CSE 326: Data Structures
Splay Trees

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Announcements (4/25/08)

- No homework assigned this week
- Midterm next Friday
  - Closed notes, book
  - It will cover everything through today, maybe part of Monday's lecture.
  - It will not be a test of your Java knowledge.
  - It will not have hard proofs.
  - I will say more next week.
- Reading for this lecture is in Weiss, Chapter 4.

Pros and Cons of AVL Trees

Arguments for AVL trees:

1. Search is O(log n) since AVL trees are always well balanced.
2. The height balancing adds no more than a constant factor to the speed of insertion and deletion; thus both are O(log n).

Arguments against using AVL trees:

1. Must store and track height info.
2. May be fine to have O(n) for a single operation if amortized cost can be O(log n) (e.g., Splay trees).
3. Very large searches are done in database systems and need efficient strategies based on disk access costs (e.g., B-trees).

Splay Trees

- Blind adjusting version of AVL trees
  - Why worry about balances? Just rotate anyway!
- Worst case time for an operation is O(n)
- *Amortized* time per operation is O(log n)

SAT/GRE Analogy question:

AVL tree **:** Splay tree **::** Leftist heap **:**

Webster's Dictionary Definition

Main Entry:

[3]splay

Function:

adjective

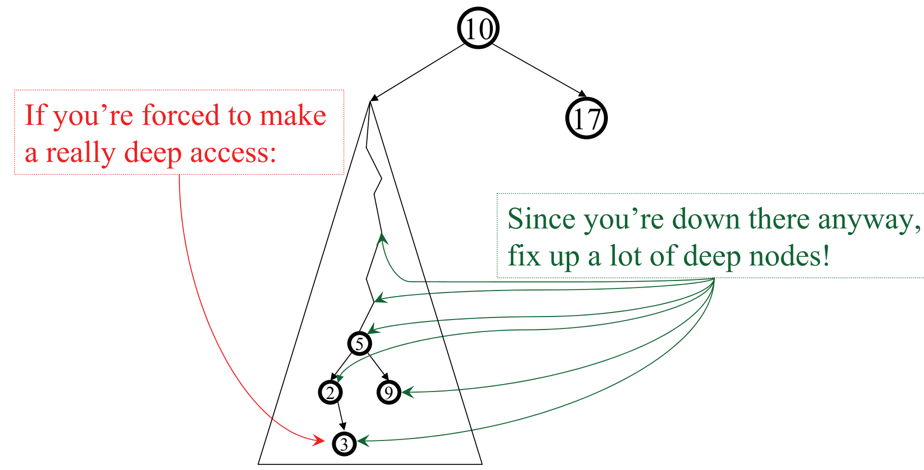
Date:

1548

1 : turned outward **<*splay* knees>**

2 : awkward, ungainly

The Splay Tree Idea



Find/Insert in Splay Trees

1. Find or insert a node ***k***
2. **Percolate *k* to the root with rotations.**

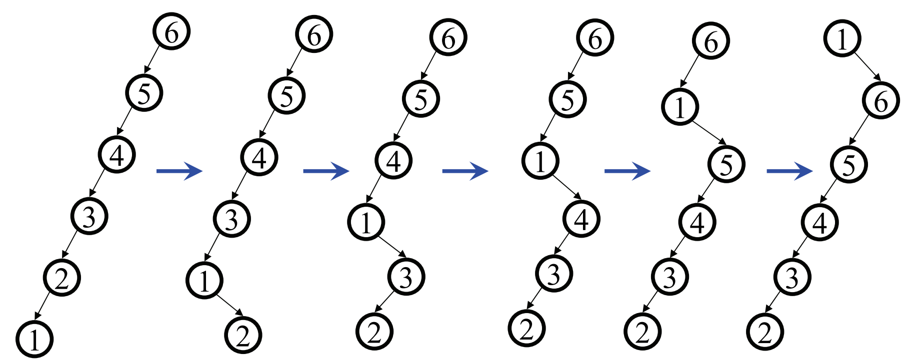
Instead of "percolate ***k*** to the root with rotations" we will often just say "*splay* ***k*** to the root".

Why could this be good??

