CSE 326: Data Structures Binary Search Trees

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Spring 2008

Announcements

- HW #3 will be assigned this afternoon, due at beginning of class next Friday.
- Project 2A due next Wed. night.

Outline

- Dictionary ADT / Search ADT
- Quick Tree Review
- Binary Search Trees

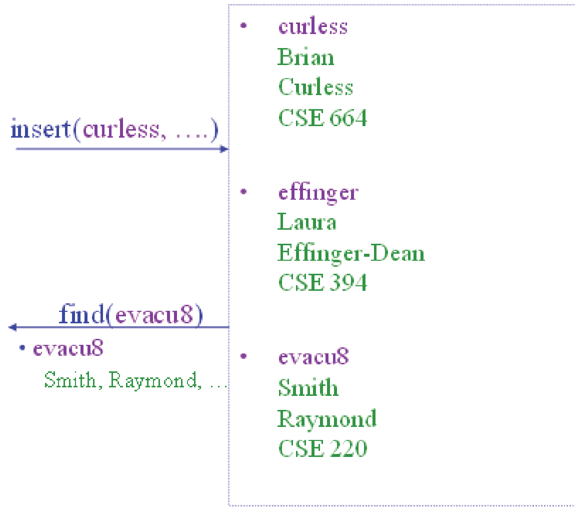
ADTs Seen So Far

- Stack
  - Push
  - Pop
- Queue
  - Enqueue
  - Dequeue
- Priority Queue
  - Insert
  - DeleteMin

Then there is decreaseKey…

The Dictionary ADT

- Data:
  - a set of (key, value) pairs
- Operations:
  - Insert (key, value)
  - Find (key)
  - Remove (key)



*The Dictionary ADT is also called the "**Map ADT**"*

A Modest Few Uses

- Sets
- Dictionaries
- Networks : Router tables
- Operating systems : Page tables
- Compilers : Symbol tables

Probably the most widely used ADT!

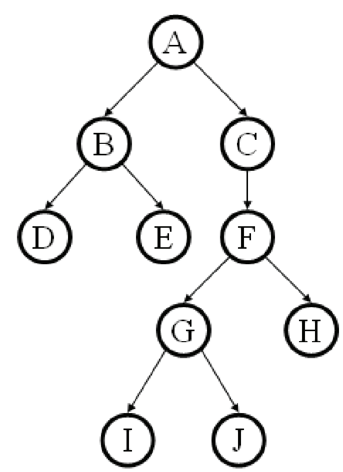
Implementations

	insert	find	delete
• Unsorted Linked-list	$O(1)$	$O(n)$	$O(n)$
• Unsorted array	$O(1)$	$O(n)$	$O(n)$
• Sorted array	$O(\log n + n) = O(n)$	$O(\log n)$	$O(n)$

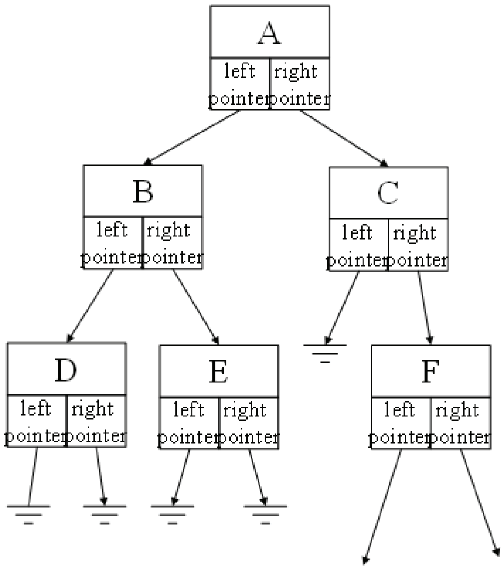
Binary Trees

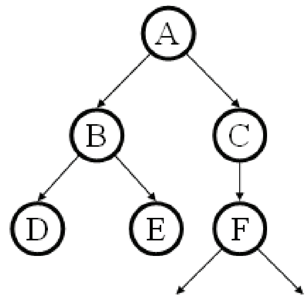
- Binary tree is
  - a root
  - left subtree *(maybe empty)*
  - right subtree *(maybe empty)*
- Representation:

Data	
left pointer	right pointer



Binary Tree: Representation



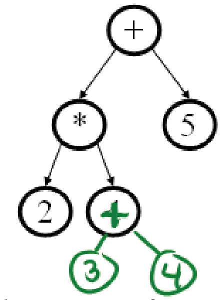


Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

Three types:

- Pre-order: Root, left subtree, right subtree
- In-order: Left subtree, root, right subtree
- Post-order: Left subtree, right subtree, root



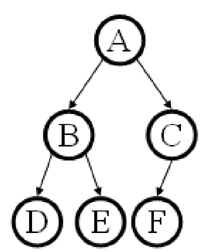
(an expression tree)

2*(3+(4))+(5)

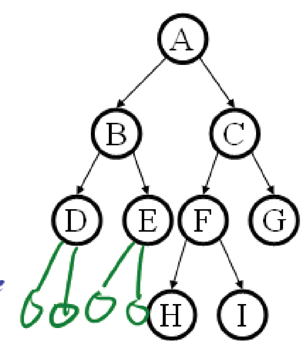
Inorder Traversal

```
void traverse(BNode t){
  if (t != NULL)
    traverse (t.left);
    process t.element;
    traverse (t.right);
  }
}
```


Binary Tree: Special Cases

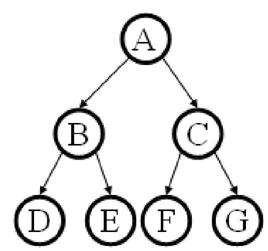


Complete Tree

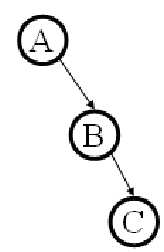


Full Tree

every node has 0 or 2 children



Perfect Tree



"List" Tree

Binary Tree: Some Numbers…

Recall: height of a tree = longest path from root to leaf.

For binary tree of height h:

- max # of leaves: 2^h
- max # of nodes: $2^{h+1}-1$
- min # of leaves: 1
- min # of nodes: $h+1$

Binary Tree: Some Numbers…

Recall: depth of a node = path length from node to root.

Consider the space (forest) of all possible binary trees of N nodes.

- Sum up the depths of every node in that forest and divide by the number of nodes.
- This is the average depth over all nodes over all binary trees of size N. How big is it?

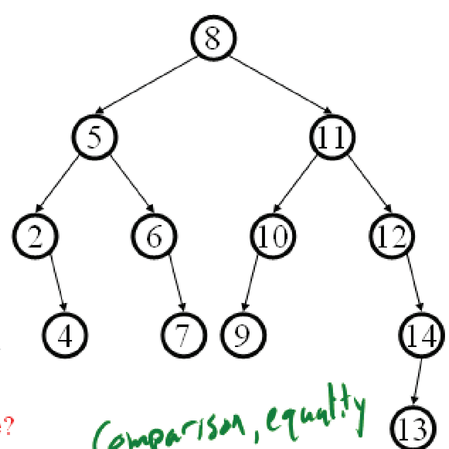
$\sqrt{N}$

What would the average depth be for a well-balanced tree? $\log(N)$

Binary Search Tree Data Structure

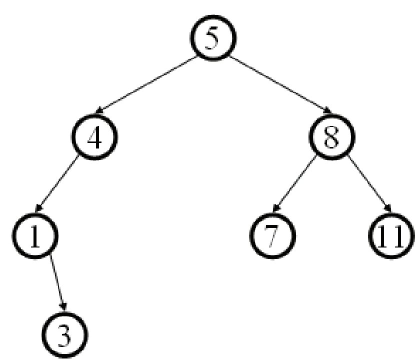
- Structural property
  - each node has ≤ 2 children
  - result:
    - storage is small
    - operations are simple
- Order property
  - all keys in left subtree smaller than root's key
  - all keys in right subtree larger than root's key
  - result: easy to find any given key
- What must I know about what I store?

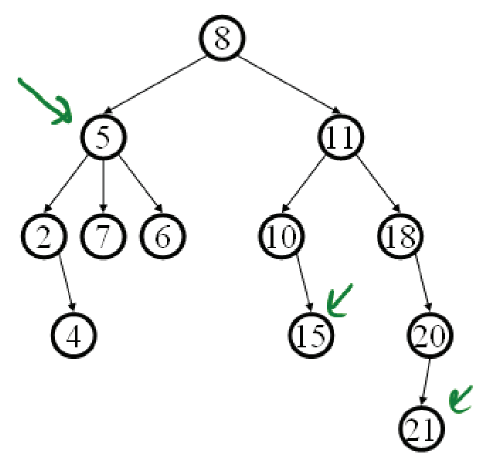
Comparison, equality



Example and Counter-Example

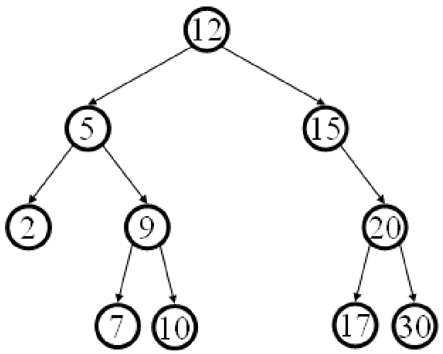
Yes





BINARY SEARCH TREES?

