CSE 326 Problem Set 6 November 10, 1997

This problem set is not to be handed in, but solving the problems will help you prepare for the final exam.

1. Determine the number of comparisons performed by Shellsort when the input array is already sorted into increasing order and the sequence of increments is $h_1, h_2, \cdots, h_t$.

2. A sorting algorithm is *stable* if elements with equal keys are left in the same order as they occur in the input. Which of the following sorting algorithms are stable: Insertion sort, Shellsort, Heapsort, Quicksort?

3. Prove that 7 comparisons are required in the worst case to sort 5 items.

4. Recall the Twenty Questions model that we used to derive a lower bound of $\lg n!$ on the worst-case number of comparisons for sorting. Using the same approach, prove that any algorithm based on comparisons for finding a given element x in an ordered list of n elements requires at least $\lceil \lg n \rceil$ comparisons in the worst case.

5. The following sequence of keys is to be partitioned into increasing runs using replacement selection: $81, 72, 4, 96, 24, 82, 66, 14, 77, 17, 13, 87, 79, 95$.A priority queue with space for 4 keys is to be used. Give the resulting runs.

6. A set of records of total length 10^7 characters is to be sorted using replacement selection followed by P-way merging. Replacement selection partitions the records into 200 runs of approximately equal length. Let us suppose that:

   (a) It takes $70 + .005n$ milliseconds to transfer n consecutive characters between disk and main memory;

   (b) $100,000$ characters of main memory are available for working storage;

   (c) .004 milliseconds of computation time are required to transmit each character from input to output;

   (d) There is to be no overlap between reading, writing or computing.

   Determine the optimal value of P and the time required for the sorting process.

7. Find a topological ordering for the digraph in Weiss, Fig. 9.79. Ignore the weights on the edges.

8. Devise an algorithm with worst-case execution time $O(\mid V \mid + \mid E \mid)$ to determine whether a digraph $G = (V, E)$ is acyclic.

9. Give an algorithm with worst-case execution time $O(|V| + |E|)$ to determine, for each vertex v in the digraph $G = (V, E)$, the number of shortest paths from the source vertex s to v. Here the length of a path is defined as the number of edges it contains.