Representation invariants and abstraction functions

CSE 331

UW CSE

ADTs and specifications

Specification: only in terms of the abstraction

- Never mentions the representation

An ADT is more than just a data structure

- data structure + a set of conventions

Why do we need to relate the specification to the representation?

Connecting specifications and implementations

Representation invariant: Object → boolean

- Indicates whether a data structure is well-formed
- Defines the set of valid values of the data structure

Abstraction function: Object → abstract value

- What the data structure means (as an abstract value)
- How the data structure is to be interpreted
- How do you compute the inverse, abstract value → Object ?

A data abstraction is defined by a specification

- A collection of procedural abstractions
  - Not a collection of procedures
- Together, these procedural abstractions provide
  - A set of values
  - ***All*** the ways of directly using that set of values
    - Creating
    - Manipulating
    - Observing
- Creators and producers: make new values
- Mutators: change the value (but don't affect **==**)
- Observers: allow one to tell values apart

Implementation of an ADT is provided by a class

To implement a data abstraction

- Select the representation of instances, the *rep*
- Implement operations in terms of that rep

Choose a representation so that

- It is possible to implement operations
- The most frequently used operations are efficient
  - But which will these be?
  - Abstraction allows the rep to change later

CharSet Abstraction

```
// Overview: A CharSet is a finite mutable set of Characters

// effects: creates a fresh, empty CharSet
public CharSet ( )

// modifies: this
// effects: this_post = this_pre U {c}
public void insert (Character c);

// modifies: this
// effects: this_post = this_pre - {c}
public void delete (Character c);

// returns: (c ∈ this)
public boolean member (Character c);

// returns: cardinality of this
public int size ( );
```

A CharSet implementation.
What client code will expose the error?

```
class CharSet {
  private List<Character> elts
    = new ArrayList<Character>();
  public void insert(Character c)   {
    elts.add(c);
  }
  public void delete(Character c)   {
    elts.remove(c);
  }
  public boolean member(Char
    return elts.contains(c)
  }
  public int size() {
    return elts.size();
  }
}
```

```
CharSet s = new CharSet();
Character a
    = new Character('a');
s.insert(a);
s.insert(a);
s.delete(a);
if (s.member(a))
    // print "wrong";
else
    // print "right";
```

Where Is the Error?

The answer to this question tells you what needs to be fixed

Perhaps **`delete`** is wrong

- It should remove all occurrences

Perhaps **`insert`** is wrong

- It should not insert a character that is already there

How can we know?

- The representation invariant tells us

The representation invariant

- States data structure well-formedness
- Captures information that must be shared across implementations of multiple operations

Write it this way:

```
class CharSet {
  // Rep invariant: elts has no nulls and no duplicates
  private List<Character> elts;
  …
```

Or, if you are the pedantic sort:

$\forall$ indices i of elts . elts.elementAt(i) $\neq$ null

$\forall$ indices i, j of elts .

$\quad$ i $\neq$ j $\Rightarrow$ $\neg$ elts.elementAt(i).equals(elts.elementAt(j))

Now, we can locate the error

```
// Rep invariant:
// elts has no nulls and no duplicates

public void insert(Character c) {
  elts.add(c);
}

public void delete(Character c) {
  elts.remove(c);
}
```

Another rep invariant example

```
class Account {
    private int balance;
    // history of all transactions
    private List<Transaction> transactions;
    …
}
```

// real-world constraints:

balance ≥ 0

balance $= \Sigma_i$ transactions.get(i).amount

// implementation-related constraints:

transactions $\neq$ null

no nulls in transactions

Listing the elements of a CharSet

Consider adding the following method to CharSet

// returns: a List containing the members of this

public List<Character> getElts();

Consider this implementation:

```
// Rep invariant: elts has no nulls and no duplicates
public List<Character> getElts()  { return elts; }
```

Does the implementation of getElts preserve the rep invariant?

... sort of

Representation exposure

Consider the client code (outside the CharSet implementation)

```
CharSet s = new CharSet();
Character a = new Character('a');
s.insert(a);
s.getElts().add(a);
s.delete(a);
if (s.member(a)) …
```

Representation exposure is external access to the rep

Representation exposure is almost always evil

If you do it, document why and how

- And feel guilty about it!

Ways to avoid rep exposure

1. Exploit immutability

```
Character choose() {
  return elts.elementAt(0);
}
```

Character is immutable.

2. Make a copy

```
List<Character> getElts() {
  return new ArrayList<Character>(elts);
  // or: return (ArrayList<Character>) elts.clone();
}
```

Mutating a copy doesn't affect the original.

Don't forget to make a copy on the way in!

3. Make an immutable copy

```
List<Character> getElts() {
  return Collections.unmodifiableList<Character>(elts);
}
```

Client cannot mutate

Still need to make a copy on the way in

Checking rep invariants

Should code check that the rep invariant holds?

- Yes, if it’s inexpensive
- Yes, for debugging (even when it’s expensive)
- It’s quite hard to justify turning the checking off
- Some private methods need not check (Why?)

