CSE 326: Data Structures
B-Trees

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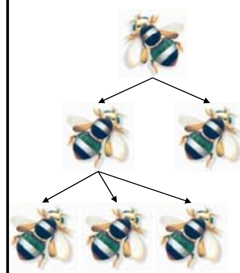
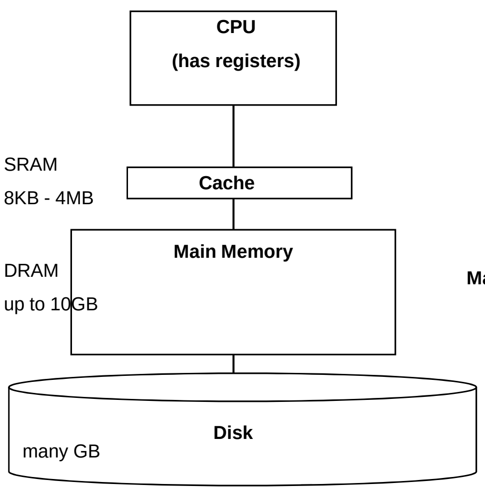
Lecture 14-15

B-Trees



Weiss Sec. 4.7



Time to access (conservative)

1 ns per instruction

Cache

2-10 ns

Main Memory

40-100 ns

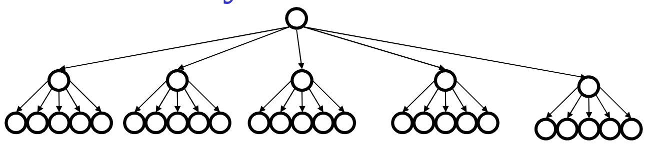
Disk a few *milli*seconds

(5-10 *Million* ns)

Trees so far

- BST
- AVL
- Splay

M-ary Search Tree



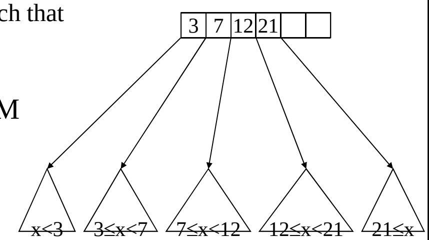
- Maximum branching factor of `M`
- Complete tree has height =

disk accesses for *find*:

Runtime of *find*:

Solution: B-Trees

- specialized *M*-ary search trees
- Each **node** has (up to) M-1 keys:
  - subtree between two keys *x* and *y* contains leaves with *values v* such that $x \leq v < y$



- Pick branching factor M such that each node takes one full {*page*, *block*} of memory

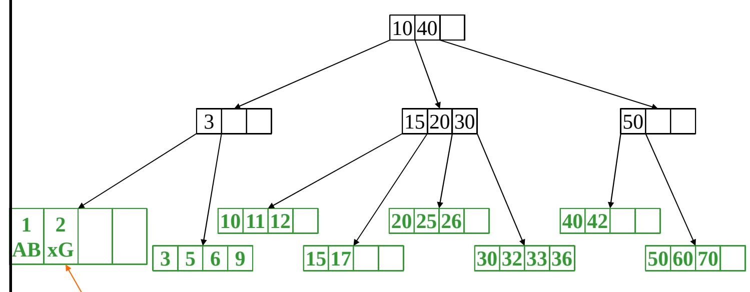
B-Trees

What makes them disk-friendly?

1. **Many keys stored in a node**
   - All brought to memory/cache in one access!
2. Internal nodes contain *only* keys;
   Only leaf nodes contain keys and actual *data*
   - The tree structure can be loaded into memory irrespective of data object size
   - Data actually resides on disk

B-Tree: Example

B-Tree with `M = 4` (# pointers in internal node)
and `L = 4` (# data items in leaf)



Data objects, that I'll ignore in slides

Note: All leaves at the same depth!

B-Tree Properties ‡

– Data is stored at the leaves

– All leaves are at the same depth and contains between $\lceil L/2 \rceil$ and L data items

– Internal nodes store up to M-1 keys

– Internal nodes have between $\lceil M/2 \rceil$ and M children

– Root (special case) has between 2 and ***M*** children (or root could be a leaf)

‡These are technically B+-Trees

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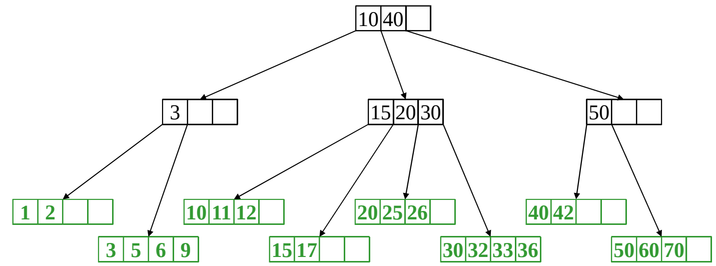
Example, Again

B-Tree with ***M* = 4**

and ***L* = 4**



(Only showing keys, but leaves also have data!)

