CSE 326 Data Structures
Midterm Review

Hal Perkins

Spring 2007

Dates

- Midterm Friday!
- Project 2 due next Wednesday
- Homework 4
  - Hmmmm…..
  - We ought to talk about this….

Logistics

- Closed Notes
- Closed Book
- Open Mind
- You may bring a calculator, though don't even think about loading it with notes or programs. And you probably won't find it of much use anyway.

Material Covered

- Everything we've talked/read in class up to AVL trees
  - And for AVL trees, up to inserting and rotations, but not implementations in Java

Material Not Covered

- We won't make you write syntactically correct Java code (pseudocode okay)
- We won't make you do a super hard proof
- We won't test you on the details of generics, interfaces, etc. in Java
  - But you should know the basic ideas since we spent a lecture on them and had to deal with them in project 2A

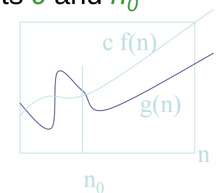
Back to our two functions f and g from before

Order Notation: Definition

O(f(*n*)) : <u>a set or class of functions</u>

$g(n) \in O(f(n))$ iff there exist consts c and n_0 such that:

$g(n) \leq c\, f(n)$ for all $n \geq n_0$



Example: $g(n) = 1000n$ vs. $f(n) = n^2$

Is $g(n) \in O(f(n))$?

Pick: n0 = 1000, c = 1

$1000n \leq 1 * n^2$ for all $n \geq 1000$

So $g(n) \in O(f(n))$

Log?

$\log_k n \in O(\log_2 n)$? $\log_k n = \log_2 n / \log_2 k$

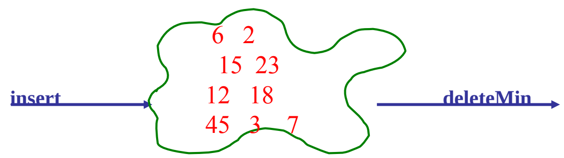
$\log_2 n^2 \in O(\log_2 n)$? $\log_2 n^2 = 2\log_2 n$

Definition of Order Notation

- Upper bound: $T(n) = O(f(n))$ Big-O
  Exist constants c and n' such that
  $T(n) \leq c\, f(n)$ for all $n \geq n'$
- Lower bound: $T(n) = \Omega(g(n))$ Omega
  Exist constants c and n' such that
  $T(n) \geq c\, g(n)$ for all $n \geq n'$
- Tight bound: $T(n) = \theta(f(n))$ Theta
  When both hold:
  $T(n) = O(f(n))$
  $T(n) = \Omega(f(n))$

Priority Queue ADT

- Checkout line at the supermarket ???
- Printer queues ???
- operations: insert, deleteMin



Implementations of Priority Queue ADT

	insert	deleteMin
Unsorted list (Array)	**O(1)**/O(N)worst-array full, should say WHY, might reject on full instead.	**O(N)** – to find value
Unsorted list (Linked-List)	**O(1)**	**O(N)** – to find value
Sorted list (Array)	O(log N) to find loc w. Bin search, but **O(N)** to move vals	O(1) to find val, but O(N) to move vals, (or **O(1)** if in reverse order
Sorted list (Linked-List)	**O(N)** to find loc, O(1) to do the insert	**O(1)**
Binary Search Tree (BST)	**O(N)**	**O(N)**
Plus – good memory usage **Binary Heap** Binary Heap	**O(log N)** **close to O(1)** **1.67 levels on average**	**O(log N)**

Tree Review

root(**T**): A

leaves(**T**): DEFJ..NI

children(B):

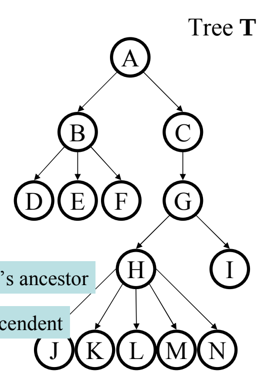
parent(H):

siblings(E):

ancestors(F): Its parent or parent's ancestor

descendents(G): Its child or child's descendent

subtree(C): Itself plus all descendents

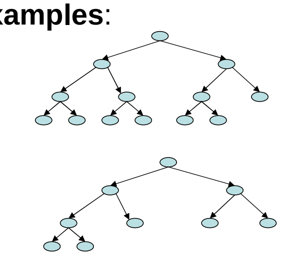


Heap **Structure** Property

- A binary heap is a ***complete*** binary tree.

Complete binary tree – binary tree that is completely filled, with the possible exception of the bottom level, which is filled left to right.

