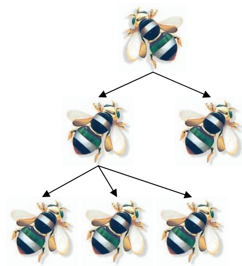
CSE 326: Data Structures
B-Trees and B+ Trees

Brian Curless
Spring 2008



Announcements (4/30/08)

- Midterm on Friday
- Special office hour: 4:30-5:30 Thursday in Jaech Gallery (6th floor of CSE building)
  - This is *instead of* my usual 11am office hour.
- Reading for this lecture: Weiss Sec. 4.7

Traversing very large datasets

Suppose we had very many pieces of data (as in a database), e.g., $n = 2^{30} \approx 10^9$.

How many (worst case) hops through the tree to find a node?

- BST
- AVL
- Splay

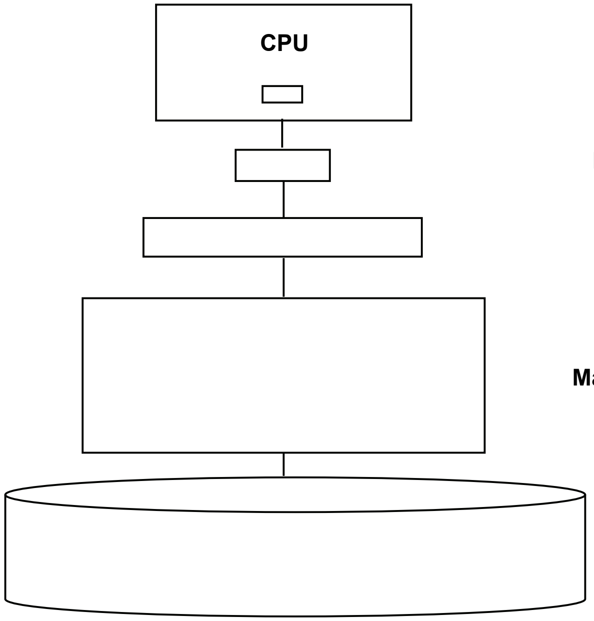
Memory considerations

What is in a tree node? In an object?

```
Node:
  Object obj;
  Node left;
  Node right;
  Node parent;

Object:
  Key key;
  …data…
```

Suppose the data is 1KB.
  How much space does the tree take?
  How much of the data can live in 1GB of RAM?



Cycles to access:

Registers	1
L1 Cache	2
L2 Cache	30
Main memory	250
Disk	Random: 30,000,000 Streamed: 5000

Minimizing random disk access

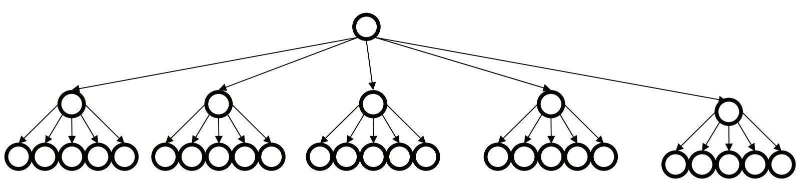
In our example, almost all of our data structure is on disk.

Thus, hopping through a tree amounts to random accesses to disk. Ouch!

How can we address this problem?

M-ary Search Tree

Suppose, *somehow*, we devised a search tree with maximum branching factor *M*:



Complete tree has height:

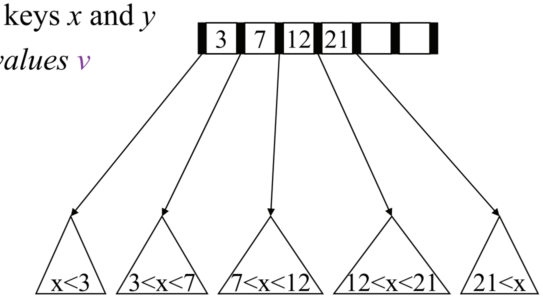
hops for *find*:

Runtime of *find*:

B-Trees

How do we make an *M*-ary search tree work?

