CSE 326: Data Structures Dynamic Programming – Floyd/Warhsall Algorithm

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Lectures 26

Single-Source Shortest Path

- Given a graph **G** = (V, E) and a single distinguished vertex **s**, find the shortest weighted path from **s** to every other vertex in **G**.

All-Pairs Shortest Path:

- Find the shortest paths between all pairs of vertices in the graph.
- How?

Analysis

- Total running time for Dijkstra's:

  $O(|V|^2 + |E|)$ (linear scan)

  $O(|V| \log |V| + |E| \log |V|)$ (heaps)

  What if we want to find the shortest path from each point to ALL other points?

Dynamic Programming

Algorithmic technique that systematically <u>records</u> the answers to sub-problems in a table and <u>re-uses</u> those recorded results (rather than re-computing them).

Simple Example: Calculating the Nth Fibonacci number.

Fib(N) = Fib(N-1) + Fib(N-2)

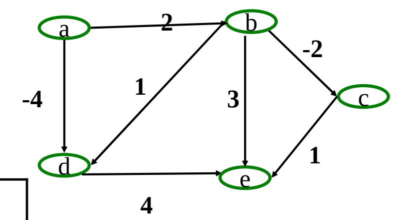
Floyd-Warshall

```
for (int k = 1; k =< V; k++)
 for (int i = 1; i =< V; i++)
  for (int j = 1; j =< V; j++)
   if ( ( M[i][k]+ M[k][j] ) < M[i][j] )
     M[i][j] =  M[i][k]+ M[k][j]
```

Invariant: After the kth iteration, the matrix includes the shortest paths for all pairs of vertices (i,j) containing only vertices 1..k as intermediate vertices

5

Initial state of the matrix:

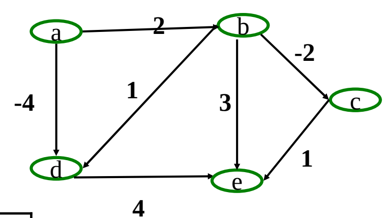


	a	b	c	d	e
a	0	2	-	-4	-
b	-	0	-2	1	3
c	-	-	0	-	1
d	-	-	-	0	4
e	-	-	-	-	0

M[i][j] = min(M[i][j], M[i][k]+ M[k][j])

6

Floyd-Warshall - for All-pairs shortest path



	a	b	c	d	e
a	0	2	0	-4	0
b	-	0	-2	1	-1
c	-	-	0	-	1
d	-	-	-	0	4
e	-	-	-	-	0

Final Matrix Contents

7