Examples:

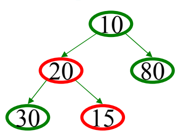


Since they have this **regular** structure property, we can take advantage of that to store them in a **compact** manner.

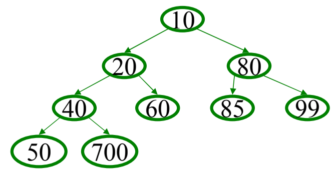
Heap Order Property

Heap order property: For every non-root node X, the value in the parent of X is less than (or equal to) the value in X.

This is the order for a MIN heap – could do the same for a max heap.



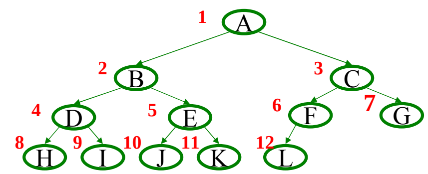
not a heap



This is a PARTIAL order (diff than BST)

For each node, its value is less than all of its descendants (no distinction between left and right)

Representing Complete Binary Trees in an Array



From node **i**:

left child: $2 * i$

right child: $(2 * i)+1$

parent: $\lfloor i / 2 \rfloor$

implicit (array) implementation:

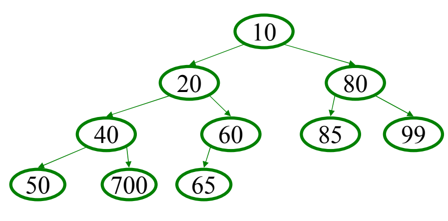
	A	B	C	D	E	F	G	H	I	J	K	L	
0	1	2	3	4	5	6	7	8	9	10	11	12	13

Heap Operations

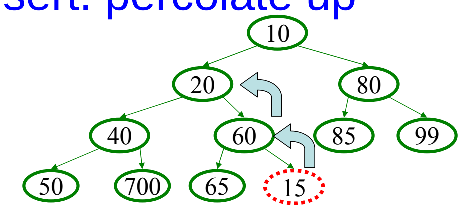
How?

- findMin:
- insert(val): percolate up.
- deleteMin: percolate down.

Is the tree unique?
Swap 85 and 99.
Swap 700 and 85?



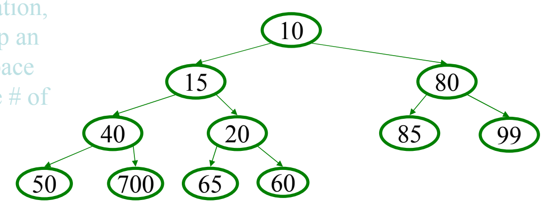
Insert: percolate up



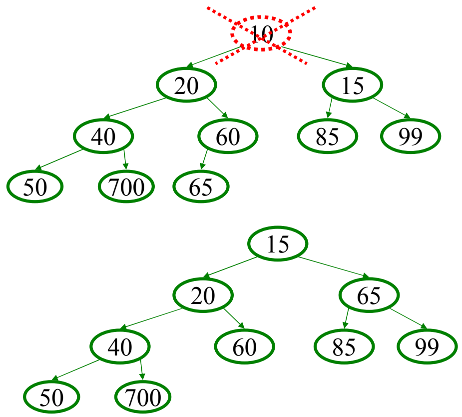
Now insert 90.
(no swaps, even though 99 is larger!)

Now insert 7.

Optimization, bubble up an empty space to reduce # of swaps



DeleteMin: percolate down



Max # of exchanges? = O(log N),

There is a good chance goes to bottom (started at bottom) vs. insert

- Could also use the percolate empty bubble down

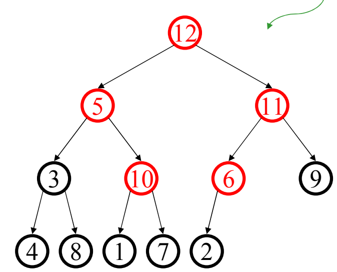
BuildHeap: Floyd's Method

0 1 2 3 10 11 12

12	5	11	3	10	6	9	4	8	1	7	2

Add elements arbitrarily to form a complete tree.
Pretend it's a heap and fix the heap-order property!