- Each **node** has (up to) M-1 keys.
- Order property:
  - subtree between two keys *x* and *y* contain leaves with *values* *v* such that $x < v < y$



B-Tree Structure Properties

Root (special case)

– has between 2 and M children (or root could be a leaf)

Internal nodes

– store up to M-1 keys

– have between $\lceil M/2 \rceil$ and M children

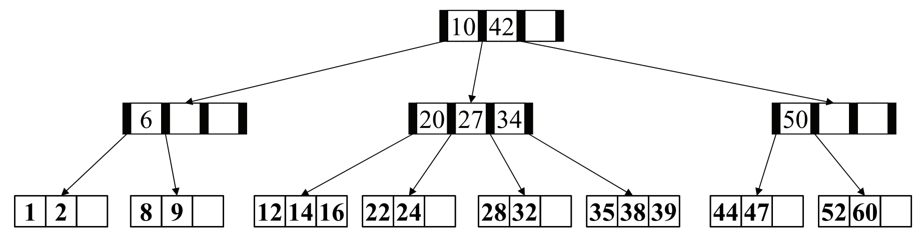
Leaf nodes

– store between $\lceil (M-1)/2 \rceil$ and M-1 sorted keys

– all at the same depth

B-Tree: Example

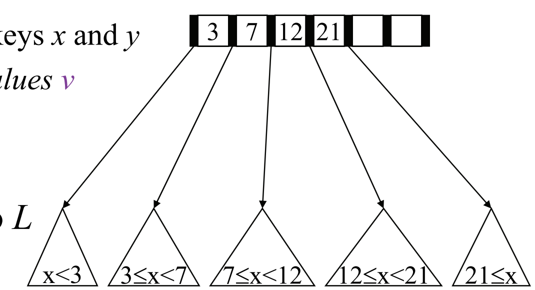
B-Tree with ***M* = 4**



B+ Trees

In a B+ tree, the internal nodes have no data – only the leaves do!

- Each internal node still has (up to) M-1 keys:
- Order property:
  - subtree between two keys x and y contain leaves with *values* v such that $x \leq v < y$
  - Note the "≤"
- Leaf nodes have up to L sorted keys.



B+ Tree Structure Properties

Root (special case)

– has between 2 and ***M*** children (or root could be a leaf)

Internal nodes

– store up to M-1 keys

– have between $\lceil M/2 \rceil$ and M children

Leaf nodes

– where data is stored

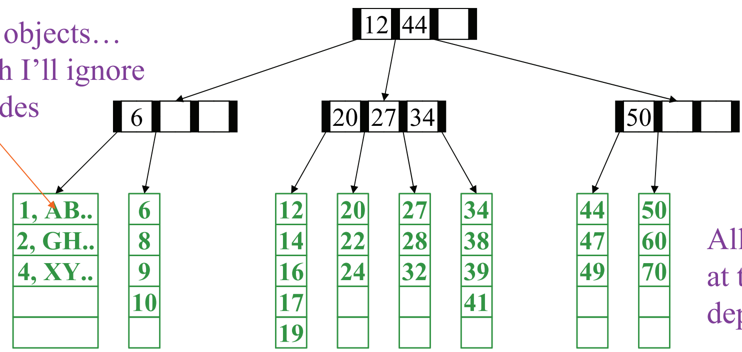
– all at the same depth

– contain between $\lceil L/2 \rceil$ and L data items

B+ Tree: Example

B+ Tree with `M = 4` (# pointers in internal node)
and `L = 5` (# data items in leaf)

Data objects… which I'll ignore in slides



All leaves at the same depth

Definition for later: "neighbor" is the next sibling to the left or right.

Disk Friendliness

What makes B+ trees disk-friendly?

