CSE 326: Data Structures

Priority Queues – Binary Heaps

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Spring 2007

Lectures 3 & 4

Outline

- Admin (proj #1)
- Math/Big-O – short summary
- Priority Queues (Binary Min Heaps)
  - Reading: Weiss, Ch. 6

Project #1 Turn-in

- The turnin page for project 1 is now linked to the project page.
- Turn in your **electronic documents** before **midnight** on Wednesday.
- Turn in **hardcopies** in sections on Thursday (whichever section you normally attend).

Simplifying Recurrences

Given a recursive equation for the running time, can sometimes simplify it for analysis.

- For an upper-bound analysis, can optionally simplify to something larger, e.g.

  $T(n) = T(\text{floor}(n/2)) + 1$ to $T(n) \le T(n/2) + 1$

- For a lower-bound analysis, can optionally simplify to something smaller, e.g.

  $T(n) = 2T(n/2 + 5) + 1$ to $T(n) \ge 2T(n/2) + 1$

The One Page Cheat Sheet

- **Calculating series:**
  e.g. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

  1. Brute force (Section 1.2.3)
  2. Induction (Section 1.2.5)
  3. Memorize simple ones!

- **Solving recurrences:**
  e.g. T(n) = T(n/2) + 1

  1. Expansion (example in class)
  2. Induction (Section 1.2.5)
  3. Telescoping (later…)

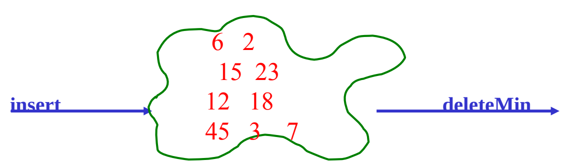
- **General proofs (Section 1.2.5)**
  e.g. *How many edges in a tree with n nodes?*

  1. Counterexample
  2. Induction
  3. Contradiction

Processor Scheduling

Priority Queue ADT

- Checkout line at the supermarket ???
- Printer queues ???
- operations: insert, deleteMin



Priority Queue ADT

1. **PQueue <u>data</u>** : collection of data with priority
2. **PQueue <u>operations</u>**
   - insert
   - deleteMin

   (also: create, destroy, is_empty)
3. **PQueue <u>property</u>**: for two elements in the queue, x and y, if x has a **<u>lower</u>** priority value than y, x will be deleted before y

Applications of the Priority Q

- Select print jobs in order of decreasing length
- Forward packets on network routers in order of urgency
- Select most frequent symbols for compression
- Sort numbers, picking minimum first

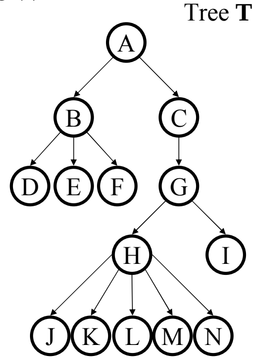
- **Anything *greedy***

Implementations of Priority Queue ADT

	insert	deleteMin
Unsorted list (Array)		
Unsorted list (Linked-List)		
Sorted list (Array)		
Sorted list (Linked-List)		
Binary Search Tree (BST)		

Tree Review

Tree **T**



root(**T**):

leaves(**T**):

children(B):

parent(H):

siblings(E):

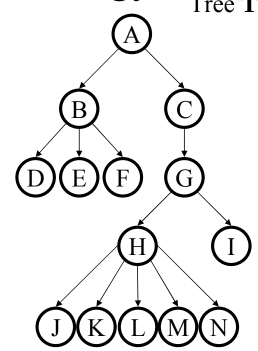
ancestors(F):

descendents(G):

subtree(C):

More Tree Terminology

Tree **T**



depth(**T**):

height(G):

degree(B):