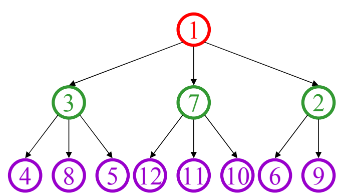
Red nodes need to percolate down



A Solution: *d*-Heaps

How does height compare to bin heap? (less)

- Each node has *d* children
- Still representible by array
- Good choices for *d*:
  - (choose a power of two for efficiency)
  - fit one set of children in a cache line
  - fit one set of children on a memory page/disk block



12	1	3	7	2	4	8	5	12	11	10	6	9

Operations on *d*-Heap

- Insert : runtime =

depth of tree decreases, $O(\log_d n)$ worst

- deleteMin: runtime =

percolateDown requires comparison to find min, $O(d \log_d n)$, worst/ave

Does this help insert or deleteMin more?

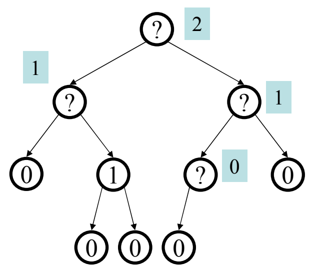
Definition: Null Path Length

null path length (npl) of a node *x* = the number of nodes between *x* and a null in its subtree

OR

npl(x) = min distance to a descendant with 0 or 1 children

- *npl*(null) = -1
- *npl*(leaf) = 0
- *npl*(single-child node) = 0



Equivalent definitions:

1. *npl(x)* is the height of largest complete subtree rooted at *x*
2. *npl(x)* = 1 + min{*npl*(left(x)), *npl*(right(x))}

Leftist Heap Properties

- Heap-order property
  - parent's priority value is ≤ to childrens' priority values
  - result: minimum element is at the root
- Leftist property
  - For every node *x*, *npl*(left(*x*)) ≥ *npl*(right(*x*))
  - result: tree is at least as "heavy" on the left as the right

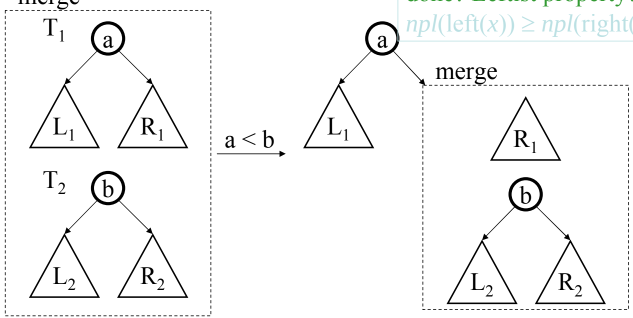
Are leftist trees…

complete? No,

balanced? no

Merging Two Leftist Heaps

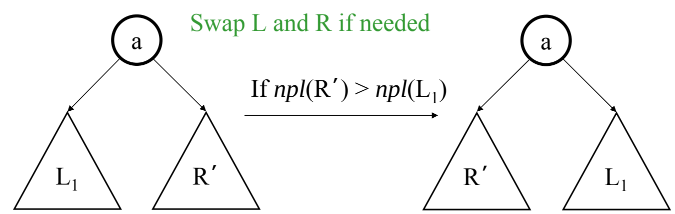
- merge(T_1,T_2) returns one leftist heap containing all elements of the two (distinct) leftist heaps T_1 and T_2



done? Leftist property?

npl(left(*x*)) ≥ *npl*(right(*x*))

Leftist Merge Continued



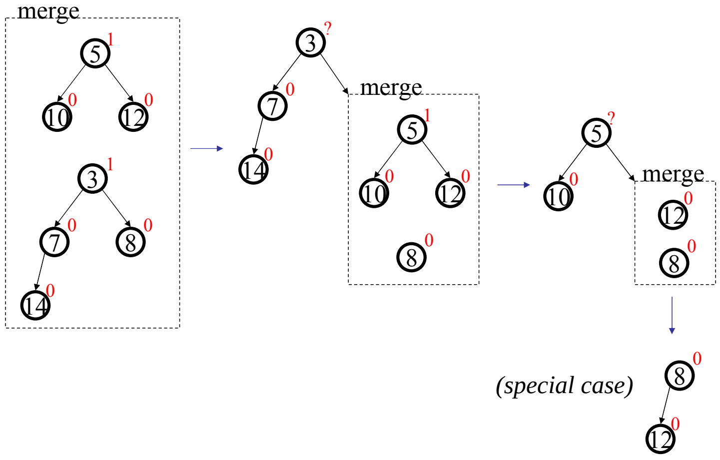
Swap L and R if needed

R′ = Merge(R_1, T_2)