1. **Many keys stored in a node**
   - All brought to memory/cache in one disk access.
2. Internal nodes contain *only* keys;
   Only leaf nodes contain keys and actual *data*
   - Much of tree structure can be loaded into memory irrespective of data object size
   - Data actually resides in disk

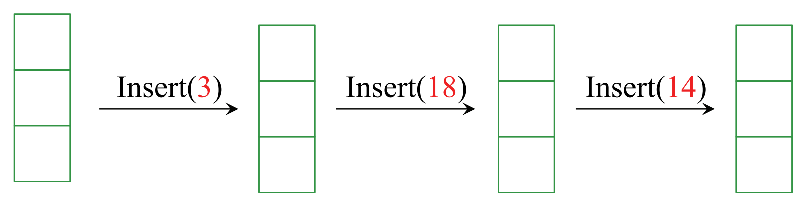
B+ trees vs. AVL trees

Suppose again we have $n = 2^{30} \approx 10^9$ items:

- Depth of AVL Tree
- Depth of B+ Tree with M = 256, L = 256

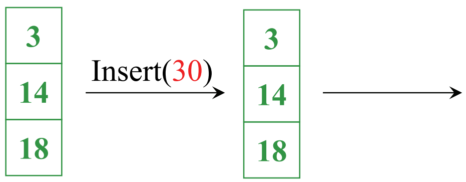
Great, but how to we actually make a B+ tree and keep it balanced…?

