CSE 326: Data Structures Sorting

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Announcements (5/9/08)

- Project 3 is now assigned.
- Partnerships due by 3pm
  - We will not assume you are in a partnership unless you sign up!
- Homework #5 will be ready after class, due in a week.
- Reading for this lecture: Chapter 7.

Sorting

- Input
  - an array A of data records
  - a key value in each data record
  - a comparison function which imposes a consistent ordering on the keys
- Output
  - reorganize the elements of A such that
    - For any i and j, if i < j then A[i] ≤ A[j]

Consistent Ordering

- The comparison function must provide a consistent *ordering* on the set of possible keys
  - You can compare any two keys and get back an indication of a < b, a > b, or a = b (trichotomy)
  - The comparison functions must be consistent
    - If `compare(a,b)` says a<b, then `compare(b,a)` must say b>a
    - If `compare(a,b)` says a=b, then `compare(b,a)` must say b=a
    - If `compare(a,b)` says a=b, then `equals(a,b) and equals(b,a)` must say a=b

Why Sort?

- Allows binary search of an N-element array in O(log N) time
- Allows O(1) time access to *k*th largest element in the array for any *k*
- Sorting algorithms are among the most frequently used algorithms in computer science

Space

- How much space does the sorting algorithm require in order to sort the collection of items?
  - Is copying needed?
    - **In-place** sorting algorithms: no copying or at most O(1) additional temp space.
  - External memory sorting – data so large that does not fit in memory

Stability

A sorting algorithm is **stable** if:

- Items in the input with the same value end up in the same order as when they began.

Input		**Unstable sort**		**Stable Sort**	
Adams	1	Adams	1	Adams	1
Black	2	Smith	1	Smith	1
Brown	4	Washington	2	Black	2
Jackson	2	Jackson	2	Jackson	2
Jones	4	Black	2	Washington	2
Smith	1	White	3	White	3
Thompson	4	Wilson	3	Wilson	3
Washington	2	Thompson	4	Brown	4
White	3	Brown	4	Jones	4
Wilson	3	Jones	4	Thompson	4

[Sedgewick]

Time

How fast is the algorithm?

- The definition of a sorted array A says that for any i<j, A[i] ≤ A[j]
- This means that you need to at least check on each element at the very minimum
  - Complexity is at least: $O(n)$
- And you could end up checking each element against every other element
  - Complexity could be as bad as: $O(n^2)$

The big question is: How close to $O(n)$ can you get?

Sorting: *The* Big *Picture*

Given *n* comparable elements in an array, sort them in an increasing order.

Simple algorithms: $O(n^2)$	Improved algorithms: $O(n^{1+q})$	Fancier algorithms: $O(n \log n)$	Comparison lower bound: $\Omega(n \log n)$	Specialized algorithms: $O(n)$	Handling huge data sets
Insertion sort Selection sort Bubble sort …	Shell sort Comb sort(?) …	Heap sort Binary tree sort Merge sort Quick sort (avg case) …		Bucket sort Radix sort	External sorting

Selection Sort: idea

1. Find the smallest element, ~~put it~~ swap into 1st
2. Find the next smallest element, put it 2nd
3. Find the next smallest, put it 3rd
4. And so on …

Try it out: Selection Sort

- 31, 16, 54, 4, 2, 17, 6

2 4 6 16 ~~54~~ 54
17 31

Selection Sort: Code

```
void SelectionSort (Array a[0..n-1]) {
    for (i=0, i<n; ++i) {
        j = Find index of
                smallest entry in a[i..n-1]
        Swap(a[i],a[j])
    }
}
```

Only n swaps
-- might be good

Runtime:

worst case : $O(n^2)$
best case : $O(n^2)$
average case : $O(n^2)$

Bubble Sort Idea

- Take a pass through the array
  - If neighboring elements are out of order, swap them.
- Take passes until no swaps needed.

Try it out: Bubble Sort

- 31, 16, 54, 4, 2, 17, 6

16 31 ~~54~~ ~~54~~ ~~54~~ ~~54~~ 54
4 2 17 6

16 4 2 17 6 31 54

Bubble Sort: Code

```
void BubbleSort (Array a[0..n-1]) {
    swapPerformed = 1
    while (swapPerformed) {
        for (i=0, i<n-1; i++) {
            swapPerformed = 0
            if (a[i] < a[i+1]) {
                Swap(a[i],a[i+1])
                swapPerformed = 1
            }
        }
    }
}
```

2,3,4,5 ~~~, n, 1

Runtime:

worst case : $O(n^2)$

best case : $O(n)$

average case : $O(n^2)$

bi-directional sort?
cocktail sort?

Insertion Sort: Idea

1. Sort first 2 elements.
2. Insert 3rd element in order.
   - (First 3 elements are now sorted.)
3. Insert 4th element in order
   - (First 4 elements are now sorted.)
4. And so on…

How to do the insertion?

Suppose my sequence is:

16, 31, 54, 78, 32, 17, 6

And I've already sorted up to 78. How to insert 32?