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B-trees vs. AVL trees

Suppose we have 100 million items (100,000,000):

- Depth of AVL Tree

- Depth of B+ Tree with M = 128, L = 64

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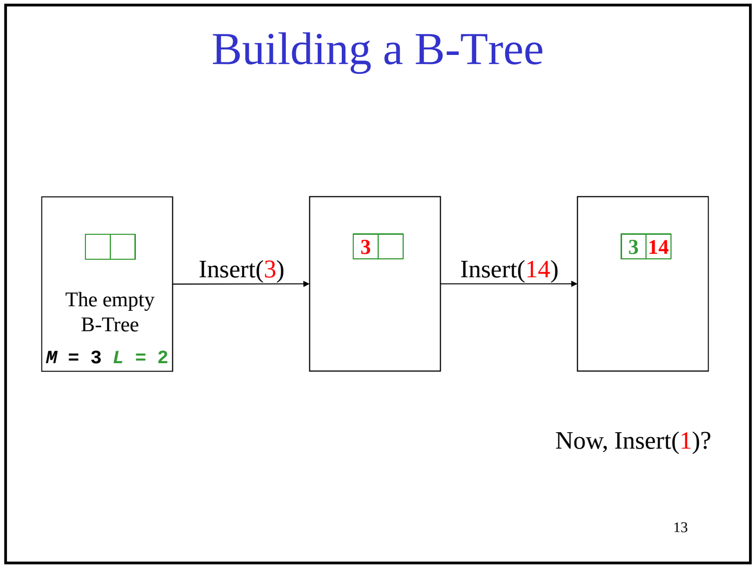
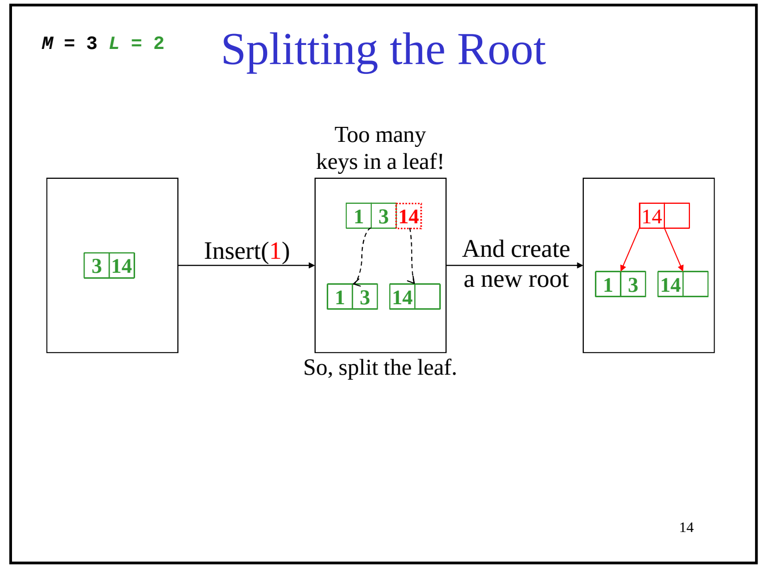
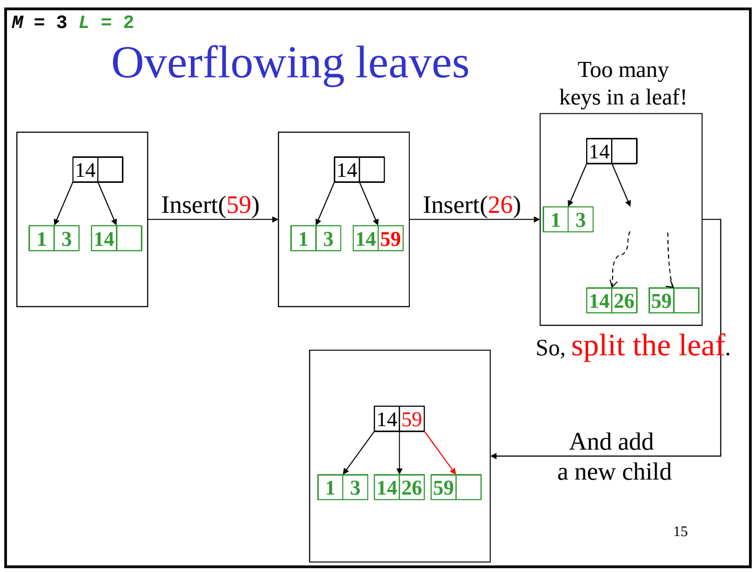
B+ Trees in Practice (From CSE 444)

