CSE 326: Data Structures

Skew Heaps

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Announcements (4/11/08)

- Written HW #2 due next Friday
- Project #2 Phase A out now
  - Partner sign-ups by 3pm today

Skew Heaps

Problems with leftist heaps
- Extra storage for npl
- Extra complexity/logic to maintain and check npl

Observation:
- Right side of leftist heap is "often" heavy and requires a switch.

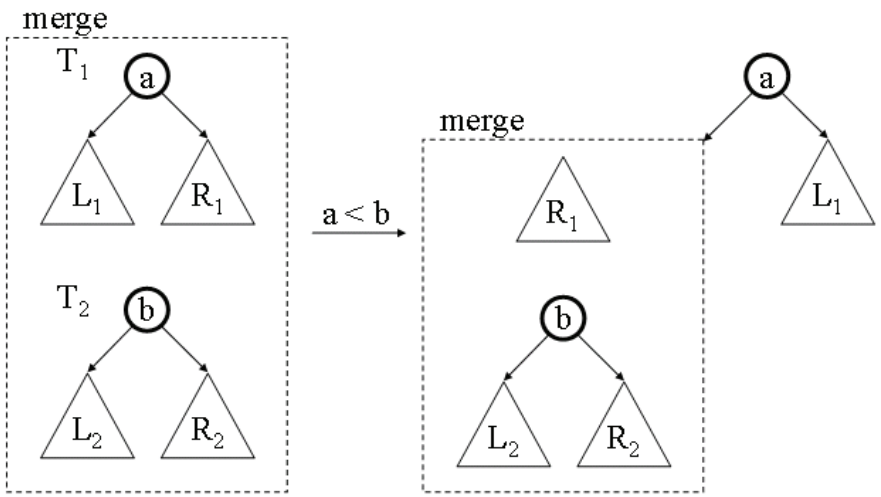
Solution: skew heaps
- "Blindly" adjusting version of leftist heaps
- Merge *always* switches children when fixing right path
- Worst case time: merge, insert, deleteMin = O(n)
- *Amortized* time: merge, insert, deleteMin = O(log n)

Amortized vs. average complexity

Recall:
- **Average-case complexity**: **avg** # steps algorithm takes on *random* inputs of size **N**
- **Amortized complexity (improved definition)**: **max** total # steps algorithm takes, in the worst case, for **M** *consecutive* operations on inputs of size **N**, divided by **M** (i.e., divide the max total by **M**).

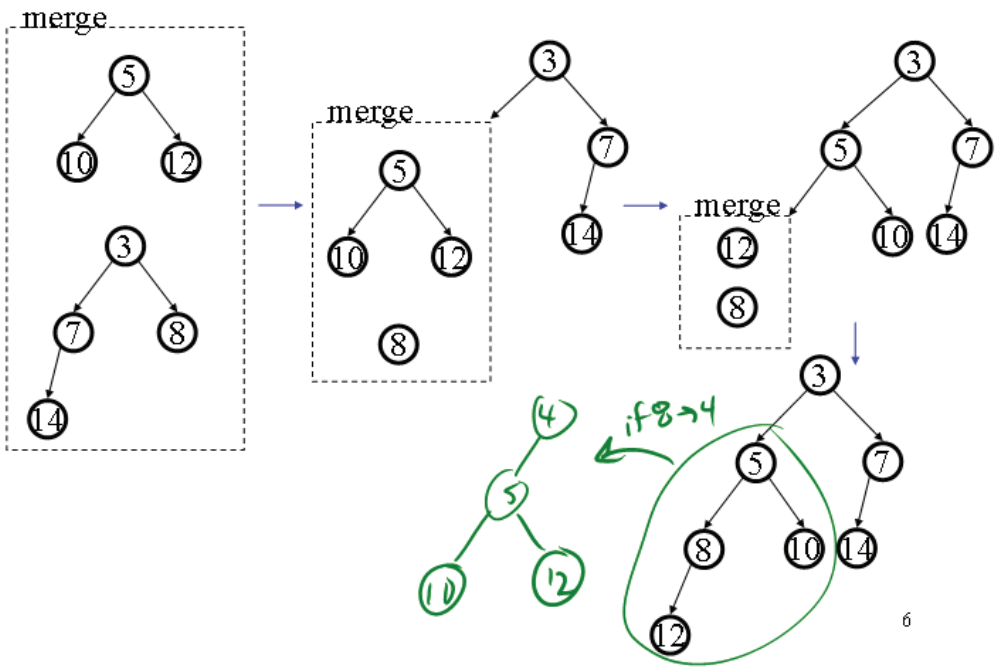
  Example: if M operations take total O(M log N) time in the worst case, *amortized* time per operation is O(log N).

Merging Two Skew Heaps



Only one step per iteration, with children *always* switched

Example



Skew Heap Code

SkewHeap

```
void merge(heap1, heap2) {
  case {
     heap1 == NULL: return heap2;
     heap2 == NULL: return heap1;
     heap1.findMin() <=heap2.findMin():
          temp = heap1.right;
          heap1.right = heap1.left;
          heap1.left = merge(heap2, temp);
          return heap1;
     otherwise:
          return merge(heap2, heap1);
  }
}
```


Runtime Analysis: Worst-case and Amortized

- No worst case guarantee on right path length!
- All operations rely on merge

  ⇒ worst case complexity of all ops = $O(n)$

- It is known: M merges take time $\Theta(M \log n)$ in the worst case

  ⇒ amortized complexity of all ops = $\Theta(\log n)$