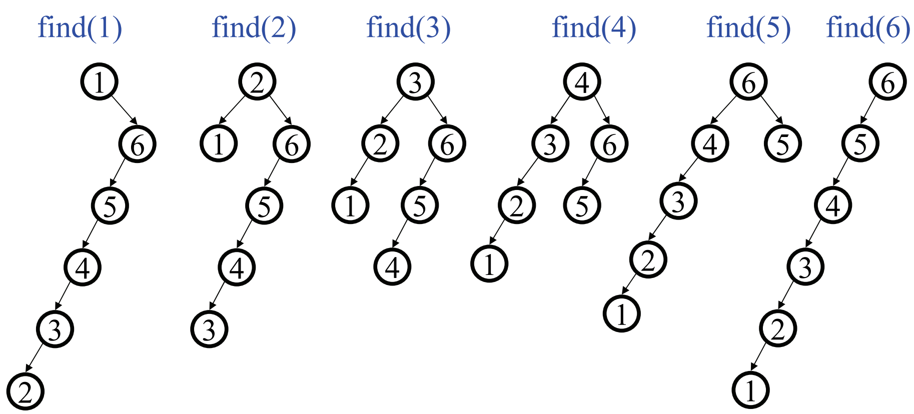
1. Helps the new root, ***k***
   - *Great if k is accessed again soon*
2. And helps many others on the path to ***k***!
   - *Great if others on the path are accessed soon*

Do it all with AVL single rotations?

Consider the ordered "list tree" at left. Now do find(1) and splay it to the root with only AVL single rotations:



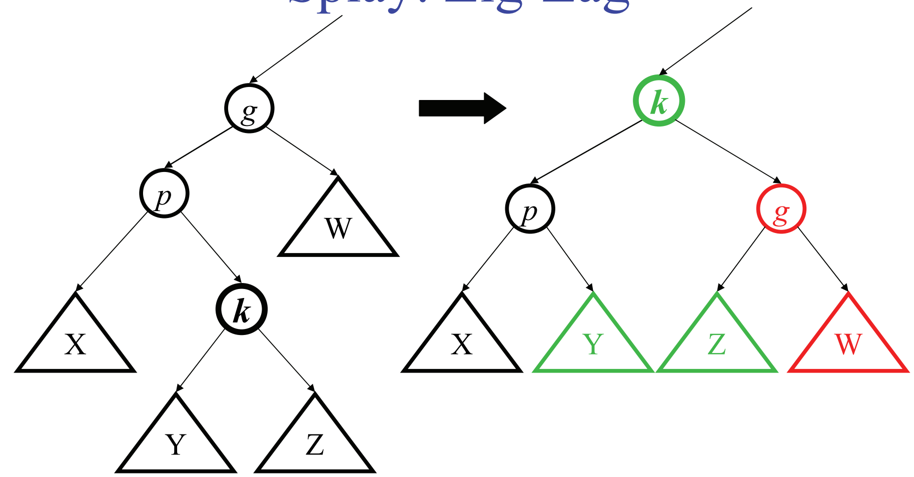
Do it all with AVL single rotations?



Cost of sequence: find(1), find(2), … find(n)?

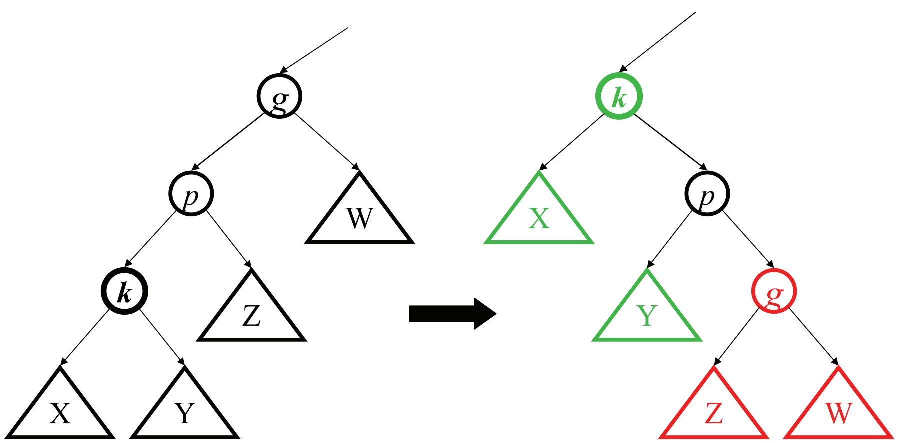
Single rotations can help, but they are not enough…