Checking the rep invariant

Rule of thumb: check on entry *and* on exit (why?)

```
public void delete(Character c) {
  checkRep();
  elts.remove(c)
  // Is this guaranteed to get called?
  // See handouts for a less error-prone way to check at exit.
  checkRep();
}
…
/** Verify that elts contains no duplicates. */
private void checkRep() {
  for (int i = 0; i < elts.size(); i++) {
    assert elts.indexOf(elts.elementAt(i)) == i;
  }
}
```

Practice defensive programming

Assume that you will make mistakes

Write and incorporate code designed to catch them

- On entry:
  - Check rep invariant
  - Check preconditions (<u>requires</u> clause)
- On exit:
  - Check rep invariant
  - Check postconditions

Checking the rep invariant helps you discover errors

Reasoning about the rep invariant helps you avoid errors

- Or prove that they do not exist!
- We will discuss such reasoning, later in the term

The rep invariant constrains structure, not meaning

New implementation of insert that preserves the rep invariant:

```
public void insert(Character c) {
  Character cc = new Character(encrypt(c));
  if (!elts.contains(cc))
    elts.addElement(cc);
}
public boolean member(Character c) {
  return elts.contains(c);
}
```

The program is still wrong

- Clients observe incorrect behavior
- What client code exposes the error?
- Where is the error?
- We must consider the meaning
- The *abstraction function* helps us

```
CharSet s = new CharSet();
Character a = new
Character('a'));
s.insert(a);
if (s.member(a))
    // print "right";
else
    // print "wrong";
```

Abstraction function: rep → abstract value

The abstraction function maps the concrete representation to the abstract value it represents

AF: Object → abstract value

AF(CharSet this) = { c | c is contained in this.elts }

  "set of Characters contained in this.elts"

  Typically *not* executable

The abstraction function lets us reason about behavior from the client perspective

Our real goal is to satisfy the specification of insert:

  // modifies: this

  // effects: $this_{post} = this_{pre} \cup \{c\}$

  public void insert (Character c);

Once again we can place the blame

  Applying the abstraction function to the result of the call to insert yields AF(elts) U {encrypt('a')}

  What if we used this abstraction function?

    AF(this) = { c | encrypt(c) is contained in this.elts }

    AF(this) = { decrypt(c) | c is contained in this.elts }

Placing the blame

Our real goal is to satisfy the specification of insert:

// modifies: this
// effects: $this_{post}$ = $this_{pre}$ U {c}
public void insert (Character c);

The AF tells us what the rep means (and lets us place the blame)

AF(CharSet this) = { c | c is contained in this.elts }

Consider a call to insert:

On entry, the meaning is AF($this_{pre}$) ≈ $elts_{pre}$
On exit, the meaning is AF($this_{post}$) = AF($this_{pre}$) U {encrypt('a')}

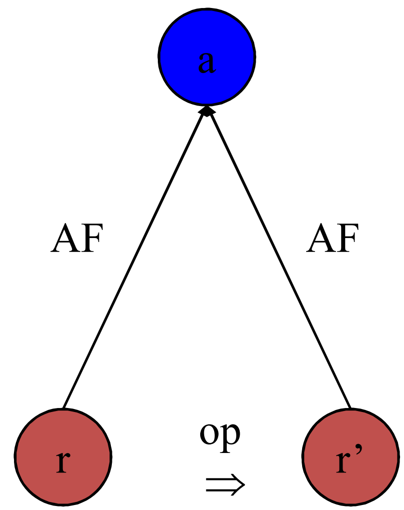
What if we used this abstraction function?

AF(this) = { c | encrypt(c) is contained in this.elts }
= { decrypt(c) | c is contained in this.elts }

Benevolent side effects

Different implementation of member:

```
boolean member(Character c1) {
  int i = elts.indexOf(c1);
  if (i == -1)
    return false;
  // move-to-front optimization
  Character c2 = elts.elementAt(0);
  elts.set(0, c1);
  elts.set(i, c2);
  return true;
}
```



Move-to-front speeds up repeated membership tests

Mutates rep, but does not change *abstract* value

- AF maps both reps to the same abstract value

The abstraction function is a function

Q: Why do we map concrete to abstract rather than vice versa?

1. It's not a function in the other direction.

   E.g., lists [a,b] and [b,a] each represent the set {a, b}

2. It's not as useful in the other direction.

   Can construct objects via the provided operators

Writing an abstraction function

- The domain: all representations that satisfy the rep invariant
- The range: can be tricky to denote
  - For mathematical entities like sets: easy
  - For more complex abstractions: give them fields
    - AF defines the value of each "specification field"
    - For "derived specification fields", see the handouts
- The overview section of the specification should provide a way of writing abstract values
  - A printed representation is valuable for debugging

Summary

- Rep invariant
  - Which concrete values represent abstract values
- Abstraction function
  - For each concrete value, which abstract value it represents
- Together, they modularize the implementation
  - Can examine operators one at a time
  - Neither one is part of the abstraction (the ADT)
- In practice
  - Always write a representation invariant
  - Write an abstraction function when you need it
    - Write an informal one for most non-trivial classes
    - A formal one is harder to write and usually less useful