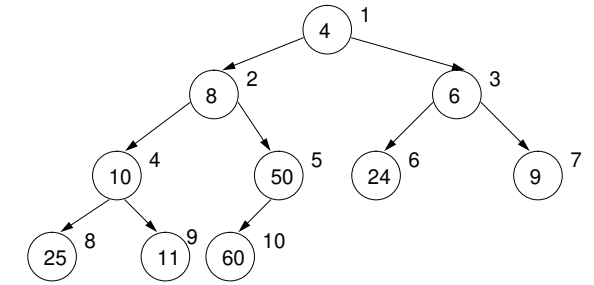
# CSE 326
# Lecture 17

Spring Quarter 1997



## Standard Binary Heap




| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 8 | 6 | 10 | 50 | 24 | 9 | 25 | 11 | 60 | | |

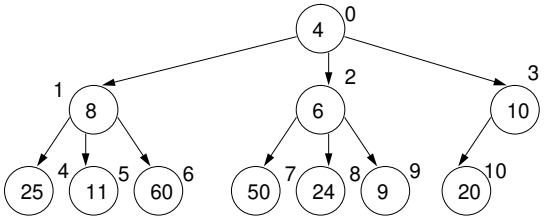


## d-Heaps

- Use a complete d-ary tree.
- Each node has d children instead of just 2.
- Store the root at 0.
- The children of i are stored in di+1, di+2, ..., di+d.
- The parent of i is stored in (i-1)/d



## 3-Heap




| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 8 | 6 | 10 | 25 | 11 | 60 | 50 | 24 | 9 | 20 | |

## Are d-Heaps Better Than Binary Heaps?

- Weiss on d-heaps

  "... the Delete_Min operation is more expensive because, even though the tree is shallower, the minimum of d children must be found, which takes d-1 comparisons using a standard algorithm."

- Weiss is wrong, probably 4-heaps are best.

  1. Theory says that 4-heaps are probably better.

  2. 4-heaps take fewer instructions per operation than 2-heaps

  3. 4 heaps can take better advantage of cache memory than 2 heaps



## The Theory

- d-heaps are better than binary heaps for insertion both in worst case and average case.
- For some d, the d-heap is better than binary heaps for delete_min.

  Assuming that all delete-min's goes to the leaves the time to delete_min is

  $$(ad + b) \log_d N$$

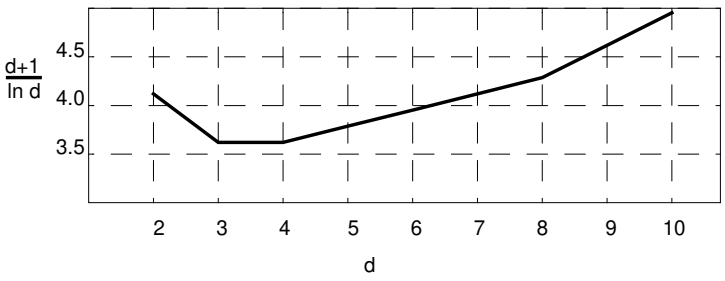
  For simplicity let a = b = 1, then the time per delete_min is

  $$(d + 1) \log_d N = (d + 1) \ln N / \ln d$$

| d | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| (d+1) / ln d | 4.33 | 3.64 | 3.61 | 3.73 | 3.91 | 4.11 | 4.33 | 4.55 | 4.78 |



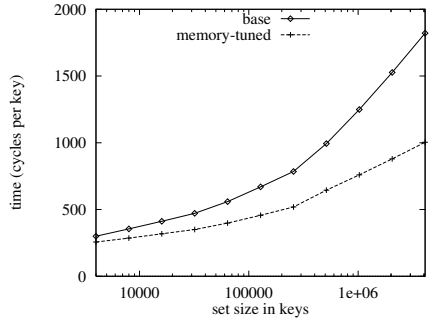
## Plot of d vs. (d+1) / ln d





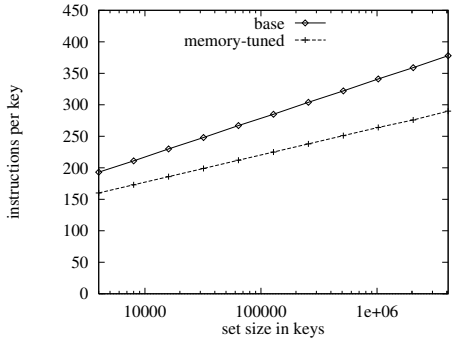

## Heapsort Execution Time
## 2-Heap vs. 4 Heap

- Sorting 8 byte integers, randomly chosen
- DEC alpha 250 with 2MB level 2 cache



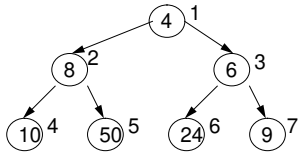
## Heapsort Instruction Count
## 2-Heap vs. 4-heap

- Atom simulator on Dec alpha 250

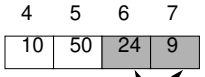




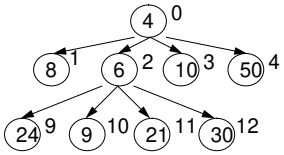

## Fully Utilizing Cache Blocks



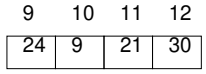

cache block read
when visiting node 2




unused keys




cache block read
when visiting node 2

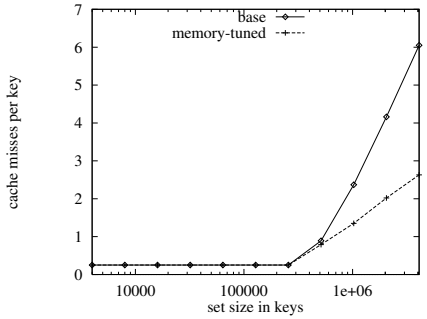



all of cache block used



## Heapsort Cache Misses
## 2-Heap vs. 4-Heap

- Atom simulator on DEC alpha 250
- 32 byte cache blocks, 4 integers per block






## Heaps on Heaps

- The binary heap

  + simple, elegant, and works great
  + building a binary heap can be done in O(n) time
  + insertion: O(log n) time worst case and constant time on average
  + delete_min: O(log n) time worst case
  + supports additional operations of decrease, increase, delete
  - may be somewhat inflexible in some applications

- The d-heap

  + has all the advantages of binary heaps
  + in some applications where performance is critical a d-heap is a better choice than a binary heap
  - slightly more complicated than binary heap
  - may also be inflexible in some applications