Splay: Zig-Zag

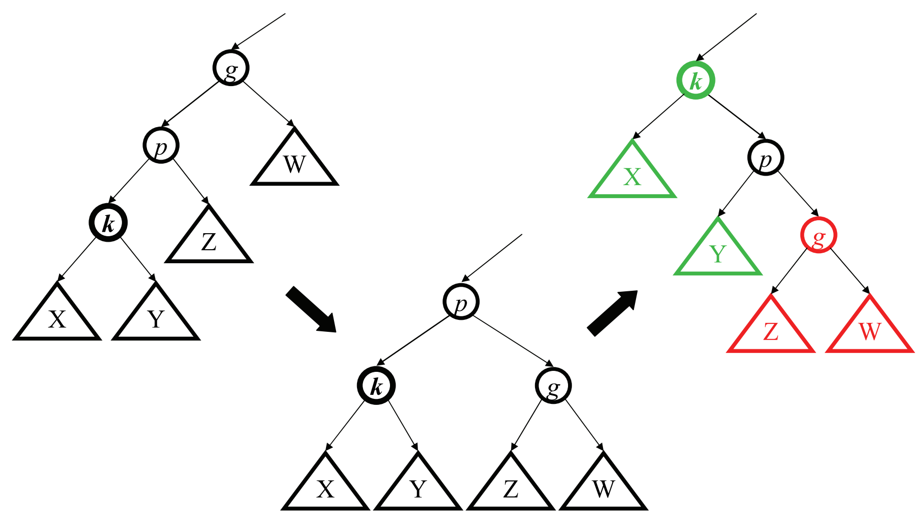


Relationship to AVL rotations?

Splay: Zig-Zig

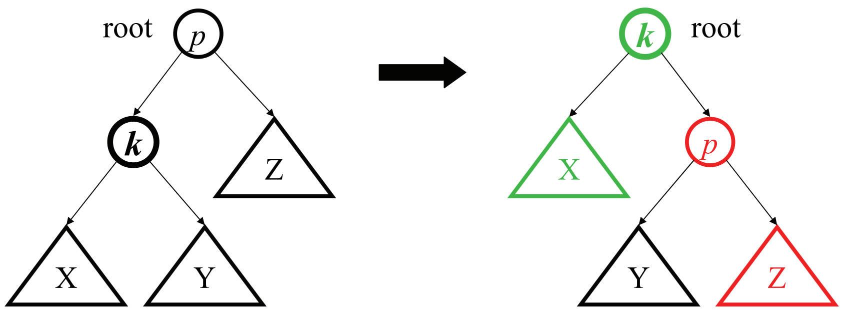


Splay: Zig-Zig



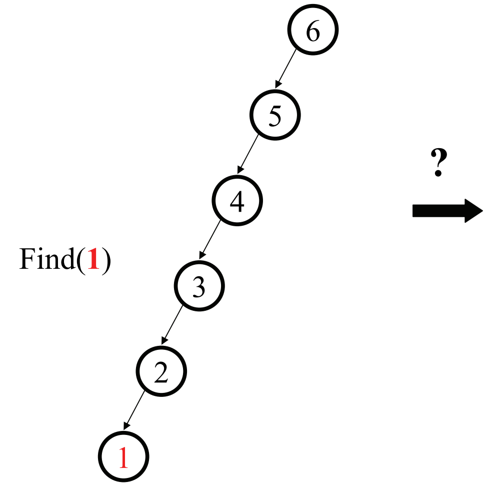
Relationship to AVL rotations?

Special Case for Root: Zig

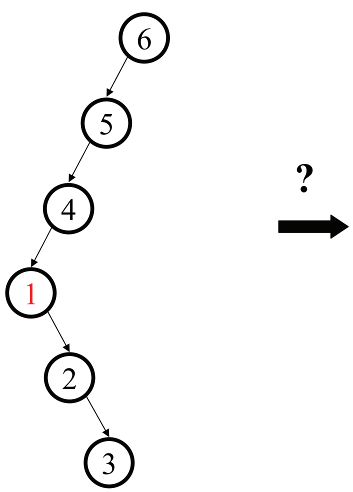


Relationship to AVL rotations?