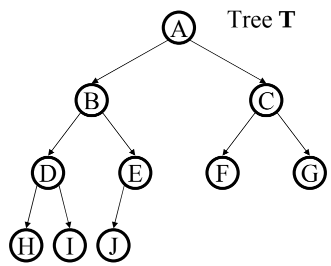
branching factor(**T**):

Some More Tree Terminology

T is *binary* if …

T is *n-ary* if …

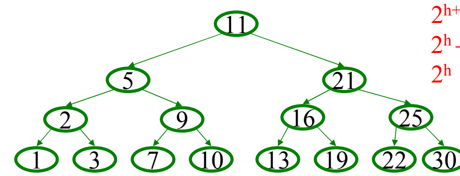
T is *complete* if …



How deep is a complete tree with *n* nodes?

Brief interlude: Some Definitions:

A ***Perfect*** binary tree – A binary tree with all leaf nodes at the same depth. All internal nodes have 2 children.



height h
$2^{h+1} - 1$ nodes
$2^h - 1$ non-leaves
2^h leaves

Full Binary Tree

- A binary tree in which each node has ***exactly zero or two children***.
- (also known as a proper binary tree)
- (we will use this later for Huffman trees)

Binary Heap Properties

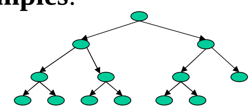
1. Structure Property
2. Ordering Property

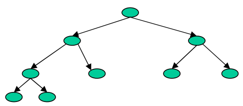
Heap **Structure** Property

- A binary heap is a ***complete*** binary tree.

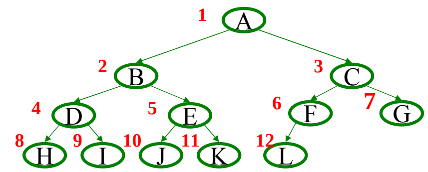
Complete binary tree – binary tree that is completely filled, with the possible exception of the bottom level, which is filled left to right.

Examples:





Representing Complete Binary Trees in an Array



From node **i**:

left child:
right child:
parent:

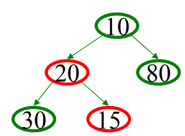
implicit (array) implementation:

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	
0	1	2	3	4	5	6	7	8	9	10	11	12	13

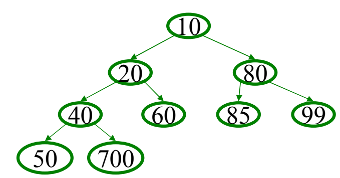
Why better than tree with pointers?

Heap **Order** Property

Heap order property: For every non-root node X, the value in the parent of X is less than (or equal to) the value in X.

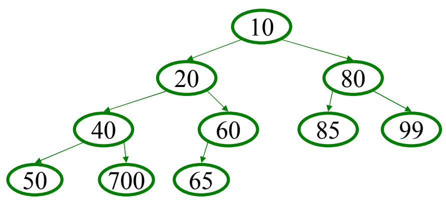


not a heap



Heap Operations

- findMin:
- insert(val): percolate up.
- deleteMin: percolate down.

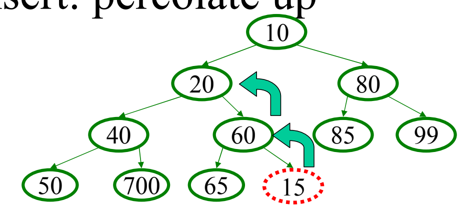


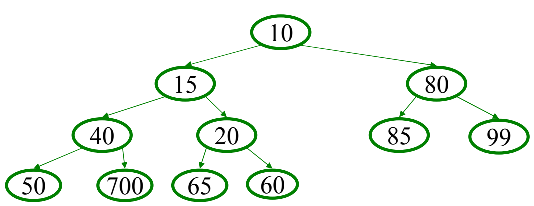
Heap – Insert(val)

Basic Idea:

1. Put val at “next” leaf position
2. Repeatedly exchange node with its parent if needed

Insert: percolate up





Insert pseudo/C++ Code (optimized)

```
void insert(Object o) {
  assert(!isFull());
  size++;
  newPos =
    percolateUp(size,o);
  Heap[newPos] = o;
}
```

```
int percolateUp(int hole,
                Object val) {
  while (hole > 1 &&
         val < Heap[hole/2])
    Heap[hole] = Heap[hole/2];
    hole /= 2;
  }
  return hole;
}
```

runtime:

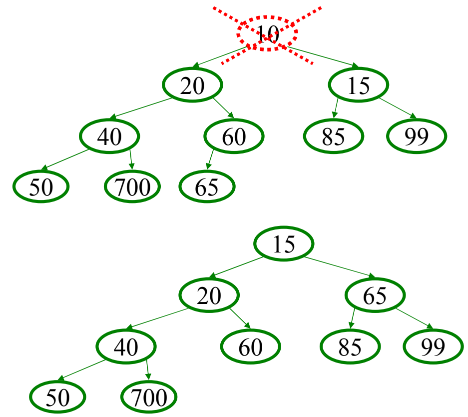
(Java code in book)

Heap – Deletemin

Basic Idea:

1. Remove root (that is always the min!)
2. Put “last” leaf node at root
3. Find smallest child of node
4. Swap node with its smallest child if needed.
5. Repeat steps 3 & 4 until no swaps needed.

DeleteMin: percolate down



DeleteMin pseudo/C++ Code (Optimized)

```
Object deleteMin() {
  assert(!isEmpty());
  returnVal = Heap[1];
  size--;
  newPos =
    percolateDown(1,
        Heap[size+1]);
  Heap[newPos] =
    Heap[size + 1];
  return returnVal;
}
```

runtime:

(Java code in book)

```
int percolateDown(int hole,
                  Object val) {
while (2*hole <= size) {
    left = 2*hole;
    right = left + 1;
    if (right ≤ size &&
        Heap[right] < Heap[left])
      target = right;
    else
      target = left;

    if (Heap[target] < val) {
      Heap[hole] = Heap[target];
      hole = target;
    }
    else
      break;
  }
  return hole;
}
```


Insert: 16, 32, 4, 69, 105, 43, 2

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**

More Priority Queue Operations

- **decreaseKey**
  - given a pointer to an object in the queue, reduce its priority value

  Solution: change priority and ____________________________

- **increaseKey**
  - given a pointer to an object in the queue, increase its priority value

  Solution: change priority and ____________________________

Why do we need a *pointer*? Why not simply data value?

More Heap Operations

decreaseKey(objPtr, amount): raise the priority of a object, percolate up

increaseKey(objPtr, amount): lower the priority of a object, percolate down

remove(objPtr): remove a object, move to top, them delete. 1) decreaseKey(**objPtr**, ∞)

2) deleteMin()

Worst case Running time for all of these:

FindMax?

ExpandHeap – when heap fills, copy into new space.

More Priority Queue Operations

- **Remove(objPtr)**
  - given a pointer to an object in the queue, remove it

  Solution: set priority to negative infinity, percolate up to root and deleteMin

- **buildHeap**

  Naïve solution:

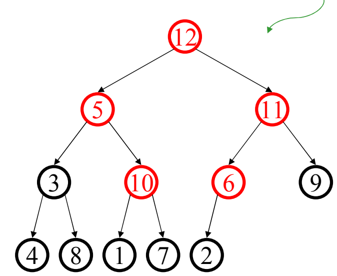
  Running time:

Can we do better?

BuildHeap: Floyd's Method

12	5	11	3	10	6	9	4	8	1	7	2

Add elements arbitrarily to form a complete tree.
Pretend it's a heap and fix the heap-order property!