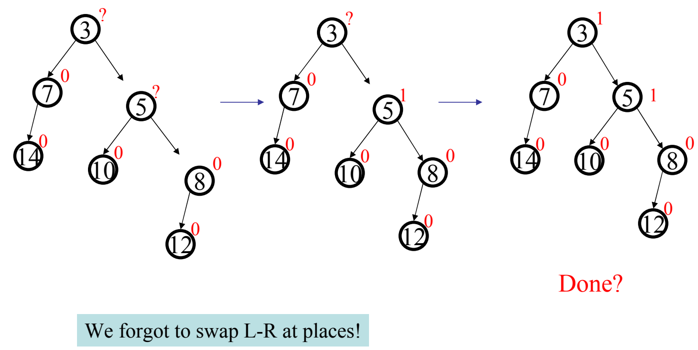
Work at each step = call to merge, swap (constant)
traverse the right path of both trees = length is at most log N

runtime: O(log n)

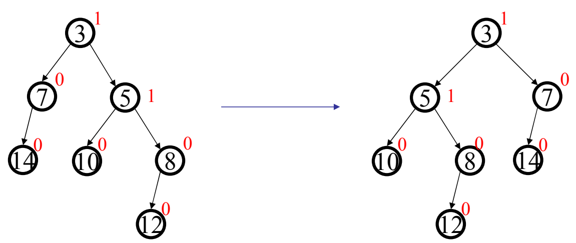
Leftist Merge Example



Sewing Up the Leftist Example



Finally…(Leftist)



Skew Heaps

Problems with <u>leftist</u> heaps

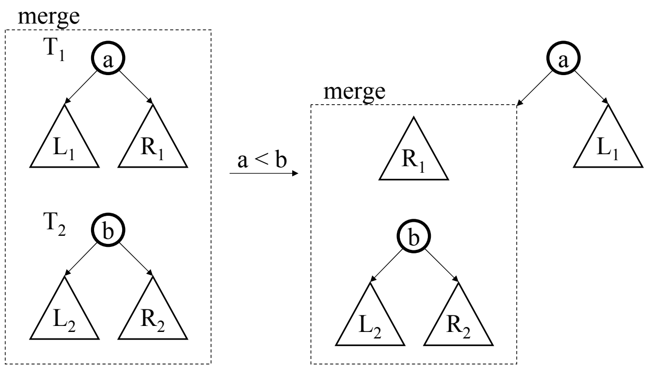
- extra storage for npl
- extra complexity/logic to maintain and check npl
- right side is “often” heavy and requires a switch

- Simple to implement,
- no npl stuff

Solution: <u>skew</u> heaps

- “blindly” adjusting version of leftist heaps
- merge *always* switches children when fixing right path
- <u>amortized time</u> for: merge, insert, deleteMin = O(log *n*)
- however, <u>worst case time</u> for all three = O(*n*)

Merging Two Skew Heaps



Only one step per iteration, with children *always* switched

Yet Another Data Structure: Binomial Queues

- Structural property
  - Forest of binomial trees with at most one tree of any height

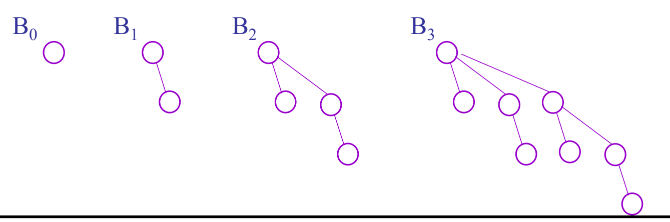
What's a forest?

What's a binomial tree?

- Order property
  - Each binomial tree has the heap-order property

The Binomial Tree, B_h

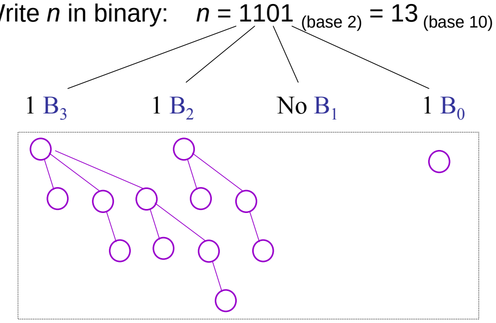
- B_h has height h and exactly 2^h nodes
- B_h is formed by making B_{h-1} a child of another B_{h-1}
- Root has exactly h children
- Number of nodes at depth d is binomial coeff. $\binom{h}{d}$
  - Hence the name; we will *not* use this last property



Binomial Queue with *n* elements

Binomial Q with *n* elements has a *unique* structural representation in terms of binomial trees!

Write *n* in binary: $n = 1101_{\text{(base 2)}} = 13_{\text{(base 10)}}$



Merging Two Binomial Queues

Essentially like adding two binary numbers!

