CSE 326: Data Structures

Skew Heaps

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Announcements (4/11/08)

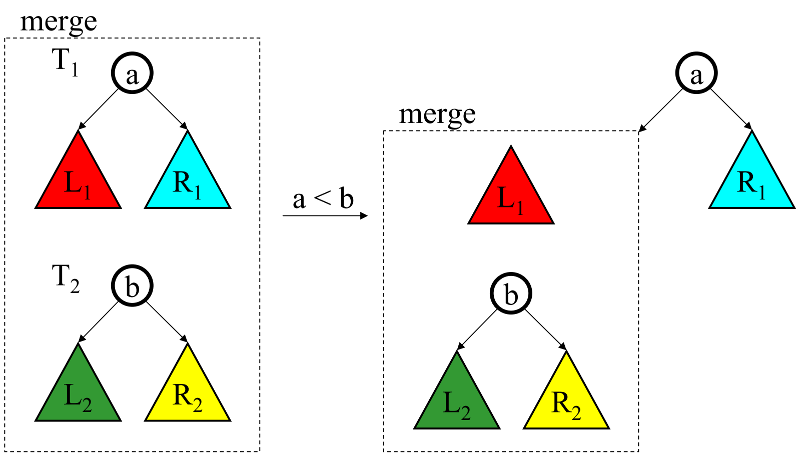
- HW 1 due now
- HW 2 out today, due next Friday
- Project #2 Phase A out now
  - Partner sign-ups by 11:59pm today

Merge

- Useful operation for priority queues
- Simplifies heap implementation
  - Implement other ops in terms of merge

How to Merge Two Binary Heaps?

Dropping the Structure Property



worst case:

Amortized Complexity

Suppose you run M times and average the running times

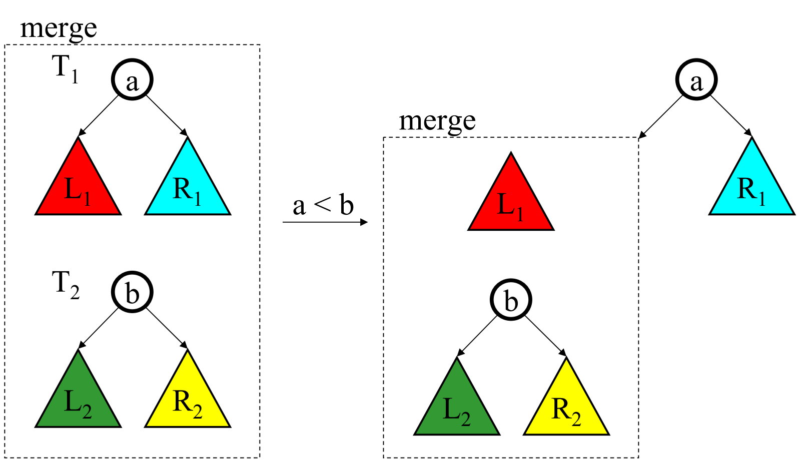
– Does it get better over time?

Amortized complexity:

max total # steps algorithm takes, in the worst case, for **M** *consecutive* operations on inputs of size **N**, divided by **M** (i.e., divide the max total by **M**).

Example: if M operations take total O(M log N) time in the worst case, *amortized* time per operation is O(log N).

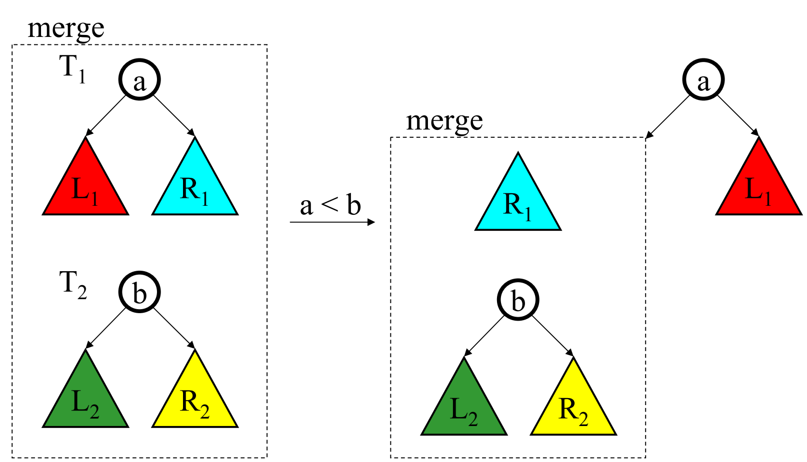
Does it get better over time?



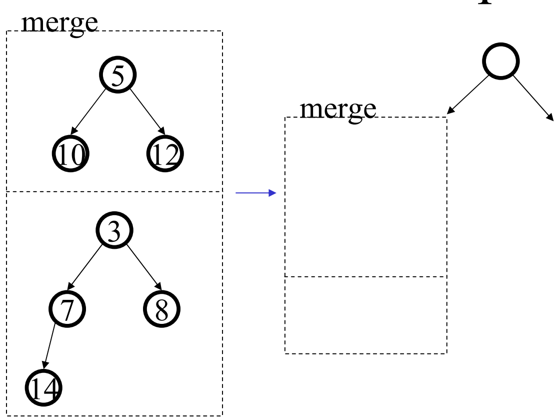
amortized worst case:

