&
           !corner1.equals(corner2)

        //     ...
}
```

ADT Example: NonNullStringList

```
public class NonNullStringList  {
      // Abstraction function:
      // ??

      // Rep invariant:
      // ??

      public void add(String s) { ... }
      public boolean remove(String s) { ... }
      public String get(int i) { ... }
}
```

NonNullStringList Implementation 1

```
public class NonNullStringList  {
      // Abstraction function:
      // Index i in arr contains the ith element in the
         list

      // Rep invariant:
      // RI = [0,count-1] != null

      private String[] arr;
      private int count;

      public void add(String s) { ... }
      public boolean remove(String s) { ... }
      public String get(int i) { ... }
}
```

Problems?

NonNullStringList Implementation 2

```
public class NonNullStringList  {
      // Abstraction function:
      // Value in the nth node after head contains the
         nth item in the list

      // Rep invariant:
      // RI = Head has size nodes after it, each whose
         value is non-null, no cycle in ListNodes

      public int size;
      public ListNode head;

      public void add(String s) { ... }
      public boolean remove(String s) { ... }
      public String get(int i) { ... }
}
```