

CSE 326 DATA STRUCTURES HOMEWORK 8 - The Last Homework!

Due: **Friday, March 12, 2010** at the beginning of class. Please put your quiz section in addition to your name at the top of your homework.

Problem 1. Topological Sort

Weiss, problem 9.1. For each step, show the in-degree array and the queue.

Problem 2. Dijkstra's Algorithm

- (a) Weiss, problem 9.5(a). Use Dijkstra's algorithm and show the results of the algorithm in the form used in lecture — a table showing for each vertex its known distance from the starting vertex and its predecessor vertex on the path.

Also show the order in which the vertices are added to the “cloud” of known vertices as the algorithm progresses.

- (b) If there is more than one minimum cost path from v to w , will Dijkstra's algorithm always find the one with the *fewest edges*? If not, explain (in a few sentences) how to modify Dijkstra's algorithm so that if there is more than one minimum path from v to w , a path with the fewest number of edges is chosen.
- (c) Give an example where Dijkstra's algorithm gives the wrong answer in the presence of a negative-cost edge but no negative-cost cycles.

Explain why Dijkstra's algorithm fails on the particular example you provide.

- (d) Suppose you are given a graph that has negative-cost edges but no negative-cost cycles. Consider the following strategy to find shortest paths in this graph: uniformly add a constant k to the cost of every edge, so that all costs become non-negative, then run Dijkstra's algorithm and return that result with the edge costs reverted back to their original values (i.e. with k subtracted).

Give an example where this technique fails and explain why it does so. (Hint: one simple example uses only three vertices.) Also, give a general explanation as to why this technique does not work.

Problem 3. Minimum Spanning Trees

- (a) Weiss, problem 9.15(a). For Prim's algorithm, start with vertex A , show the resulting table (see Figure 9.55 as an example), and indicate the order in which vertices are added. For Kruskal's algorithm, produce a table, similar to Figure 9.56. Ties may be broken arbitrarily.
- (b) Weiss, problem 9.15(b).
- (c) Under what conditions is a Minimum Spanning Tree unique? This should require only a sentence or two to explain.