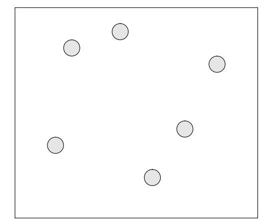
CSE 326
Lecture 13

Spring Quarter 1997

Multidimensional Search Trees

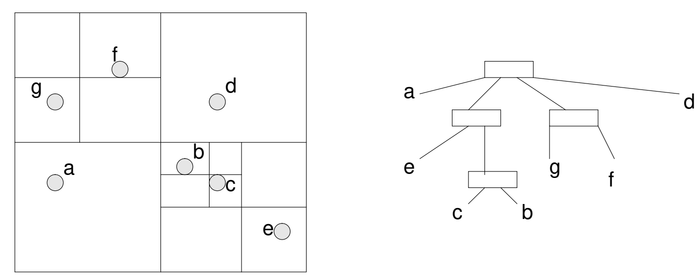
- Example: two dimensions. We want to store a set of ordered pairs (x_1,y_1), (x_2,y_2), (x_3,y_3), ... , (x_n,y_n). There is no specific order to the points as there is with say integers or character strings.



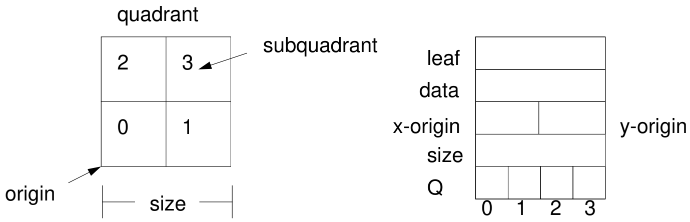
Application of Multidimensional Search

- Astronomy - simulation of galaxies, 3 dimensions.
- Protein folding in molecular biology - 3 dimensions.
- Lossy data compression - 4 to 16 dimensions.
- Image processing - 2 dimensions
- Graphics - 2 or 3 dimensions
- Animation-video - 3 to 4 dimensions
- Geographical Databases - 2 or 3 dimensions
- Web searching - 200 dimensions

Quad Trees



Quad Tree Node



- leaf is boolean. If leaf is true then data holds data or pointer to data.
- origin is the bottom left corner of the quadrant.
- size is the width and height of the quadrant.
- Q is an array which contains pointers to other active subquadrants.

Find in Quad Trees

- Find(x,y,n) finds the node which has the pair (x,y) in it and returns null if there is no such node.
- Find(x,y,n) {

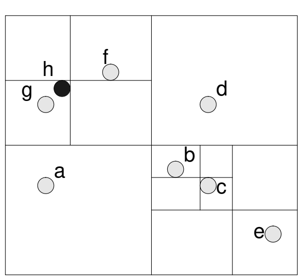
```
case
   n = null: return null;
   n.leaf = true:  if n.data = (x,y) then return(n) else return null
   otherwise:   return(find(x,y,Q[quadrant(x,y,n)])
endcase
```

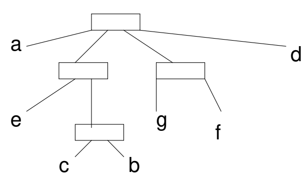
- quadrant(x,y,n) {

  return((x ≥ n.x_origin + size/2) + 2(y ≥ n.y_origin + size/2)) }

Insert

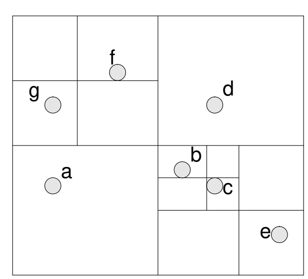
- Insert - Find the leaf quadrant where point belongs.
  If quadrant occupied then split into new quadrants, this may propagate

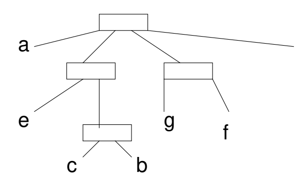




Delete

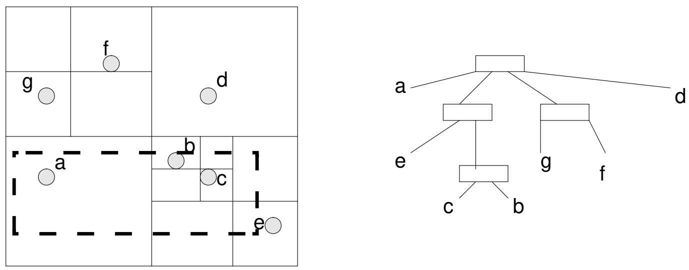
- Delete - Find the point in its leaf quadrant.
  Delete the point.
  If the parent has one point, then delete the parent.
  This may propagate.





