CSE 326

How to Explain Inverse Ackerman's Function in a Data Structures Class

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Option 1

DON'T

Option 2

- Ackerman's function is defined as

$$A(m,n) = \begin{cases} n+1 & \text{if } m = 0 \\ A(m-1,1) & \text{if } m > 0 \text{ and } n = 0 \\ \mathrm{A(m-1, A(m,n-1))} & \text{if } m > 0 \text{ and } n > 0 \end{cases}$$

- Therefore, its functional inverse, $\alpha(m,n)$, is

$$\alpha(m,n) = \min\{i \geq 1 \mid A(i, \lfloor m/n \rfloor) > \log n\}$$

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- That should make things clear

Option 3

A(m,n) grows veeeeery quickly

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Now let's move on to the next topic

Credit: Raimund Seidel, Understanding the Inverse Ackerman Function, EWCG '06 invited talk: http://cgi.di.uoa.gr/~ewcg06/invited/Seidel.pdf

Option 4

- Simplify a bit:
  - Define $f(n) = A(n,n)$
  - Define $\alpha(n) = f^{-1}(n)$
  - Then $\alpha(f(n)) = n$

Ackerman Values (Wikipedia)

Values of $A(m, n)$

$m \backslash n$	0	1	2	3	4	n
0	1	2	3	4	5	$n+1$
1	2	3	4	5	6	$n+2$
2	3	5	7	9	11	$2n+3$
3	5	13	29	61	125	$2^{(n+3)}-3$
4	13	65533	$2^{65536}-3$	$2^{2^{65536}}-3$	$A(3, A(4, 3))$	$\underbrace{2^{2^{\cdot^{\cdot^{2}}}}}_{n+3 \text{ twos}}-3$
5	65533	$\underbrace{2^{2^{\cdot^{\cdot^{2}}}}}_{65536 \text{ twos}}-3$	$A(4, A(5, 1))$	$A(4, A(5, 2))$	$A(4, A(5, 3))$	$A(4, A(5, n-1))$
6	$A(5, 1)$	$A(5, A(6, 0))$	$A(5, A(6, 1))$	$A(5, A(6, 2))$	$A(5, A(6, 3))$	$A(5, A(6, n-1))$

$\alpha(n) = f^{-1}(n)$
$f(n) = A(n,n)$

But it's easier to understand log*(n)

log* x = number of times you need to compute log to bring value down to at most 1

E.g. log* 2 = 1

log* 4 = log* 2^2 = 2

log* 16 = log* 2^{2^2} = 3 (log log log 16 = 1)

log* 65536 = log* $2^{2^{2^2}}$ = 4 (log log log log 65536 = 1)

log* 2^{65536} = …………… = 5

$\alpha(n)$ grows even slower than this

Option N

Move on to the next topic