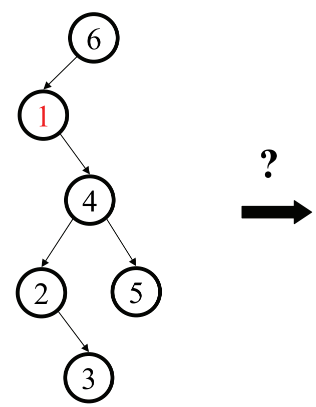
Splaying Example: Find(1)



Find(1) continued…

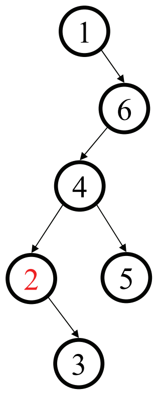


Find(1) finished



Another Splay: Find(2)

Find(**2**)



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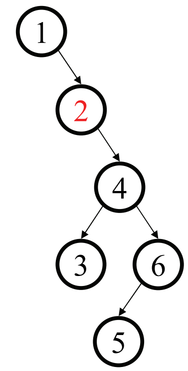
Find(2) finished



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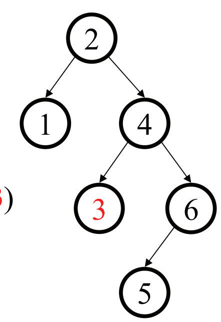
Another Splay: Find(3)

Find(**3**)



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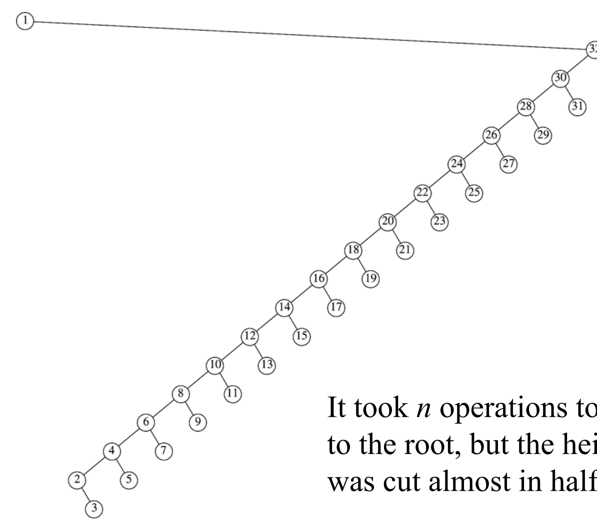
A bigger, badder example

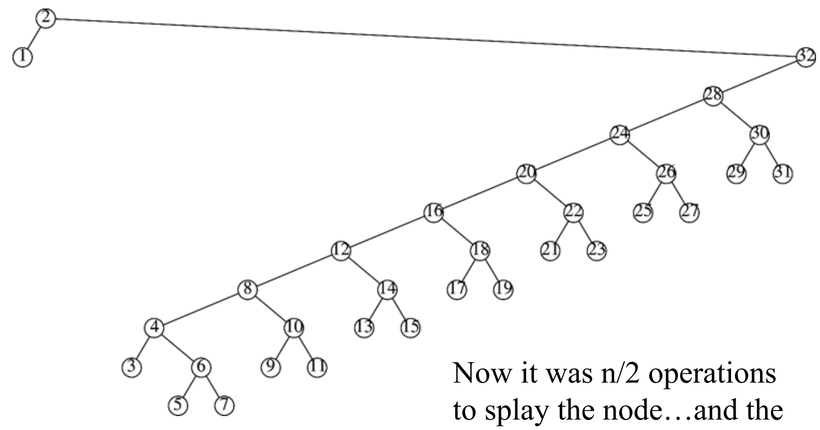
Consider a “list tree” from 1-32. Doing find(1) gives:



It took *n* operations to bring that node to the root, but the height of the tree was cut almost in half!

A bigger, badder example

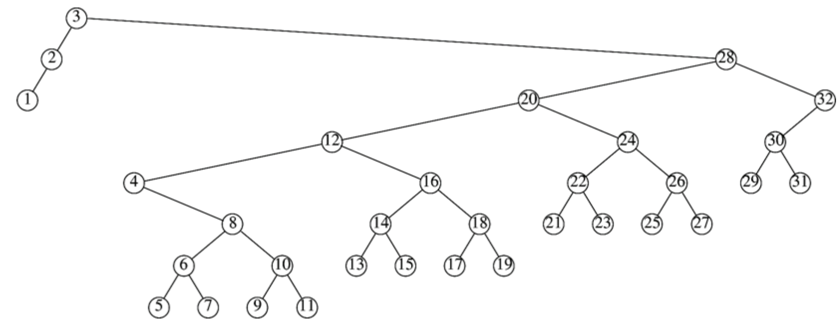
Now do find(2):



Now it was n/2 operations
to splay the node…and the
tree height is cut in ~half again!