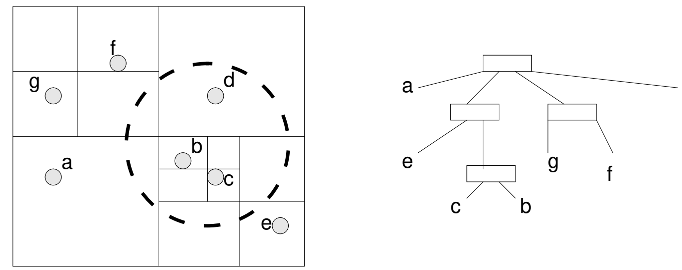
Rectangular Range Query

- Given a rectangle, find all the points inside the rectangle

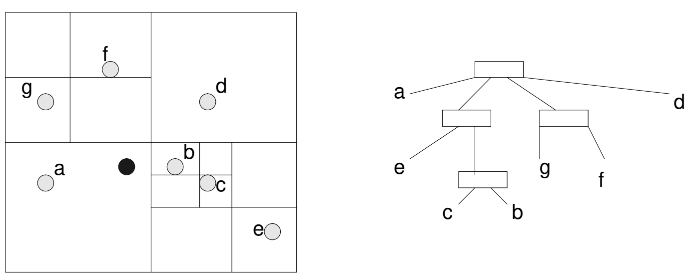


Circular Range Query

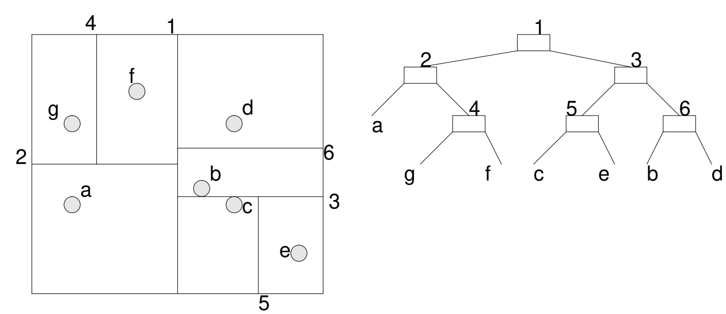
- Given a circle, find all the points inside the circle.



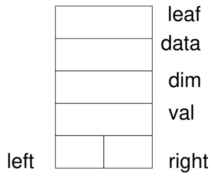
Nearest Neighbor Search



k-d Trees



K-d Trees



- Unlike Quad Trees, the decision has to be made in which dimension to split the data, and where within that dimension the split line should lie. This depends on the application somewhat but a good policy is:
  1. Choose the dimension with the widest spread.
  2. Split the points approximately in half.

Find in K-d Trees

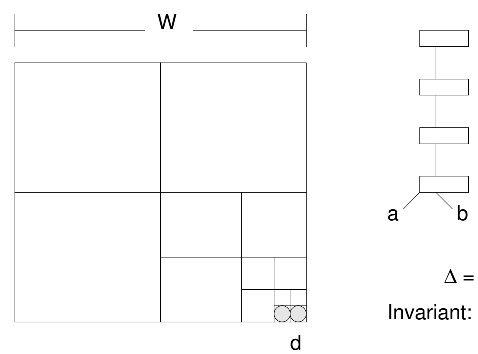
- Find(x,y,n) finds the node which has the pair (x,y) in it and returns null if there is no such node.
- Find(x,y,n) {

```
case
   n = null: return null
   n.leaf = true:  if n.data = (x,y) then return(n) else return null
   otherwise:
   case
    n.dim =1: if x < n.val then return(Find(x,y,n.left))
                 else return(Find(x,y,n.right))
    n.dim =2: if y < n.val then return(Find(x,y,n.left))
                 else return(Find(x,y,n.right))
   endcase
endcase
```


Quad Trees vs. K-d Trees

- Quad trees - spacially balanced trees
  - Number of nodes is $O(n(1 + \log \Delta/n))$ where n is the number of points and Δ is the ratio of the width original quadrant and the smallest distance between two points.
  - Height of the tree is $O(\log n + \log \Delta)$.
  - Supports insert, delete, find, nearest neighbor, range queries
  - For higher dimensions are somewhat bulky
- K-d trees - density balanced trees
  - Number of nodes is O(n) where n is the number of points
  - Height of the tree is O(log n)
  - Supports insert, delete, find, nearest neighbor, range queries.
  - Leaner in higher dimensions

Pathological Example



$\Delta = W/d$

Invariant: $\Delta \geq \text{sqrt}(2)/2$