Building a B+ Tree with Insertions

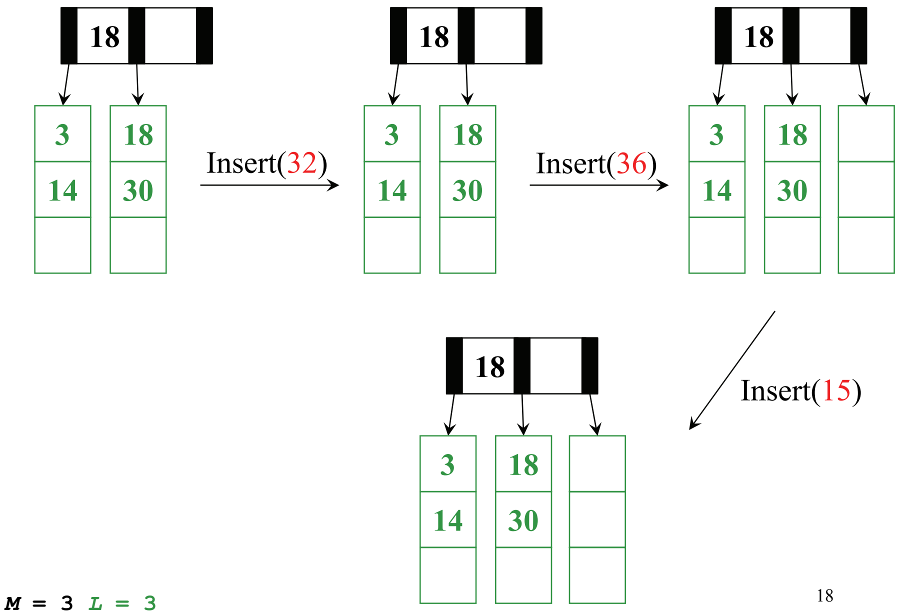


The empty B-Tree

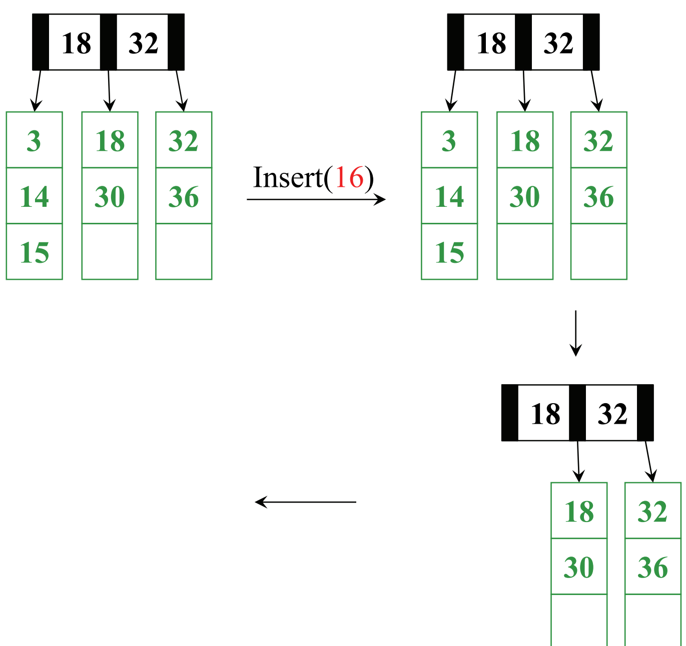
`M = 3 L = 3`



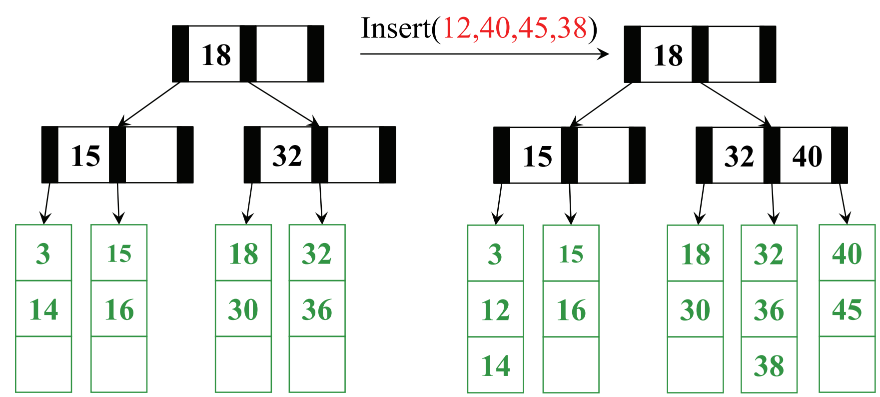
M = 3 *L* = 3



M = 3 *L* = 3



M = 3 *L* = 3



M = 3 *L* = 3

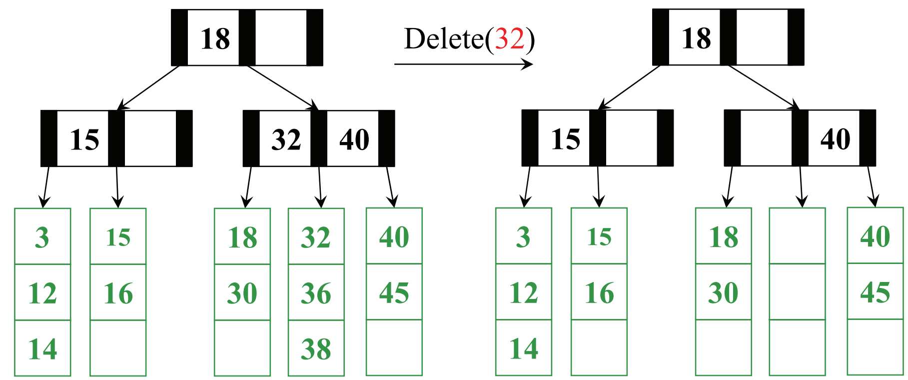
Insertion Algorithm

1. Insert the key in its leaf in sorted order
2. If the leaf ends up with L+1 items, **overflow**!
   - Split the leaf into two nodes:
     - original with $\lceil (L+1)/2 \rceil$ items
     - new one with $\lfloor (L+1)/2 \rfloor$ items
   - Add the new child to the parent
   - If the parent ends up with $M+1$ children, **overflow**!
3. If an internal node ends up with M+1 children, **overflow**!
   - Split the node into two nodes:
     - original with $\lceil (M+1)/2 \rceil$ children
     - new one with $\lfloor (M+1)/2 \rfloor$ children
   - Add the new child to the parent
   - If the parent ends up with $M+1$ items, **overflow**!
4. Split an overflowed root in two and hang the new nodes under a new root