Skew Heaps

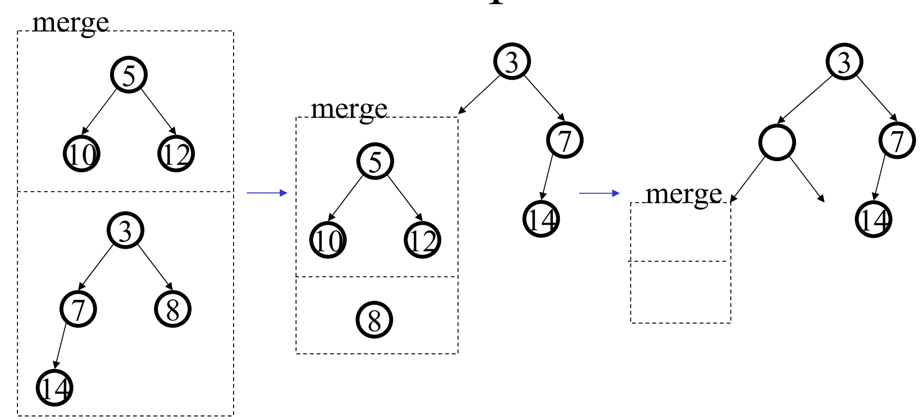
swap left-right subtrees of ⓐ before merge



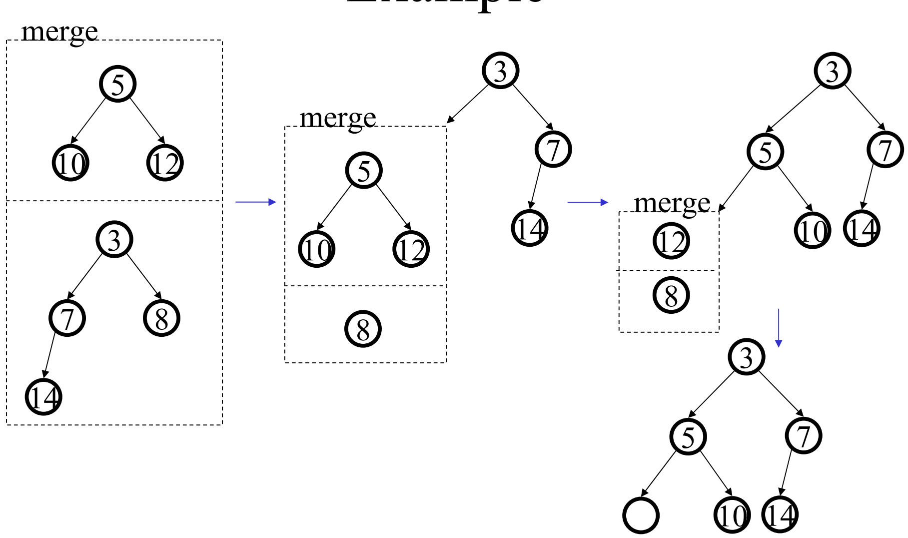
Example



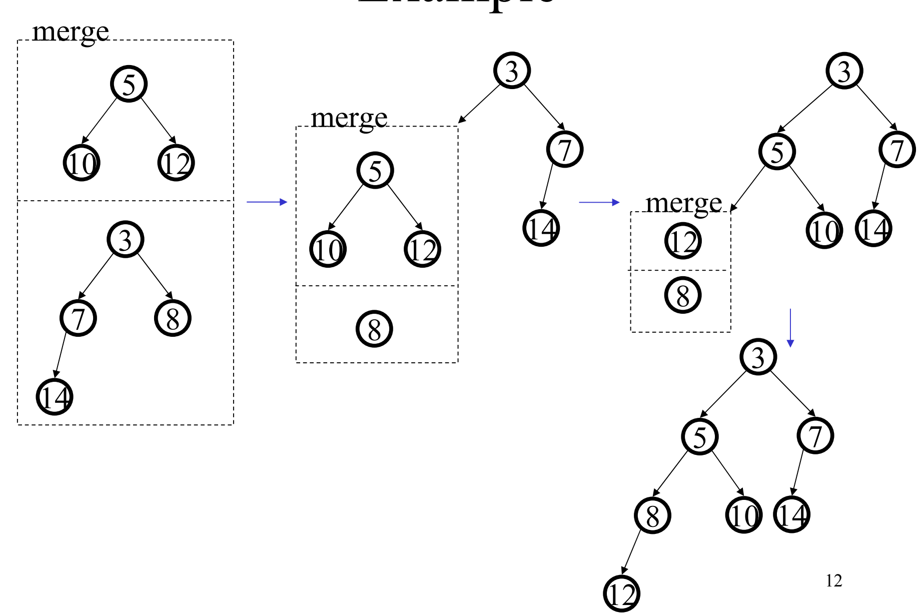
Example



Example



Example



Runtime Analysis

- All operations rely on merge

  ⇒ worst case complexity of all ops =

- It is known: M merges take time $\Theta(M \log n)$ in the worst case

  ⇒ amortized complexity of all ops =

Skew Heap Code

```
SkewHeap merge(heap1, heap2) {
  case {
     heap1 == NULL: return heap2;
     heap2 == NULL: return heap1;
     heap1.findMin() <= heap2.findMin():
          temp = heap1.right;
          heap1.right = heap1.left;
          heap1.left = merge(heap2, temp);
          return heap1;
     otherwise:
          return merge(heap2, heap1);
  }
}
```