- Typical order: 100. Typical fill-factor: 67%.
  - average fanout = 133
- Typical capacities:
  - Height 4: 133^4 = 312,900,700 records
  - Height 3: 133^3 = 2,352,637 records
- Can often hold top levels in buffer pool:
  - Level 1 = 1 page = 8 Kbytes
  - Level 2 = 133 pages = 1 Mbyte
  - Level 3 = 17,689 pages = 133 MBytes


Building a B-Tree
The empty B-Tree
M = 3 L = 2
Insert(3)
3
Insert(14)
3 14
Now, Insert(1)?
13


M = 3 L = 2
Splitting the Root
3 14
Insert(1)
Too many keys in a leaf!
1 3 14
1 3 14
So, split the leaf.
And create a new root
14
1 3 14
14


M = 3 L = 2
Overflowing leaves
14
1 3 14
Insert(59)
14
1 3 14 59
Insert(26)
Too many keys in a leaf!
14
1 3
14 26 59
So, split the leaf.
And add a new child
14 59
1 3 14 26 59
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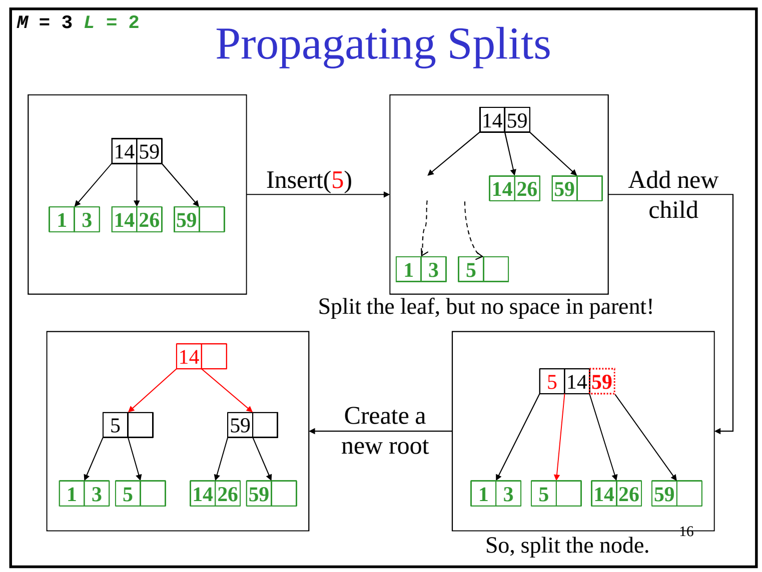
M = 3 L = 2
Propagating Splits
14 59
1 3 14 26 59
Insert(5)
14 59
14 26 59
1 3 5
Add new child
Split the leaf, but no space in parent!
5 14 59
1 3 5 14 26 59
So, split the node.
Create a new root
14
5
59
1 3 5 14 26 59
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Insertion Algorithm

1. Insert the key in its leaf
2. If the leaf ends up with L+1 items, **overflow**!
   - Split the leaf into two nodes:
     - original with $\lceil (L+1)/2 \rceil$ items
     - new one with $\lfloor (L+1)/2 \rfloor$ items
   - Add the new child to the parent
   - If the parent ends up with ***M*+1** items, **overflow**!
3. If an internal node ends up with M+1 items, **overflow**!
   - Split the node into two nodes:
     - original with $\lceil (M+1)/2 \rceil$ items
     - new one with $\lfloor (M+1)/2 \rfloor$ items
   - Add the new child to the parent
   - If the parent ends up with ***M*+1** items, **overflow**!
4. Split an overflowed root in two and hang the new nodes under a new root

This makes the tree deeper!