1. Combine the two forests
2. For k from 1 to maxheight {
   a. $m \leftarrow$ total number of B_k's in the two BQs
   b. if m=0: continue;
   c. if m=1: continue;
   d. if m=2: combine the two B_k's to form a B_{k+1}
   e. if m=3: retain one B_k and combine the other two to form a B_{k+1}

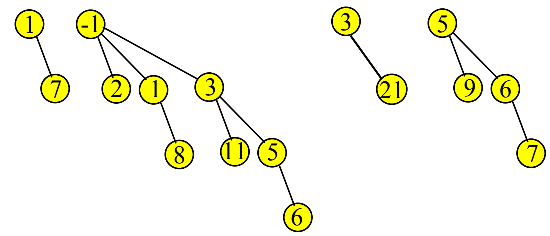
}

# of 1's
0+0 = 0
1+0 = 1
1+1 = 0+c
1+1+c = 1+c

Claim: When this process ends, the forest has at most one tree of any height

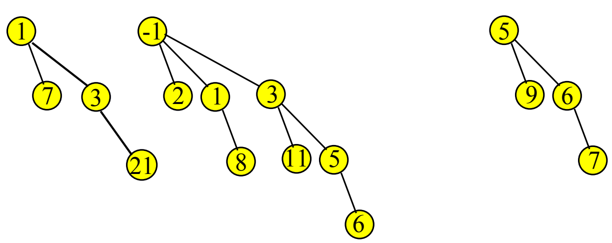
Example: Binomial Queue Merge

H1: H2:



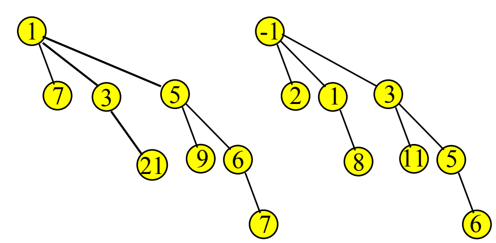
Example: Binomial Queue Merge

H1: H2:

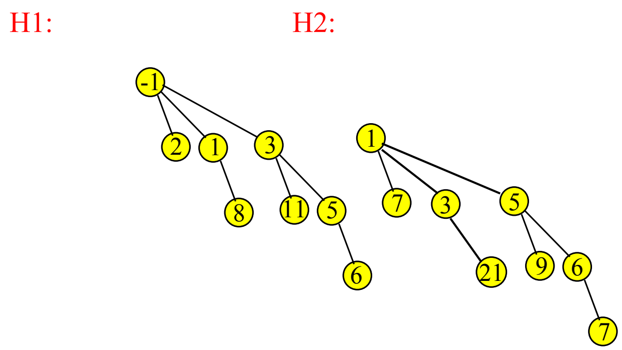


Example: Binomial Queue Merge

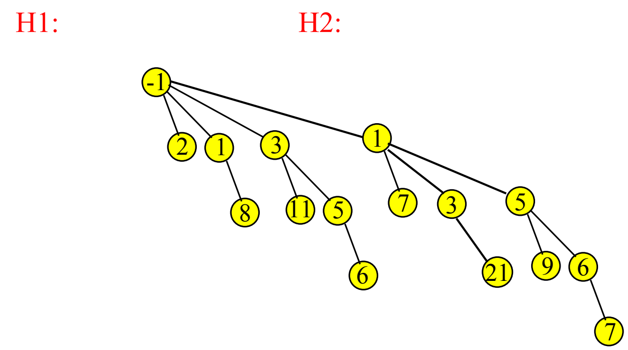
H1: H2:



Example: Binomial Queue Merge



Example: Binomial Queue Merge

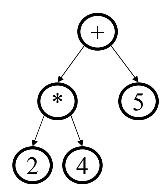


More Recursive Tree Calculations: Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

Three types:

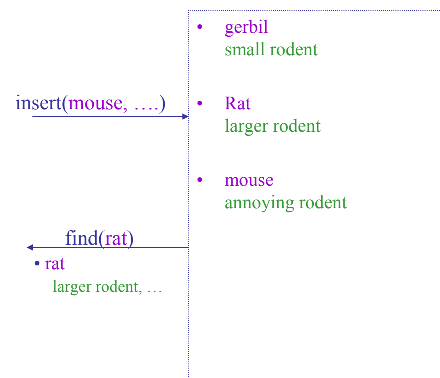
- Pre-order: Root, left subtree, right subtree
- In-order: Left subtree, root, right subtree
- Post-order: Left subtree, right subtree, root



