CSE 326 Problem Set 3 Due: November 3, 1997

1. The height of a tree is defined as the maximum number of edges in a path from the root to a leaf. By convention, the height of an empty tree is -1. A k-*AVL tree* is a binary search tree in which the heights of the left subtree and the right subtree of any node differ by at most k. In particular, the AVL trees that we have discussed in lecture are the 1-AVL trees.

   (a) Let b_h be the minimum number of nodes in a 2-AVL tree of height h. Give a recurrence relation for b_h.

   (b) For suitable constants A, x and B, where $A > 0$ and $x > 1$, prove: for all $h \geq 1$, $b_h \geq Ax^h + B$; x can be chosen as the largest root of a suitable polynomial of degree three.

   (c) Prove that the maximum possible height of a 2-AVL tree with n nodes is $O(\log n)$.

2. Let T be the tree shown below.

```
          20
         /  \
        10   30
       / \  / \
      5 15 25 35
     / / \     \
    3 12 18     40
      /
     11
```

   (a) Treating T as an AVL tree, show the steps in performing the operation $DELETE(20, T)$.

   (b) Treating T as a splay tree, show the steps in performing the operation $DELETE(11, T)$.

3. The zig-zig and zig-zag operations in splay trees can each be performed by the successive application of two single rotations (as defined in Weiss, Section 4.4.1). In splay trees zig-zag and zig-zig operations are done very often, so it makes sense to hand optimize the code.

   (a) Design zig-zag and zig-zig algorithms which minimize the number of pointer assignments. Assume you start with a pointer to the top node in the structure to be zig-zaged or zig-ziged. For each algorithm there are two mirror image cases (right zig-zig and left zig-zig, for example), and you may omit the details of one of the two.

(b) Compare the numbers of pointer assignments in your algorithm with the numbers of pointer assignments in algorithms that do two single rotations.

4. Describe the trees which result from the following sequences of operations:

   (a) Starting with an empty AVL tree, the keys $1, 2, 3, \cdots, 2^k - 1$ are inserted in increasing order.

   (b) Starting with an empty splay tree, the keys $1, 2, 3, \cdots, 2^k - 1$ are inserted in increasing order. Then the operation $find(2^k - 1)$ is performed.