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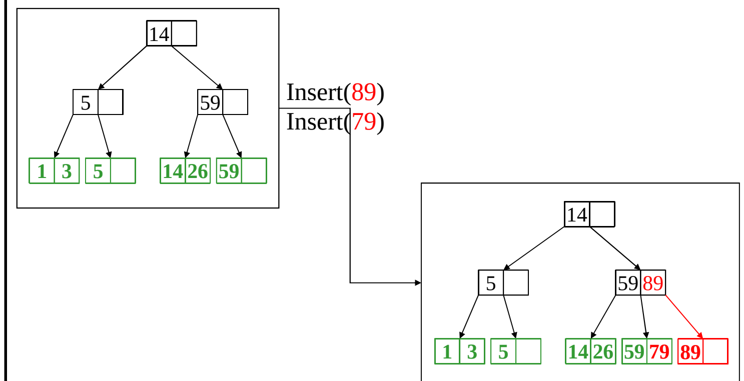
***M* = 3 *L* = 2**

After More Routine Inserts



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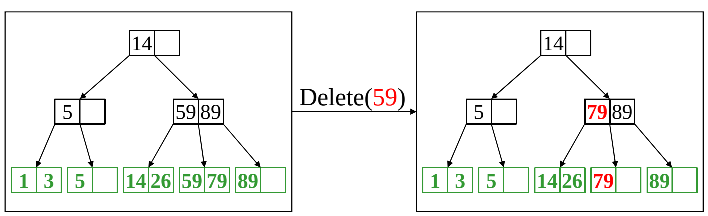
***M* = 3 *L* = 2**

Deletion

1. Delete item from leaf
2. Update keys of ancestors if necessary



What could go wrong?

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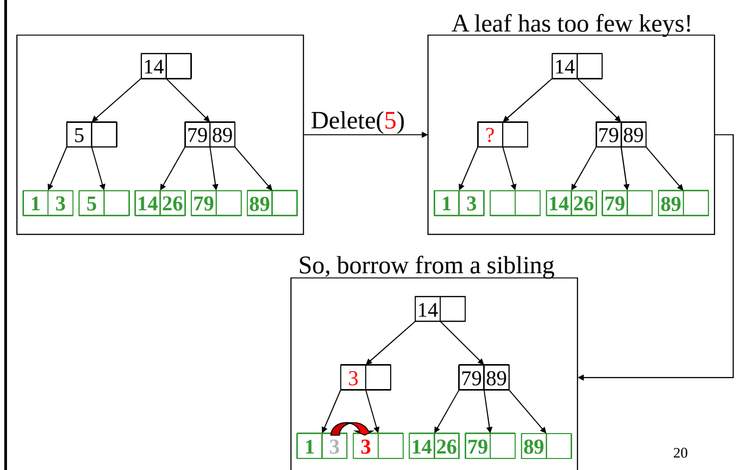
***M* = 3 *L* = 2**

Deletion and Adoption



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Does Adoption Always Work?

- What if the sibling doesn't have enough for you to borrow from?

  e.g. you have $\lceil L/2 \rceil$-1 and sibling has $\lceil L/2 \rceil$?

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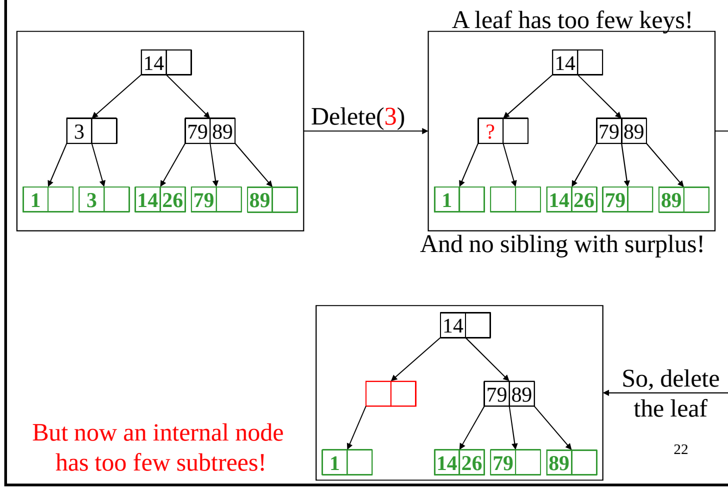
$M = 3$ $L = 2$

Deletion and Merging



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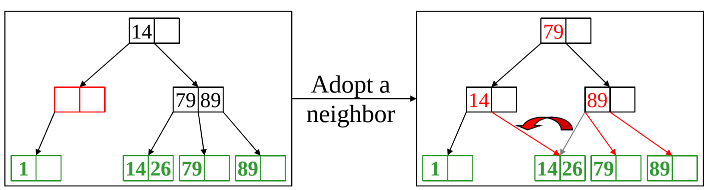
$M = 3$ $L = 2$

