CSE 326
Lecture 11

Spring Quarter 1995

Computing the Height of Each Node

- We have a new field "height" for each node

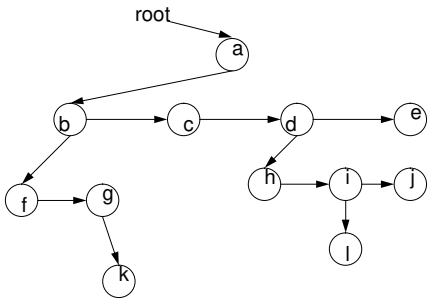
```
Height(n) {

local node pointer c;

if not(n=null) then
  if n.first_child = nil then n.height := 0 else {
     h := 0;
     c := n.first_child;
     while not (c=null) do {
        Height(c);
        h := max(h,c.height);
        c := c.next_sibling};
     n.height := h +1 }}}
```

- Call Height(root) to correctly set all height fields

Height Example



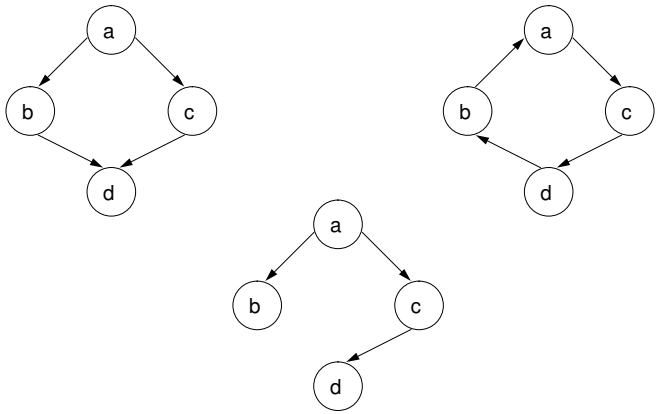
Depth First Search vs. Tree Traversal

- Preorder tree traversal is essentially depth first search of a tree.
- Tree Traverse a node:

  Visit the node
  Recursively Tree Traverse the children of the node
- Depth First Search a node:

  Visit the node
  Mark the node
  Recursively Depth First Search the unmarked nodes adjacent to the node.
- Since Depth First Search is searching a graph there is the possibility that an attempt might be made to visit a node more than once. Thus, in Depth First Search we must mark node that have already been visited.
- There is no need to mark nodes in the Tree Traversal because there is no possibility that a node will be visited a second time.

Examples



Binary Search Tree

- A binary search tree has a number at each node and the tree has the following property.

  For every node n, all the numbers in the left subtree of n are less than or equal to the number at n, and all the numbers in the right subtree of n are greater than or equal to the number at n.

  Concisely:

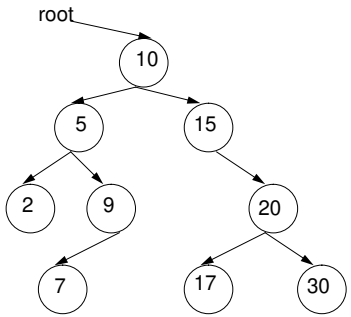
  left subtree of node < node < right subtree of node

- Why have binary search trees?

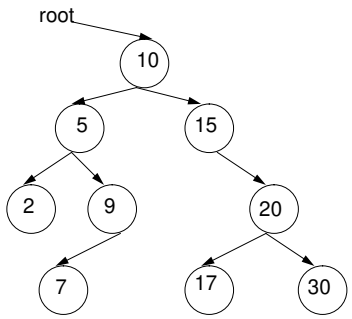
  1. Dynamic like a list - can handle inserts and deletes.

  2. Potentially efficient like a sorted array - can do fast search, O(log n) time.

Example Binary Search Tree



Example of Listing in Order



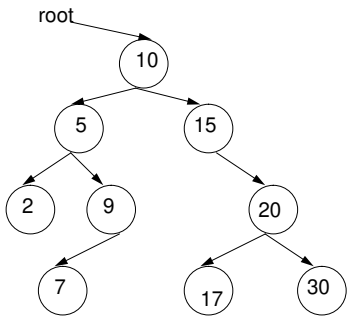
Listing the Data in Order

- List(n)

  ```
  if not(n=null) then {
    List(left_child(n));
    print(n.data);
    List(right_child(n)) }}
  ```

- Call List(root) to list all the data in order.

Search Example



Search

- Find(k,n) returns a pointer to the node with data k in the subtree rooted at n or null if there is no such node.
- Find(k,n) {

  ```
  case:
    n = null: return(null);
    n.data = k : return(n);
    n.data > k : return(Find(k,n.left_child));
    n.data < k : return(Find(k,n.right_child))
  endcase}
  ```

- Call Find(k,root) to search the entire tree.