Find in BST, Recursive




```
Node Find(Object key,
               Node root) {
  if (root == NULL)
    return NULL;

  if (key < root.key)
    return Find(key,
                   root.left);
  else if (key > root.key)
    return Find(key,
                   root.right);
  else
    return root;
}
```

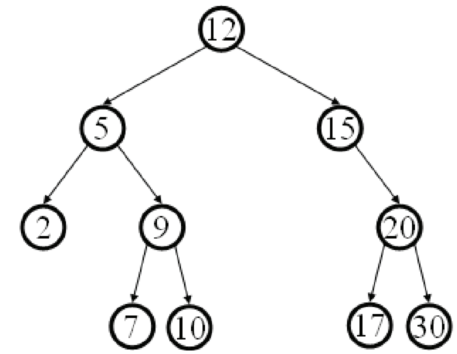
Runtime: O(n)

Find in BST, Iterative

```
Node Find(Object key,
               Node root) {

  while (root != NULL &&
           root.key != key) {
    if (key < root.key)
      root = root.left;
    else
      root = root.right;
  }

  return root;
}
```



Runtime: O(n)

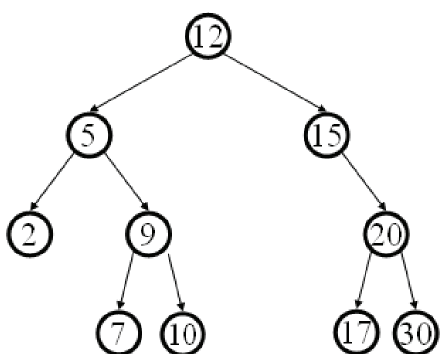
Bonus: FindMin/FindMax

- Find minimum

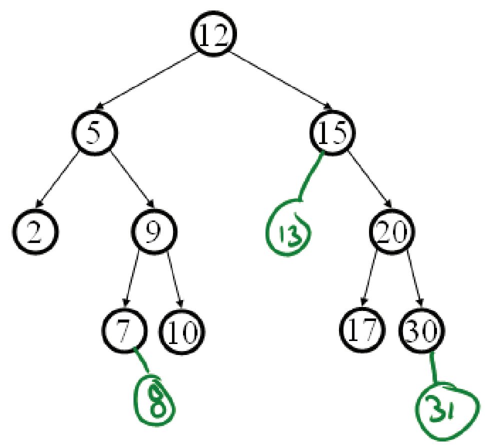
go left!

- Find maximum

go right!



Insert in BST



Insert(13)
Insert(8)
Insert(31)

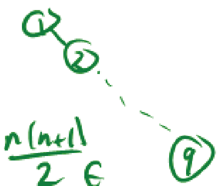
Insertions happen only at the leaves – easy!

Runtime: O(n)

BuildTree for BST

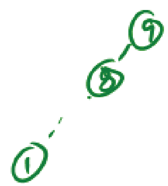
- Suppose keys 1, 2, 3, 4, 5, 6, 7, 8, 9 are inserted into an initially empty BST.

If inserted in given order, what is the tree? What big-O runtime for this kind of sorted input?



$\sum_{i=0}^{n} i = \frac{n(n+1)}{2} \in O(n^2)$

If inserted in reverse order, what is the tree? What big-O runtime for this kind of sorted input?



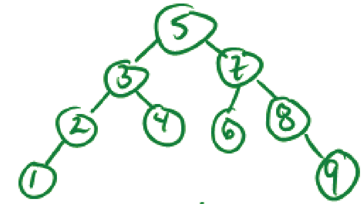
$O(n^2)$

BuildTree for BST

- Suppose keys 1, 2, 3, 4, 5, 6, 7, 8, 9 are inserted into an initially empty BST.

5, 3, 7, 2, 8, 4, 6, 1, 9

  – If inserted median first, then left median, right median, etc., what is the tree? What is the big-O runtime for this kind of sorted input?