(an expression tree)

The Dictionary ADT

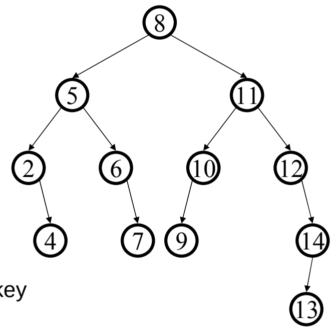
- Data:
  - a set of (key, value) pairs
- Operations:
  - Insert (key, value)
  - Find (key)
  - Remove (key)



*The Dictionary ADT is sometimes called the "**Map ADT**"*

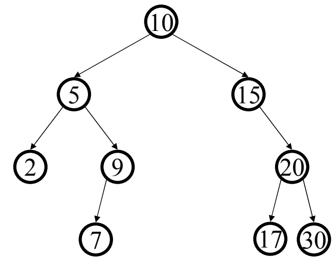
Binary Search Tree Data Structure

- Structural property
  - each node has ≤ 2 children
  - result:
    - storage is small
    - operations are simple
    - average depth is small
- Order property
  - all keys in left subtree smaller than root's key
  - all keys in right subtree larger than root's key
  - result: easy to find any given key
- What must I know about what I store?



Comparison, equality testing

Find in BST, Recursive

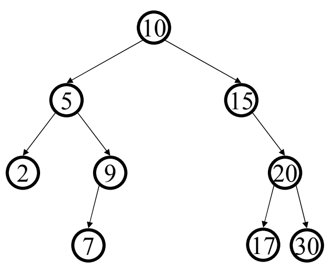



```
Node Find(Object key,
                Node root) {
  if (root == NULL)
    return NULL;

  if (key < root.key)
    return Find(key,

   root.left);
  else if (key > root.key)
    return Find(key,

   root.right);
  else
    return root;
```

Runtime:

Θ(depth) = Θ(n) worst, Θ(log n) avg

Insert in BST



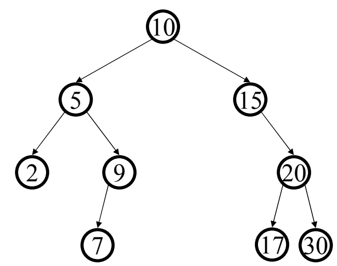
Insert(13)
Insert(8)
Insert(31)

Insertions happen only at the leaves – easy!

Runtime:

O(depth) = O(n) worst, O(log n) avg

Deletion in BST

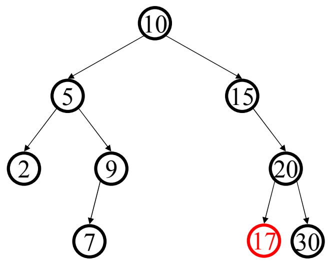


Why might deletion be harder than insertion?

May be in middle, instead of at leaf

Non-lazy Deletion – The Leaf Case

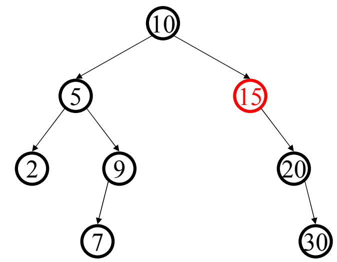
Delete(17)



Easy – prune

Deletion – The One Child Case

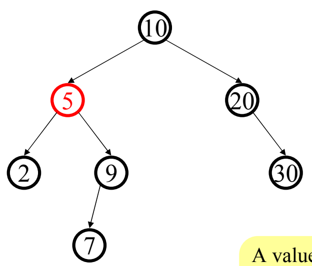
Delete(15)



Pull up child – will this always work?

Deletion – The Two Child Case

Delete(5)



What can we replace 5 with?

A value guaranteed to be between the two subtrees!
- *succ* from right subtree
- *pred* from left subtree

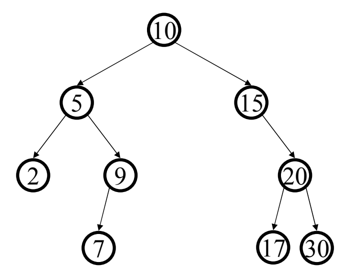
How long do these operations take? (find, insert, delete)

Lazy Deletion

Instead of physically deleting nodes, just mark them as deleted

+ simpler
+ physical deletions done in batches
+ some adds just flip deleted flag

– extra memory for deleted flag
– many lazy deletions slow finds
– some operations may have to be modified (e.g., min and max)



Balanced BST

Observation
- BST: the shallower the better!
- For a BST with *n* nodes
  - Average height is O(log *n*)
  - Worst case height is O(*n*)
- Simple cases such as insert(1, 2, 3, ..., n) lead to the worst case scenario

Solution: Require a **Balance Condition** that
1. ensures depth is O(log *n*) – strong enough!
2. is easy to maintain – not too strong!

