CSE 326: Data Structures Sorting

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Lecture 17-18

Sorting: *The Big Picture*

Given n comparable elements in an array, sort them in an increasing (or decreasing) order.

Simple algorithms: $O(n^2)$	Fancier algorithms: $O(n \log n)$	Comparison lower bound: $\Omega(n \log n)$	Specialized algorithms: $O(n)$	Handling huge data sets
Insertion sort Selection sort Bubble sort Shell sort …	Heap sort Merge sort Quick sort …		Bucket sort Radix sort	External sorting

Insertion Sort: Idea

- At the k^{th} step, put the k^{th} input element in the correct place among the first k elements
- Result: After the k^{th} step, the first k elements are sorted.

Runtime:

worst case :

best case :

average case :

Selection Sort: idea

- Find the smallest element, put it 1st
- Find the next smallest element, put it 2nd
- Find the next smallest, put it 3rd
- And so on …

Selection Sort: Code

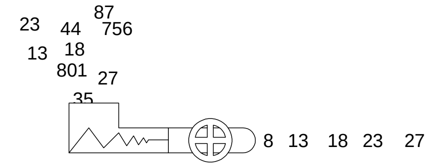
```
void SelectionSort (Array a[0..n-1]) {
      for (i=0, i<n; ++i) {
            j = Find index of smallest entry in a[i..n-1]
            Swap(a[i],a[j])
      }

}
```

Runtime:

- worst case :
- best case :
- average case :

HeapSort: Using Priority Queue ADT (heap)



Shove all elements into a priority queue,
take them out smallest to largest.

Runtime:

Merge Sort

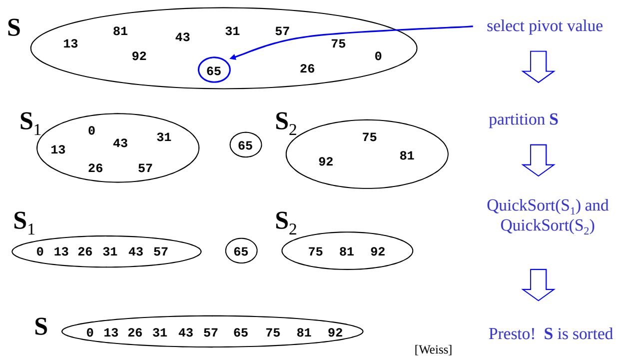
```
MergeSort (Array [1..n])
1. Split Array in half
2. Recursively sort each half
3. Merge two halves together
```

```
Merge (a1[1..n],a2[1..n])
i1=1, i2=1
While (i1<n, i2<n) {
      if (a1[i1] < a2[i2]) {
            Next is a1[i1]
            i1++
      } else {
            Next is a2[i2]
            i2++
      }
}
Now throw in the dregs…
```

“The 2-pointer method”

Merge Sort: Complexity

The steps of QuickSort



QuickSort Example

0	1	2	3	4	5	6	7	8	9
8	1	4	9	**0**	3	5	2	7	**6**

0	1	4	9	7	3	5	2	(6)	8
i								j	

•Choose the pivot as the median of three.

•Place the pivot and the largest at the right and the smallest at the left

QuickSort Example

i								j	
0	1	4	9	7	3	5	2	(6)	8
			i					j	
0	1	4	**9**	7	3	5	2	(6)	8
			i				j		
0	1	4	9	7	3	5	**2**	(6)	8
			i				j		
0	1	4	**2**	7	3	5	**9**	(6)	8

•Move i to the right to be larger than pivot.
•Move j to the left to be smaller than pivot.
•Swap

QuickSort Example

				i			j		
0	1	4	2	**7**	3	5	9	(6)	8
				i		j			
0	1	4	2	7	3	**5**	9	(6)	8
				i		j			
0	1	4	2	**5**	3	**7**	9	(6)	8
						i j			
0	1	4	2	5	3	**7**	9	(6)	8
					j	i			
0	1	4	2	5	**3**	7	9	(6)	8
					j	i			
0	1	4	2	5	3	**6**	9	**7**	8

S_1 < pivot   pivot S_2 > pivot

Recursive Quicksort

```
Quicksort(A[]: integer array, left,right : integer): {
pivotindex : integer;
if left + CUTOFF ≤ right then
  pivot := median3(A,left,right);
  pivotindex := Partition(A,left,right-1,pivot);
  Quicksort(A, left, pivotindex – 1);
  Quicksort(A, pivotindex + 1, right);
else
  Insertionsort(A,left,right);
}
```

Don’t use quicksort for small arrays.
CUTOFF = 10 is reasonable (or 6 or…).

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QuickSort: Best case complexity

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QuickSort: Worst case complexity

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QuickSort: Average case complexity

Turns out to be O(n log n)

See Section 7.7.5 for an idea of the proof.
Don’t need to know proof details for this course.

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Features of Sorting Algorithms

- In-place
  - Sorted items occupy the same space as the original items. (No copying required, only O(1) extra space if any.)
- Stable
  - Items in input with the same value end up in the same order as when they began.

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Sort Properties

Are the following:	**stable?**			**in-place?**	
Insertion Sort?	No	Yes	Can Be	No	Yes
Selection Sort?	No	Yes	Can Be	No	Yes
MergeSort?	No	Yes	Can Be	No	Yes
QuickSort?	No	Yes	Can Be	No	Yes

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How fast can we sort?

- Heapsort, Mergesort, and Quicksort all run in O(N log N) best case running time
- Can we do any better?
- No, if the basic action is a comparison.

