Graphs II
Chapter 9 in Weiss

CSE 326
Data Structures
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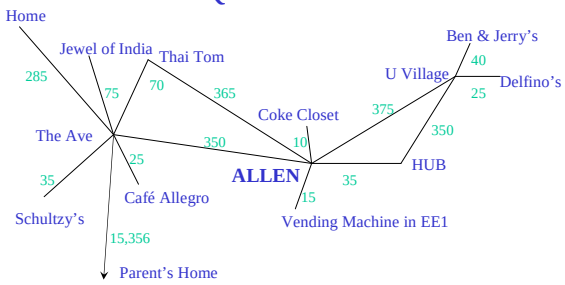
Today's Outline

- **Announcements**
  - **Project 3 Code due Wed March 3 by 11pm**
  - **Written Homework #7 due Fri March 5**
  - **Project 3 Benchmarking & Written (and Above & Beyond) due Fri March 5 by 11pm**
- **Today's Topics:**
  - **Graphs**
    - **Shortest Path Algorithms**
      - **Dijkstra's Algorithm**

Graphs so far

- Representations
- Topological Sort
- Finding paths
  - DFS
  - BFS
  - Dijkstra

Weighted SSSP: The Quest For Food



Can we calculate shortest distance to all nodes from Allen Center?

Dijkstra, Edsger Wybe

Legendary figure in computer science; was a professor at University of Texas.

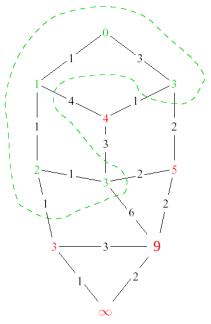
Supported teaching introductory computer courses without computers (pencil and paper programming)

Supposedly wouldn't (until very late in life) read his e-mail; so, his staff had to print out messages and put them in his box.

E.W. Dijkstra (1930-2002)

1972 Turning Award Winner, Programming Languages, semaphores, and …

Dijkstra's Algorithm: Idea

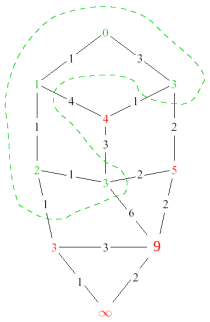


Adapt BFS to handle weighted graphs

Two kinds of vertices:

- Finished or known vertices
  - Shortest distance has been computed
- Unknown vertices
  - Have tentative distance

Dijkstra's Algorithm: Idea



At each step:

1) Pick closest unknown vertex
2) Add it to known vertices
3) Update distances

Dijkstra's Algorithm: Pseudocode

Initialize the cost of each node to ∞

Initialize the cost of the source to 0

While there are unknown nodes left in the graph
- Select an unknown node *b* with the lowest cost
- Mark *b* as known
- For each node *a* adjacent to *b*
  - *a*'s cost = min(*a*'s old cost, *b*'s cost + cost of (*b*, *a*))

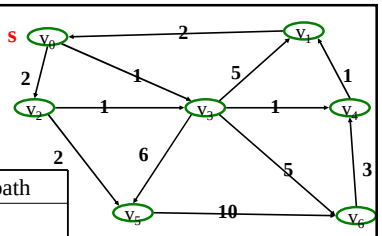

```
void Graph::dijkstra(Vertex s){
  Vertex v,w;

  Initialize s.dist = 0 and set dist of all other
  vertices to infinity

  while (there exist unknown vertices, find the
  one b with the smallest distance)
    b.known = true;

    for each a adjacent to b
      if (!a.known)
        if (b.dist + Cost_ba < a.dist){
          decrease(a.dist to= b.dist + Cost_ba);
          a.path = b;
        }
  }
}
```

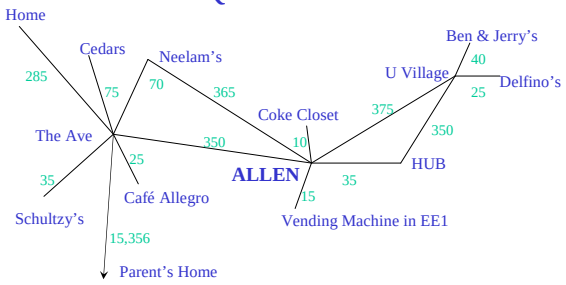

Activity



V	Known	Dist	path
v0			
v1			
v2			
v3			
v4			
v5			
v6			

Order declared Known:

Weighted SSSP: The Quest For Food



If just wanted to know shortest path to Ben and Jerry's could stop once Ben and Jerry's is "known"

Dijkstra's Alg: Implementation

Initialize the cost of each node to ∞

Initialize the cost of the source to 0

While there are unknown nodes left in the graph
- Select the unknown node *b* with the lowest cost
- Mark *b* as known
- For each node *a* adjacent to *b*
  - *a*'s cost = min(*a*'s old cost, *b*'s cost + cost of (*b*, *a*))

Running time?

Dijkstra's Algorithm: a Greedy Algorithm

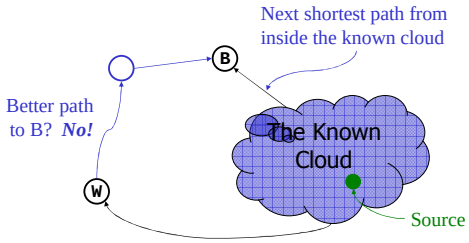
Greedy algorithms always make choices that *currently* seem the best

- Short-sighted – no consideration of long-term or global issues
- Locally optimal - does not always mean globally optimal!!

Dijkstra's Algorithm: Summary

- Classic algorithm for solving SSSP in weighted graphs *without negative weights*
- A *greedy* algorithm (irrevocably makes decisions without considering future consequences)
- Intuition for correctness:
  - shortest path from source vertex to itself is 0
  - cost of going to adjacent nodes is at most edge weights
  - cheapest of these must be shortest path to that node
  - update paths for new node and continue picking cheapest path

Correctness: The Cloud Proof



How does Dijkstra's decide which vertex to add to the Known set next???

- If path to **B** is shortest, path to **W** must be *at least as long* *(or else we would have picked **W** as the next vertex)*
- So any path *through* **W** to **B** *cannot* be any shorter!

Correctness: Inside the Cloud

Prove by induction on # of nodes in the cloud:

Initial cloud is just the source with shortest path 0

Assume: Everything inside the cloud has the correct shortest path

Inductive step: Only when we prove the shortest path to some node ***v*** (which is *not* in the cloud) is correct, we add it to the cloud

When does Dijkstra's algorithm not work?

Dijkstra's vs BFS

At each step:

1) Pick closest unknown vertex
2) Add it to finished vertices
3) Update distances

Dijkstra's Algorithm

At each step:

1) Pick vertex from queue
2) Add it to visited vertices
3) Update queue with neighbors

Breadth-first Search

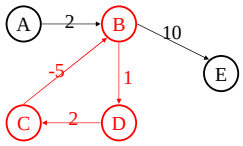
Some Similarities:

Activity

Negative-weight edges

- Why doesn't Dijkstra's work on graphs with negative-weight edges?
- Any ideas on how we could fix this?

The Trouble with Negative Weight Cycles



What's the shortest path from A to E?

Problem?

Analysis

- Total running time for Dijkstra's:
  $O(|V|^2 + |E|)$ (linear scan)
  $O(|V| \log |V| + |E| \log |V|)$ (heaps)

  What if we want to find the shortest path from each point to ALL other points?