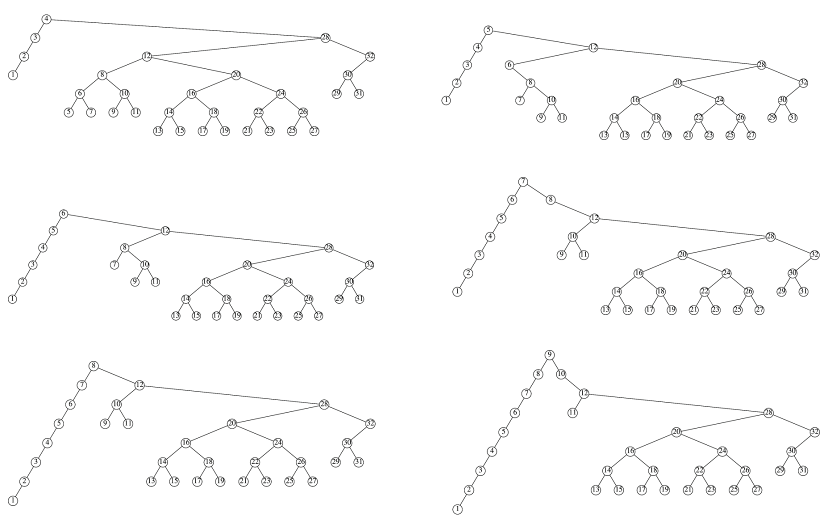
A bigger, badder example

Now do find(3):



The height is cut in ~half again.

A bigger, badder example



Why Splaying Helps

- If a node *x* on the access path is at depth *d* before the splay, it's typically at about depth *d/2* after the splay

- Overall, nodes which are low on the access path tend to move closer to the root

- Splaying gets amortized O(log n) performance.

Practical Benefit of Splaying

- No heights to maintain, no imbalance to check for
  - Less storage per node, easier to code
- Data accessed once, is often soon accessed again
  - Splaying does implicit *caching* by bringing it to the root

Splay Operations: Find, Insert

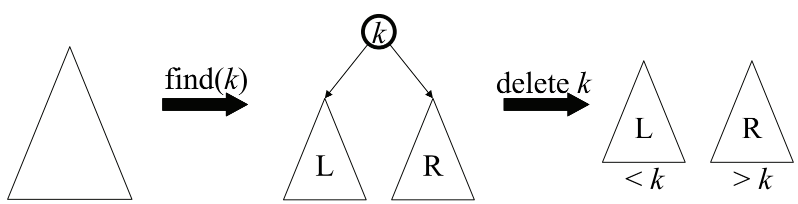
Find(x):

- Find the node in normal BST manner
- Splay the node to the root
  - if node <u>not</u> found, splay what would have been its parent

Insert(x):

- Insert the node in normal BST manner
- Splay the node to the root

Splay Operations: Remove

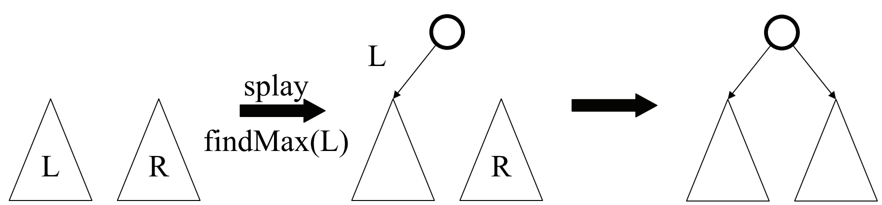


Now what?

Join

Join(L, R):

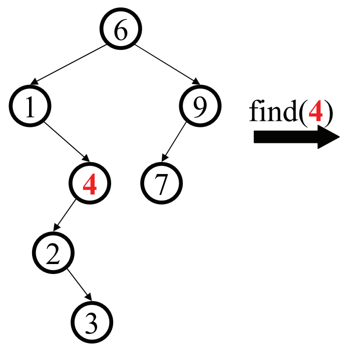
given two trees such that (stuff in L) < (stuff in R), merge them:



Splay on the maximum element in L, then attach R

Delete Example

Delete(**4**)



Splay Tree Summary

All operations are in amortized O(log *n*) time

Splaying can be done top-down; this may be better because:

- only one pass
- no recursion or parent pointers necessary
- (we didn't cover top-down in class)

Splay trees are *very* effective search trees

- Relatively simple
- No extra fields required
- Excellent *locality* properties in the following sense: frequently accessed keys are cheap to find