Adelson-Velskii and Landis

The AVL Balance Condition

Left and right subtrees of *every node* have equal *heights* **differing by at most 1**

Define: **balance**(x) = height(x.left) – height(x.right)

AVL property: **–1 ≤ balance(x) ≤ 1, for every node x**

- Ensures small depth
  - Will prove this by showing that an AVL tree of height h must have a lot of (i.e. O(2^h)) nodes
- Easy to maintain
  - Using single and double rotations

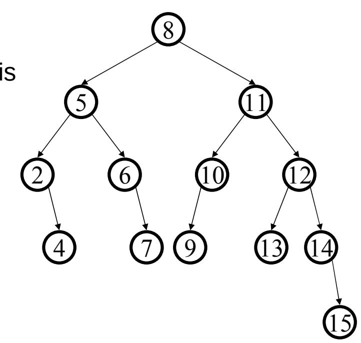
The AVL Tree Data Structure

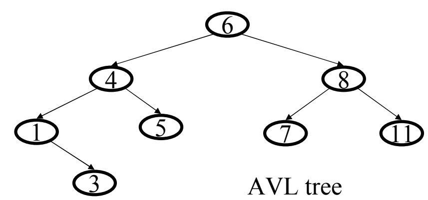
Structural properties
1. Binary tree property
2. Balance property: balance of every node is between -1 and 1

Result:

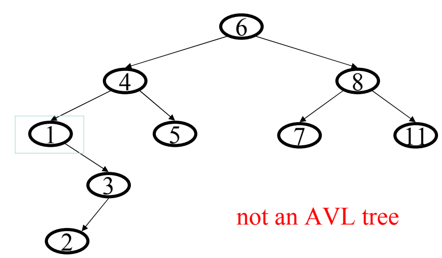
Worst case depth is O(log *n*)

Ordering property
- Same as for BST





AVL tree



not an AVL tree

AVL tree insert

Draw 1-4 pic

Let *x* be the node where an imbalance occurs.

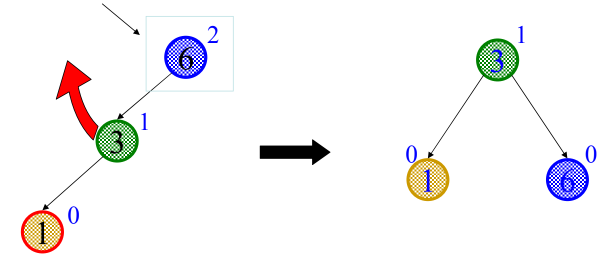
Four cases to consider. The insertion is in the

1. left subtree of the left child of *x.*
2. right subtree of the left child of *x.*
3. left subtree of the right child of *x.*
4. right subtree of the right child of *x.*

Idea: Cases 1 & 4 are solved by a single rotation.
Cases 2 & 3 are solved by a double rotation.

Fix: Apply Single Rotation

AVL Property violated at this node (x)

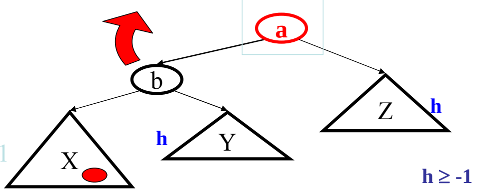


Single Rotation:
1. Rotate between x and child

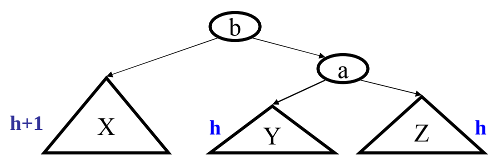
Single rotation in general

- Before red dot, X = h,
- Q:height of tree is? h+2,
-After red dot, X = h+1

Case 1, same for case 4



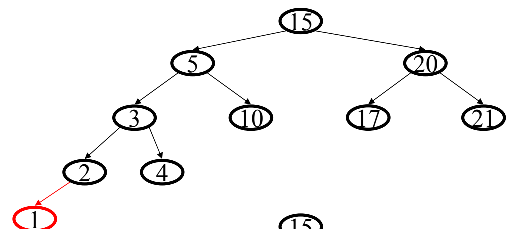
X < b < Y < a < Z

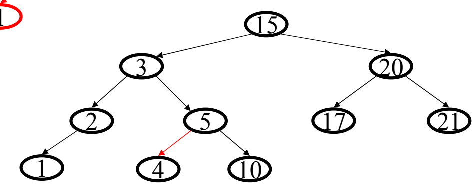


h+ 2 h+ 2 none

Height of tree before? Height of tree after? Effect on Ancestors?

