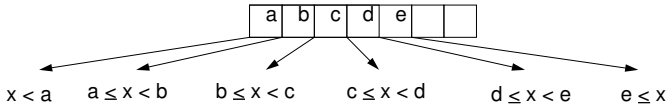
CSE 326
Lecture 12

B-Trees

Winter Quarter 1999

B - Trees

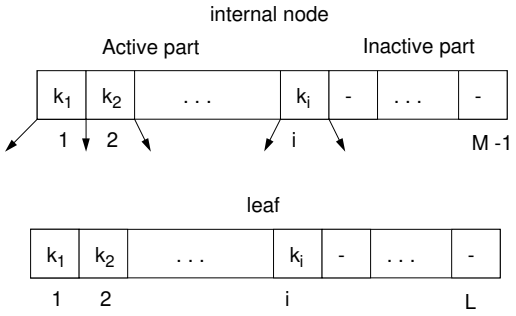
- B-trees are multiway trees which are not binary



- B-tree nodes contain search keys only, actual data is stored at the leaves.
- The subtree between keys x and y has smallest key equal to x.
- B-tree developed for database applications where minimizing the number of disk accesses is important. B-tree nodes are large enough to balance the disk access time with the internal search time within a B-tree node. Typically, a B-tree node fits into one page of memory which is unit of access to the disk.

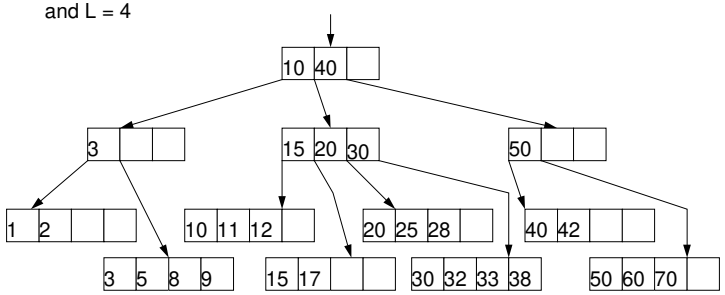
Properties of B-Trees

- The root is either a leaf or has between 2 and M children
- All non-leaf node (except the root) have between $\lceil M/2 \rceil$ and M children.
- All leaves are at the same depth and contain between $\lceil L/2 \rceil$ and L keys.



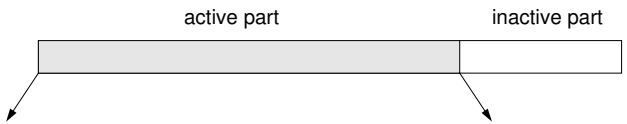
Example

B-tree with M = 4
and L = 4

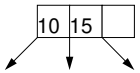


Find in a B-Tree

- If M is large then binary search within a B-tree node.



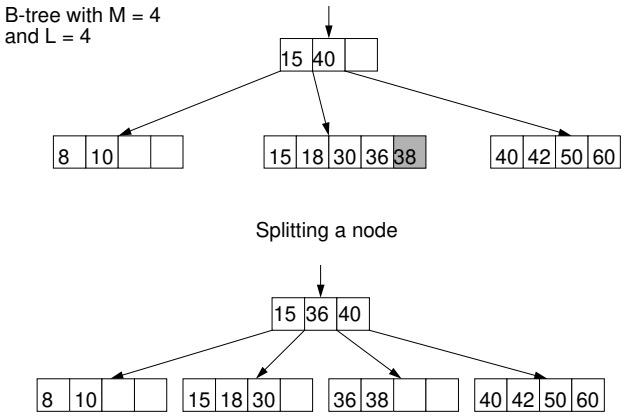
- If M is small then linear search is fine.



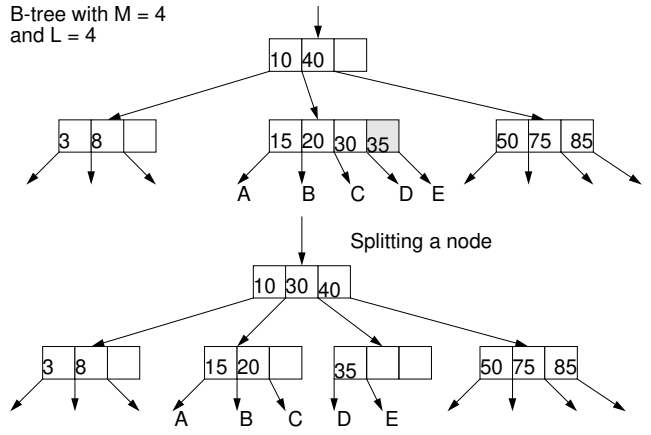
Insertion in a B-tree

- Begin by inserting in the proper leaf.
- If the leaf ends up with L+1 items, it has overflowed.
- An overflowed leaf node is split into two nodes with $\lceil (L+1)/2 \rceil$ in one and $\lfloor (L+1)/2 \rfloor$ in the other. A new child is added to the parent.
- An overflowed internal node with M+1 children is split into two nodes, one with $\lceil (M+1)/2 \rceil$ children and the other with $\lfloor (M+1)/2 \rfloor$ children. A new child is added to the parent.
- If overflow propagates to the root, then the node is split into two and a new root with two children is formed.

Insertion with Overflow - Leaf Case



Insertion with Overflow - Internal Node Case



Example of Insertion

- Insert 1,2,3,4 5,6 7,8, ... into a B-tree with M = 3 and L = 3.