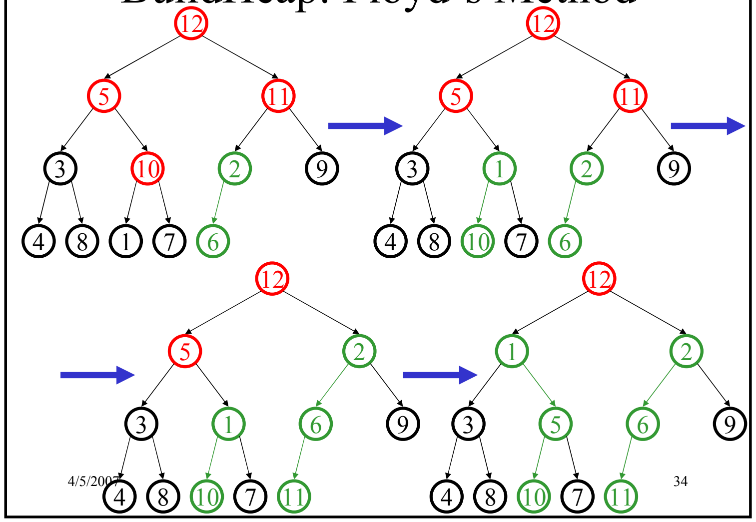


Buildheap pseudocode

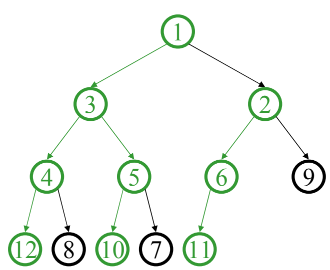
```
private void buildHeap() {
  for ( int i = currentSize/2; i > 0; i-- )
     percolateDown( i );
}
```

runtime:

BuildHeap: Floyd's Method



Finally…



runtime:

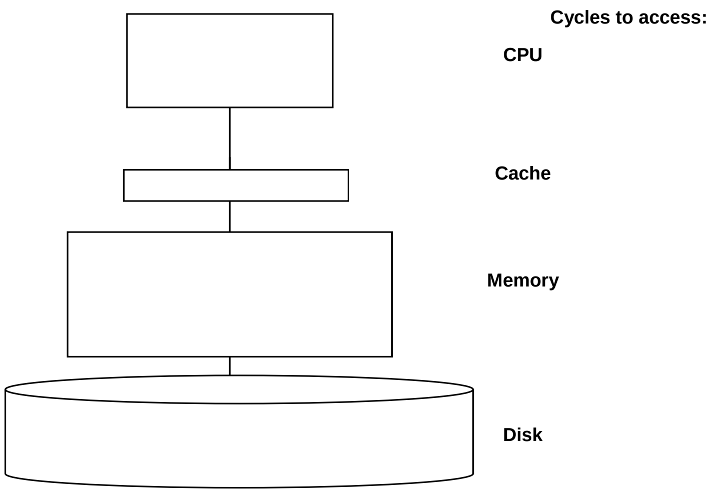
Facts about Heaps

Observations:

- finding a child/parent index is a multiply/divide by two
- operations jump widely through the heap
- each percolate step looks at only two new nodes
- inserts are at least as common as deleteMins

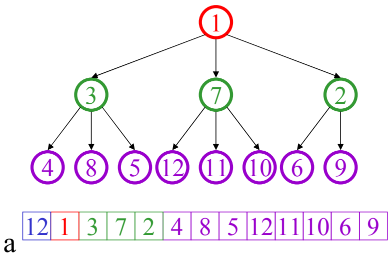
Realities:

- division/multiplication by powers of two are equally fast
- looking at only two new pieces of data: bad for cache!
- with huge data sets, disk accesses dominate



A Solution: *d*-Heaps

- Each node has *d* children
- Still representable by array
- Good choices for *d*:
  - (choose a power of two for efficiency)
  - fit one set of children in a cache line
  - fit one set of children on a memory page/disk block



Operations on *d*-Heap

- Insert : runtime =
- deleteMin: runtime =

Does this help insert or deleteMin more?

One More Operation

- Merge two heaps. Ideas?