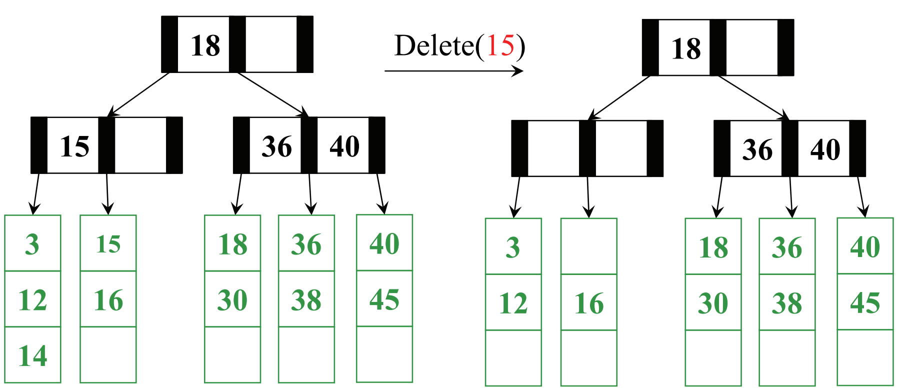
This makes the tree deeper!

5. Propagate keys up tree.

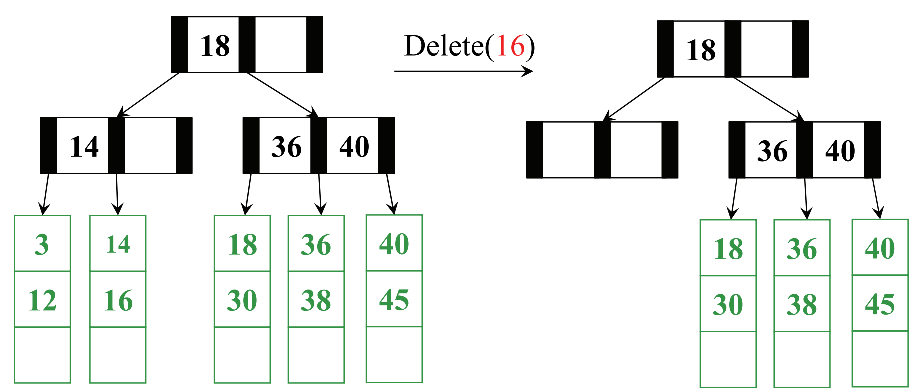
And Now for Deletion…



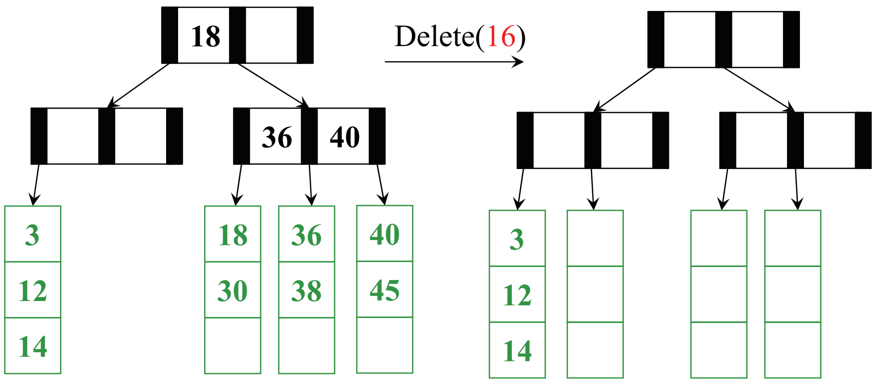
`M = 3 L = 3`



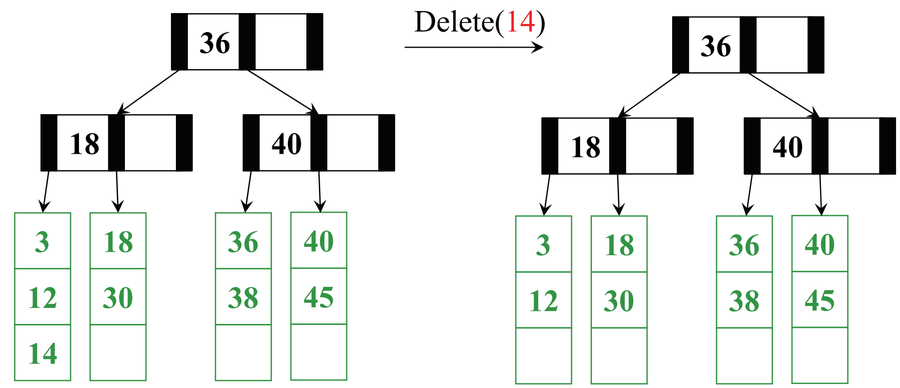
`M = 3 L = 3`



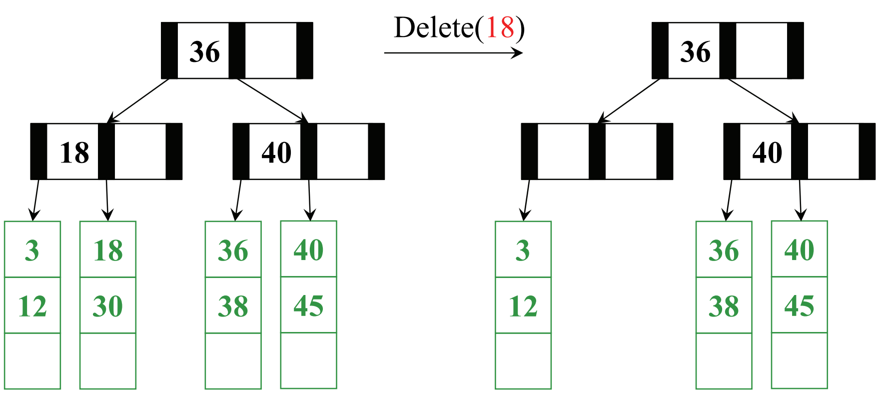
`M = 3 L = 3`



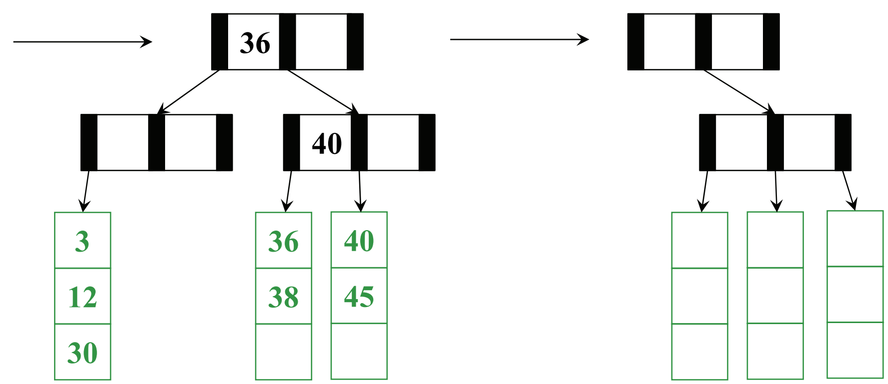
M = 3 *L* = 3



M = 3 *L* = 3



M = 3 *L* = 3



M = 3 *L* = 3

Deletion Algorithm

1. Remove the key from its leaf

2. If the leaf ends up with fewer than ⌈**`L/2`**⌉ items, **underflow**!
   - Adopt data from a neighbor; update the parent
   - If adopting won't work, delete node and merge with neighbor
   - If the parent ends up with fewer than ⌈**`M/2`**⌉ children, **underflow**!

Deletion Slide Two

3. If an internal node ends up with fewer than ⌈**`M/2`**⌉ children, **underflow**!
   - Adopt from a neighbor; update the parent
   - If adoption won't work, merge with neighbor
   - If the parent ends up with fewer than ⌈**`M/2`**⌉ children, **underflow**!

4. If the root ends up with only one child, make the child the new root of the tree

This reduces the height of the tree!

5. Propagate keys up through tree.

Thinking about B+ Trees

- B+ Tree insertion can cause (expensive) splitting and propagation
- B+ Tree deletion can cause (cheap) adoption or (expensive) deletion, merging and propagation
- Propagation is rare if ***M*** and ***L*** are large *(Why?)*
- Pick branching factor ***M*** and data items/leaf ***L*** such that each node takes one full page/block of memory/disk.

Tree Names You Might Encounter

FYI:

- B-Trees with **`M = 3`**, **`L = x`** are called **2-3 trees**
  - Nodes can have 2 or 3 keys
- B-Trees with **`M = 4`**, **`L = x`** are called **2-3-4 trees**
  - Nodes can have 2, 3, or 4 keys