Example

1	2	3	8	7	9	10	12	23	18	15	16	17	14
1	2	3	7	8	9	10	12	23	18	15	16	17	14
1	2	3	7	8	9	10	12	18	23	15	16	17	14
1	2	3	7	8	9	10	12	18	15	23	16	17	14
1	2	3	7	8	9	10	12	15	18	23	16	17	14
1	2	3	7	8	9	10	12	15	18	16	23	17	14
1	2	3	7	8	9	10	12	15	16	18	23	17	14

Example

1	2	3	7	8	9	10	12	15	16	18	17	23	14
1	2	3	7	8	9	10	12	15	16	17	18	23	14
1	2	3	7	8	9	10	12	15	16	17	18	14	23
1	2	3	7	8	9	10	12	15	16	17	14	18	23
1	2	3	7	8	9	10	12	15	16	14	17	18	23
1	2	3	7	8	9	10	12	15	14	16	17	18	23
1	2	3	7	8	9	10	12	14	15	16	17	18	23

Try it out: Insertion sort

- 31, 16, 54, 4, 2, 17, 6

16, 31, 54

4 16 31 54

2 4 16 31 54

⋮

Insertion Sort: Code

```
void InsertionSort (Array a[0..n-1]) {
    for (i=1, i<n; i++) {
        for (j=i, j>0; j--) {
            if (a[j] < a[j-1])
                Swap(a[i],a[j])
            else
                break
        }

}
```

2, 3, 4 ... n, 1

j-1

Note: can instead move the "hole" to minimize copying, as with a binary heap.

Runtime:

worst case : $O(n^2)$

best case : $O(n)$

average case : $O(n^2)$

Shell Sort: Idea

A small element at end of list takes a long time to percolate to front.

Idea: take bigger steps at first to percolate faster.

1. Choose offset k:
   a. Insertion sort over array: a[0], a[k], a[2k], a[3k], …
   b. Insertion sort over array: a[1], a[1+k], a[1+2k], a[1+3k], …
   c. Insertion sort over array: a[2], a[2+k], a[2+2k], a[2+3k], …
   d. Do this until all elements touched
2. Choose smaller offest m, where m is smaller than k, and do another set of insertion sort passes, stepping by m through the array.
3. Repeat for smaller offsets until last pass uses offset = 1

[Named after the algorithm's inventor, Donald Shell.]

Try it out: Shell Sort

- Offsets: 3, 2, 1
- Input array: 31, 16, 54, 4, 2, 17, 6

4 2 17 6 16 54 31

4 2 16 6 17 54 31

2 4 6 16 17 31 54

Shell Sort: Code

```
void ShellSort (Array a[0..n-1]) {
    determine good offsets based on n
    for (i=0, i<numOffsets; i++) {
        for (j=0, j<offsets[i]; j++) {
            insertionSort(a, j, offsets[i])
        }
}

void InsertionSkipSort (Array a[0..n-1],
                        Int start, Int offset) {
    Do insertion sort on array
       a[start], a[start+offset], a[start+2*offset],...
}
```

Shell Sort Offsets

The key to good Shell sort performance: **good offsets**.

Shell started the offset at ceil(n/2) and halved the offset each time. **Not good**.

Sedgewick proposed this offset sequence:

- Lowest offset is 1.
- Others are: $1 + 3 \cdot 2^{i} + 4^{i+1}$ for $i \geq 0$
- Looks like: 1, 8, 23, 77, 281, 1073, 4193, …
- (Put in the offset array in reverse order to work with pseudocode on previous slide.)

Result:

- Worst case complexity is $O(n^{4/3})$
- Average case is believed to be $O(n^{7/6})$

Comb Sort

Could you do something like Shell Sort with bubble sort instead of insertion sort?

Yes! Called "Comb Sort". Dobosiewicz Sort

Complexity not well understood.

offset = offset/1.3

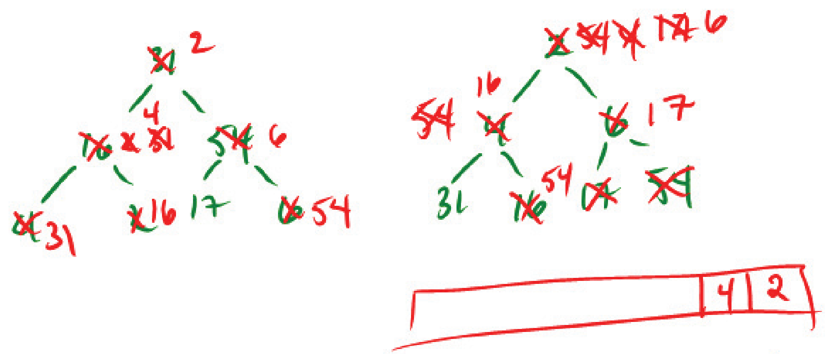
Heap Sort: Sort with a Binary Heap

1. BuildHeap( ) — O(n)
2. DeleteMin( ) until empty — O(n log n)

Runtime: O(n log n)

Try it out: Heap Sort

- 31, 16, 54, 4, 2, 17, 6



Max heap avoids reverse order output

Binary Tree Sort

1. Insert all elements into ~~BST~~ AVL tree. $O(n \log n)$
2. Do in-order traversal. $O(n)$

Runtime: $O(n \log n)$

Try it out: Binary Tree Sort

- 31, 16, 54, 4, 2, 17, 6