$O(n \log n)$

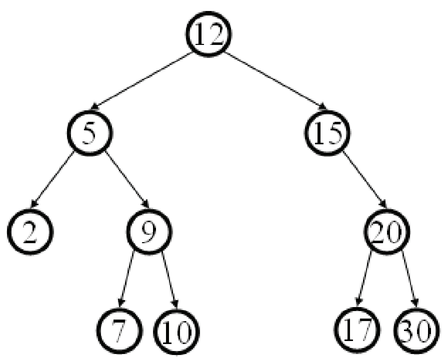
$\Omega(\frac{n}{2} \log \frac{n}{2})$

$\Theta(n \log n)$

$\Omega(\frac{n}{2}(\log n - \log 2))$

$\Omega(n \log n)$

Deletion in BST



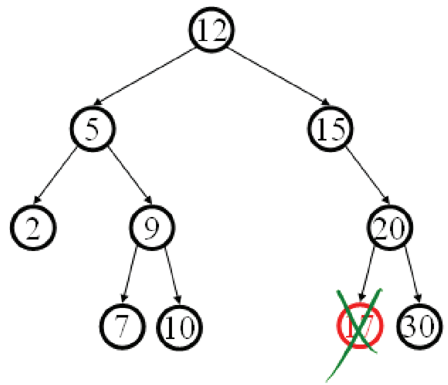
Why might deletion be harder than insertion?

Deletion

- Removing an item disrupts the tree structure.
- Basic idea: find the node that is to be removed. Then "fix" the tree so that it is still a binary search tree.
- Three cases:
  – node has no children (leaf node)
  – node has one child
  – node has two children

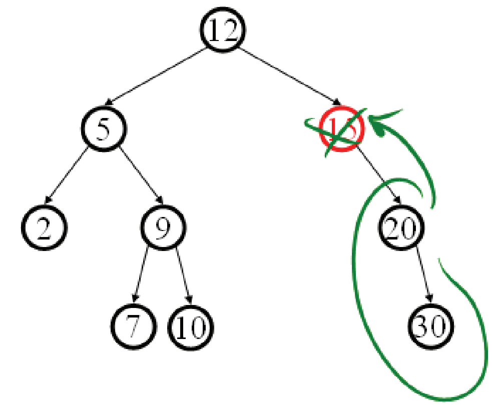
Deletion – The Leaf Case

Delete(17)



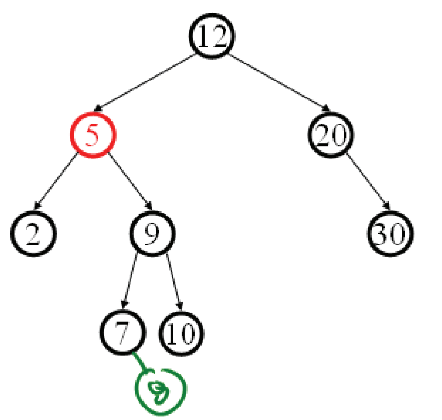
Deletion – The One Child Case

Delete(15)



Deletion – The Two Child Case

Delete(5)



What can we replace 5 with?

Deletion – The Two Child Case

Idea: Replace the deleted node with a value guaranteed to be between the two child subtrees

Options:

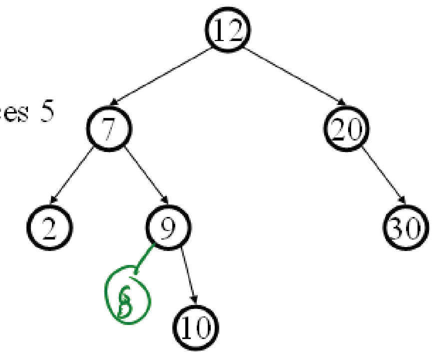
- *succ* from right subtree: findMin(t.right)
- *pred* from left subtree : findMax(t.left)

Now delete the original node containing *succ* or *pred*

- Leaf or one child case – easy!

Finally…

7 replaces 5



Original node containing
7 gets deleted

Balanced BST

Observations

- BST: the shallower the better!
- For a BST with n nodes
  - Average depth (averaged over all possible insertion orderings) is O(log n)
  - Worst case maximum depth is O(n)
- Simple cases such as insert(1, 2, 3, ..., n) lead to the worst case scenario

Solution: Require a **Balance Condition** that

1. ensures depth is O(log n) – strong enough!
2. is easy to maintain – not too strong!