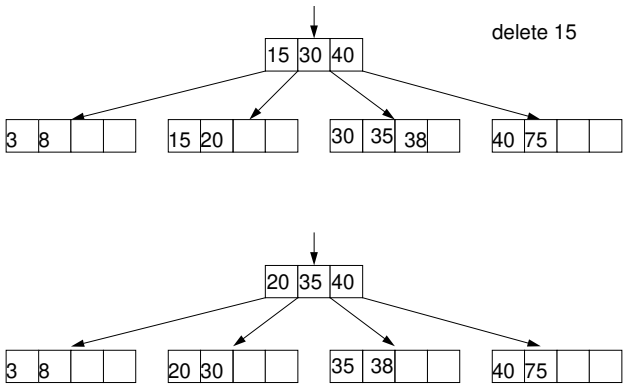
B-Tree Deletion

- In deletion a leaf may underflow, that is, have < $\lceil L/2 \rceil$ items.

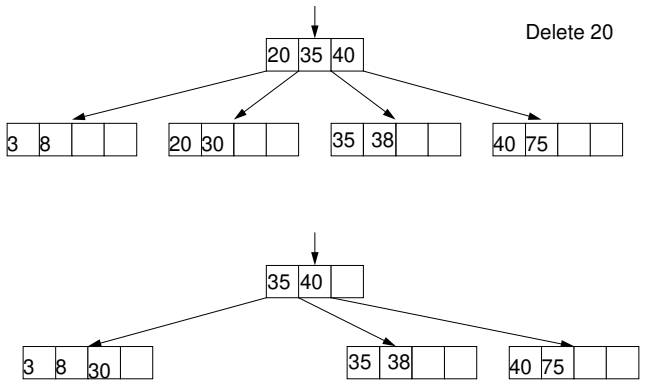
  1. Borrow from a sibling.

  2. If borrowing is impossible then give all to siblings evenly and physically delete the node. Deletion may propagate to the parent.

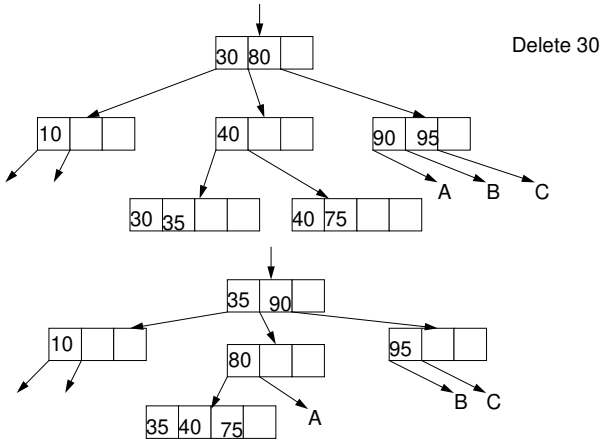
Borrowing



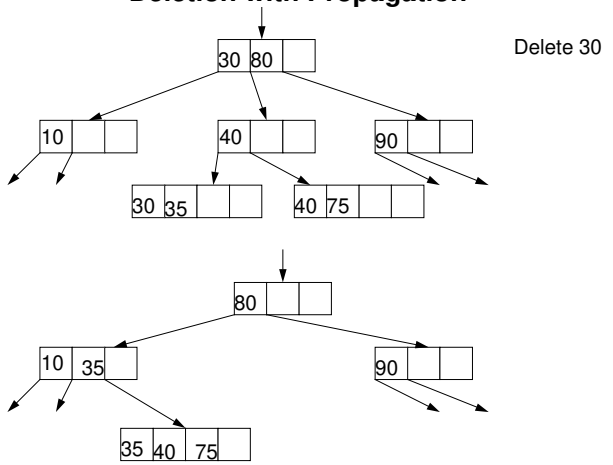
Elimination without Propagation



Deletion with Borrowing at an Internal Node



Deletion with Propagation



Summary of B-Trees

- B-trees with M=3 are often called 2-3 trees and B-trees with M=4 are often called 2-3-4 trees.
- B-trees are used when accessing a new node can be costly, like in large data bases. The root and next level are often kept in main memory.
- B-tree insertion causes splitting which may propagate.
- B-tree deletion causes borrowing (good) or physical deletion which may propagate. Propagation is quite rare if the order of the tree is large.
- In standard B-trees there can be thrashing with repeated insertions followed by deletions. This can be overcome with B* trees.
- If order of the tree is large like 64 or 128, then only a few levels are need to store very large data sets. For example, if M= L = 128 then a B-tree of height 4 will store at least 30 million items.

Summary of Search Trees

- Plain vanilla binary search trees

  Good on average - simple to understand and use.
- AVL trees - height balanced trees

  Good in worst case - more complicated.
- Splay trees

  Good amortized cost and good locality properties - slightly more complicated than AVL.
- B-trees

  Good in worst case when node access is expensive - most complicated of the algorithms. Heavily used in database applications.

Summary of Dictionary Data Structures

	Find	Insert	Delete	Find-min	List-all	n Finds
Sorted Array	log n	n	n	1	n	n log n
Ordered List	n	n	n	1	n	n^2
Binary Search Tree	n	n	n	n	n	n^2
AVL Tree	log n	log n	log n	log n	n	n log n
Splay Tree[1]	n	n	n	n	n	n log n
B-tree[2]	log n	log n	log n	log n	n	n log n

1. Splay Trees efficiently supports the additional operations of split and merge.

2. B-trees are suitable for out of memory applications while other data structures are for in memory applications.