Single rotation example

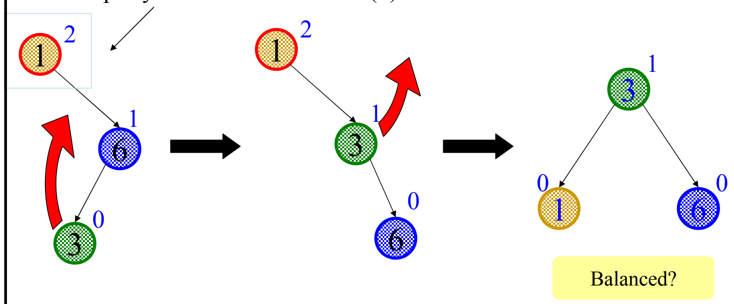




Fix: Apply Double Rotation

Intuition: 3 must become root

AVL Property violated at this node (x)



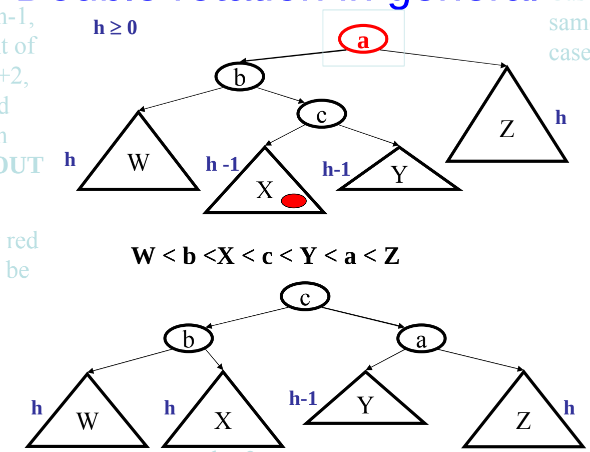
Double Rotation
1. Rotate between x's child and grandchild
2. Rotate between x and x's new child

Double rotation in general

-**Before** red dot, X = h-1,
- Q:height of tree is? h+2,
-**After** red dot, X = h
CROSSOUT -1

-Actually red dot could be at X or Y

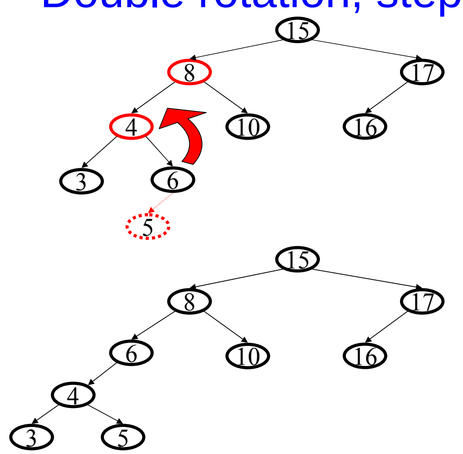
Case 2, same for case 3



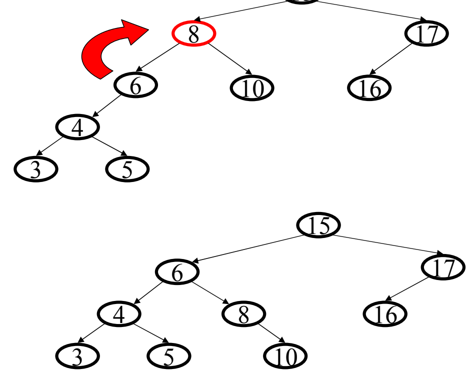
h+ 2 h+ 2 none

Height of tree before? Height of tree after? Effect on Ancestors?

Double rotation, step 1

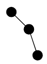


Double rotation, step 2



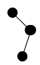
Insertion into AVL tree

1. Find spot for new key
2. Hang new node there with this key
3. Search back up the path for imbalance
4. If there is an imbalance:



case #1: Perform single rotation and exit

Zig-zig



case #2: Perform double rotation and exit

Zig-zag

Both rotations keep the subtree height unchanged.
Hence only one (sinlge or double) rotation is sufficient!