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Sorting Model

- Recall our basic assumption: we can only compare two elements at a time
  - we can only reduce the possible solution space by half each time we make a comparison
- Suppose you are given N elements
  - Assume no duplicates
- How many possible orderings can you get?
  - Example: a, b, c (N = 3)

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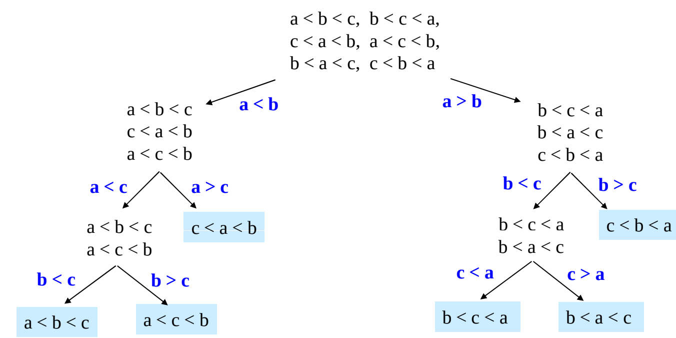
Permutations

- How many possible orderings can you get?
  - Example: a, b, c (N = 3)
  - (a b c), (a c b), (b a c), (b c a), (c a b), (c b a)
  - 6 orderings = 3•2•1 = 3! (ie, "3 factorial")
  - All the possible permutations of a set of 3 elements
- For N elements
  - N choices for the first position, (N-1) choices for the second position, …, (2) choices, 1 choice
  - N(N-1)(N-2)⋯(2)(1)= N! possible orderings

Decision Tree



The leaves contain all the possible orderings of a, b, c

Lower bound on Height

- A binary tree of height h has **at most** *how many* leaves?

  L

- A binary tree with L leaves has height **at least**:

  h

- The decision tree has how many leaves:
- So the decision tree has height:

  h

log(*N*!) is Ω(*N*log*N*)

$$\log(N!) = \log\big(N\cdot(N-1)\cdot(N-2)\cdots(2)\cdot(1)\big)$$
$$= \log N + \log(N-1) + \log(N-2) + \cdots + \log 2 + \log 1$$

select just the first N/2 terms

$$\geq \log N + \log(N-1) + \log(N-2) + \cdots + \log\frac{N}{2}$$

each of the selected terms is ≥ logN/2

$$\geq \frac{N}{2}\log\frac{N}{2}$$
$$\geq \frac{N}{2}(\log N - \log 2) = \frac{N}{2}\log N - \frac{N}{2}$$
$$= \Omega(N\log N)$$

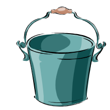
Ω(N log N)

- Run time of any comparison-based sorting algorithm is **Ω(N log N)**
- Can we do better if we don't use comparisons?

BucketSort (aka BinSort)

If all values to be sorted are *known* to be between 1 and *K*, create an array `count` of size *K*, **increment** counts while traversing the input, and finally output the result.

Example *K*=5. Input = (5,1,3,4,3,2,1,1,5,4,5)



`count` array	
1	
2	
3	
4	
5	

Running time to sort n items?

BucketSort Complexity: O(*n*+*K*)

- Case 1: *K* is a constant
  - BinSort is linear time
- Case 2: *K* is variable
  - Not simply linear time
- Case 3: *K* is constant but large (e.g. 2^{32})
  - ???

Fixing impracticality: RadixSort

- Radix = "The base of a number system"
  - We'll use 10 for convenience, but could be anything
- Idea: BucketSort on each **digit**, least significant to most significant (lsd to msd)

Radix Sort Example (1st pass)

Input data

478
537
9
721
3
38
123
67

Bucket sort by 1's digit

0	1	2	3	4	5	6	7	8	9
	721		3 123				537 67	478 38	9

After 1st pass

721
3
123
537
67
478
38
9

This example uses B=10 and base 10 digits for simplicity of demonstration. Larger bucket counts should be used in an actual implementation.

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Radix Sort Example (2nd pass)

After 1st pass

721
3
123
537
67
478
38
9

Bucket sort by 10's digit

0	1	2	3	4	5	6	7	8	9
03 09		721 123	537 38			67	478		

After 2nd pass

3
9
721
123
537
38
67
478

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Radix Sort Example (3rd pass)

After 2nd pass

3
9
721
123
537
38
67
478

Bucket sort by 100's digit

0	1	2	3	4	5	6	7	8	9
003 009 038 067	123			478	537		721		

After 3rd pass

3
9
38
67
123
478
537
721

Invariant: after k passes the low order k digits are sorted.

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RadixSort

- Input:126, 328, 636, 341, 416, 131, 328

BucketSort on lsd:

0	1	2	3	4	5	6	7	8	9

BucketSort on next-higher digit:

0	1	2	3	4	5	6	7	8	9

BucketSort on msd:

0	1	2	3	4	5	6	7	8	9

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Radixsort: Complexity

- How many passes?
- How much work per pass?
- Total time?
- Conclusion?
- In practice
  - RadixSort only good for large number of elements with relatively small values
  - Hard on the cache compared to MergeSort/QuickSort

Internal versus External Sorting

- Need sorting algorithms that minimize disk/tape access time
- **External sorting** – Basic Idea:
  - Load chunk of data into RAM, sort, store this "run" on disk/tape
  - Use the Merge routine from Mergesort to merge runs
  - Repeat until you have only one run (one sorted chunk)
  - Text gives some examples

    (also see CSE 444)