Iterative Searching

- Find(k) {

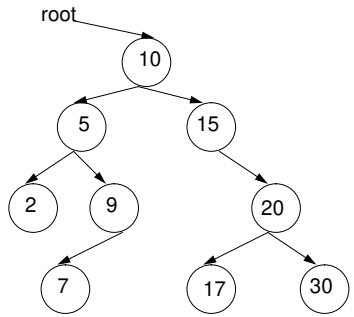
  ```
  local node pointer n, boolean not_found;

  n := root;
  found := false;
  while not(found) do {
    case:
      n = null: found := true;
      n.data = k : found := true;
      n.data > k : n := n.left_child;
      n.data < k : n := n.right_child;
    endcase}
  return(n)}
  ```

- End recursion principle:

  An end recursion is a recursive call after which nothing in done by the caller. Such recursions can be eliminated by making forming a loop and making the proper assignments to the parameters.

Insert Example



Insert

- Insert(k,n) inserts a new node in the subtree rooted at n with data equal to k. If there is already such a node nothing is done.
- Insert(k,n) {

```
case
  n = null: root := New_node(k);
  n.data > k : if n.left_child = null then n.left_child := New_node(k)
               else Insert(k,n.left_child));
  n.data < k : if n.right_child = null then n.right_child := New_node(k)
               else Insert(k,n.right_child));
endcase}
```

- New_node(k) returns a pointer to

Building a Binary Search Tree From Scratch

- Suppose the data $a_1, a_2, a_3, ..., a_n$ is inserted into an initially empty binary search tree. What can happen.

1. $a_1, a_2, a_3, ..., a_n$ are in increasing order.
2. $a_1, a_2, a_3, ..., a_n$ are in decreasing order
3. a_1 is the median of all, a_2 is the median of elements less than a_1, a_3 is the median of elements greater than a_1, etc.
4. $a_1, a_2, a_3, ..., a_n$ are randomly ordered.

Examples of Building from Scratch

- 1, 2, 3, 4, 5, 6, 7, 8, 9
- 5, 3, 7, 2, 4, 6, 8, 1, 9

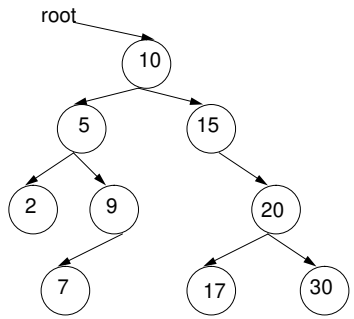
Analysis of Building a Binary Search Tree

- Worst case is $O(n^2)$.

$$1 + 2 + 3 + \ldots + n = O(n^2)$$

- Average case is O(n log n) assuming all orderings of insertions are equally likely.

Examples of Finding Minimum and Maximum



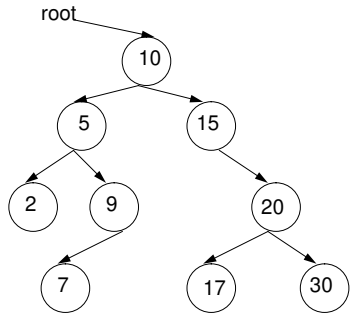
Minimum and Maximum

- Min(n) returns a pointer to the node in the subtree rooted at n with the smallest data.
- Min(n) {

  if n.left = null then return(n)
  else return Min(n.left) }
- To find the minimum in the entire tree call Min(root).
- Max(n) is similar but looking right instead of left.
- Min and Max exhibit end recursion so iterative programs are easy.
- Min(n) {

  while not(n.left = null) do n := n.left;
  return(n)D}

Finding the Successor and Predecessor



Successor and Predecessor

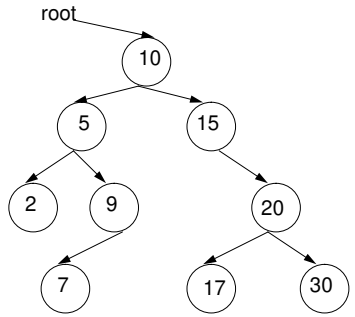
- Successor(n) finds the node whose data is next largest after n's data in the subtree rooted at n. Assume that n has a successor.
- Successor(n) {

  return(Min(n.right)) }
- Iterative version

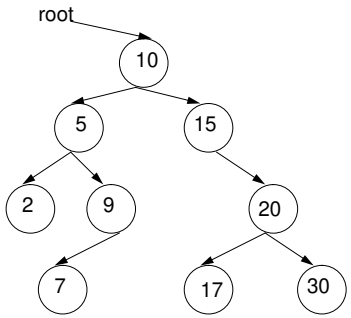
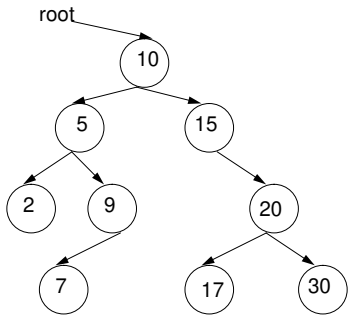
  Successor(n) {

  n := n.right;
  while not(n.left = null) do n := n.left;
  return(n) }
- For Predecessor just reverse the roles of left and right.

Deletion - Leaf Case



Deletion - One Child Case



Deletion - Two Child Case