Deletion with Propagation (More Adoption)



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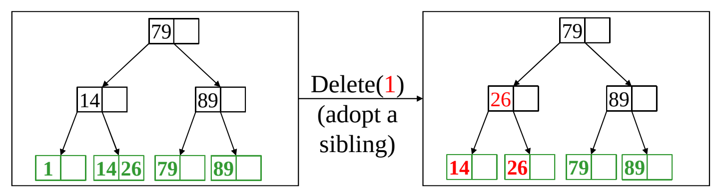
$M = 3$ $L = 2$

A Bit More Adoption



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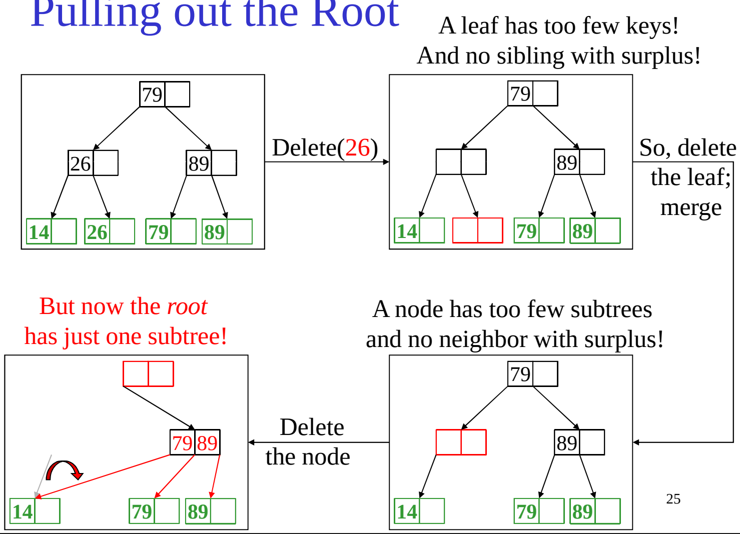
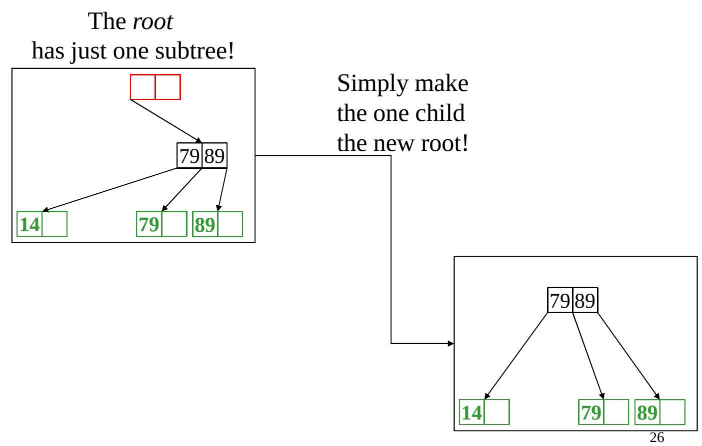
M = 3 L = 2

Pulling out the Root



M = 3 L = 2

Pulling out the Root (continued)



Deletion Algorithm

1. Remove the key from its leaf
2. If the leaf ends up with fewer than $\lceil L/2 \rceil$ items, **underflow**!
   - Adopt data from a sibling; update the parent
   - If adopting won't work, delete node and merge with neighbor
   - If the parent ends up with fewer than $\lceil M/2 \rceil$ items, **underflow**!

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Deletion Slide Two

3. If an internal node ends up with fewer than $\lceil M/2 \rceil$ items, **underflow**!
   - Adopt from a neighbor; update the parent
   - If adoption won't work, merge with neighbor
   - If the parent ends up with fewer than $\lceil M/2 \rceil$ items, **underflow**!
4. If the root ends up with only one child, make the child the new root of the tree

This reduces the height of the tree!

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Thinking about B-Trees

- B-Tree insertion can cause (expensive) splitting and propagation
- B-Tree deletion can cause (cheap) adoption or (expensive) deletion, merging and propagation
- Propagation is rare if ***M*** and ***L*** are large
  (Why?)
- If ***M* = *L* = 128**, then a B-Tree of height 4 will store at least 30,000,000 items

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Tree Names You Might Encounter

FYI:

- B-Trees with ***M* = 3**, ***L* = x** are called **2-3 trees**
  - Nodes can have 2 or 3 keys
- B-Trees with ***M* = 4**, ***L* = x** are called **2-3-4 trees**
  - Nodes can have 2, 3, or 4 keys

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