CSE 326: Data Structures Dictionaries for Data Compression

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Autumn 2007

Dictionary Coding

- Does not use statistical knowledge of data.
- Encoder: As the input is processed develop a dictionary and transmit the index of strings found in the dictionary.
- Decoder: As the code is processed reconstruct the dictionary to invert the process of encoding.
- Examples: LZW, LZ77, Sequitur,
- Applications: Unix Compress, gzip, GIF

LZW Encoding Algorithm

```
Repeat
  find the longest match w in the dictionary
  output the index of w
  put wa in the dictionary where a was the
                unmatched symbol
```

LZW Encoding Example (1)

Dictionary

0	a
1	b

a b a b a b a b a

LZW Encoding Example (2)

Dictionary

0	a
1	b
2	ab

a b a b a b a b a

0

LZW Encoding Example (3)

Dictionary

0	a
1	b
2	ab
3	ba

a b a b a b a b a

0 1

LZW Encoding Example (4)

Dictionary

0 a
1 b
2 ab
3 ba
4 aba

a b a b a b a b a
0 1 2

LZW Encoding Example (5)

Dictionary

0	a
1	b
2	ab
3	ba
4	aba
5	abab

a b a b a b a b a

0 1 2 4

LZW Encoding Example (6)

Dictionary

0	a
1	b
2	ab
3	ba
4	aba
5	abab

a b a b a b a b a

0 1 2 4 3

LZW Decoding Algorithm

- Emulate the encoder in building the dictionary. Decoder is slightly behind the encoder.

```
initialize dictionary;
decode first index to w;
put w? in dictionary;
repeat
   decode the first symbol s of the index;
   complete the previous dictionary entry with s;
   finish decoding the remainder of the index;
   put w? in the dictionary where w was just decoded;
```

LZW Decoding Example (1)

Dictionary

0	a
1	b
2	a?

<u>0</u> 1 2 4 3 6

a

LZW Decoding Example (2a)

Dictionary

0 a
1 b
2 ab

0 1 2 4 3 6
a b

LZW Decoding Example (2b)

Dictionary

0	a
1	b
2	ab
3	b?

0 1 2 4 3 6

a b

LZW Decoding Example (3a)

Dictionary

0	a
1	b
2	ab
3	ba

0 1 2 4 3 6

a b a

LZW Decoding Example (3b)

Dictionary

0	a
1	b
2	ab
3	ba
4	ab?

0 1 2 4 3 6

a b ab

LZW Decoding Example (4a)

Dictionary

0	a
1	b
2	ab
3	ba
4	aba

0 1 2 4 3 6

a b ab a

LZW Decoding Example (4b)

Dictionary

0 a
1 b
2 ab
3 ba
4 aba
5 aba?

0 1 2 4 3 6
a b ab aba

LZW Decoding Example (5a)

Dictionary

0	a
1	b
2	ab
3	ba
4	aba
5	abab

0 1 2 4 3 6
a b ab aba b

LZW Decoding Example (5b)

Dictionary

0	a
1	b
2	ab
3	ba
4	aba
5	abab
6	ba?

<u>0</u> <u>1</u> <u>2</u> <u>4</u> <u>3</u> 6

a b ab aba ba

LZW Decoding Example (6a)

Dictionary

0	a
1	b
2	ab
3	ba
4	aba
5	abab
6	bab

<u>0</u> <u>1</u> <u>2</u> <u>4</u> <u>3</u> <u>6</u>
a b ab aba ba b

LZW Decoding Example (6b)

Dictionary

0	a
1	b
2	ab
3	ba
4	aba
5	abab
6	bab
7	bab?

0 1 2 4 3 6

a b ab aba ba bab

Decoding Exercise

Base Dictionary

0	a
1	b
2	c
3	d
4	r

0 1 4 0 2 0 3 5 7

Bounded Size Dictionary

- Bounded Size Dictionary
  - n bits of index allows a dictionary of size 2^n
  - Doubtful that long entries in the dictionary will be useful.
- Strategies when the dictionary reaches its limit.
  1. Don’t add more, just use what is there.
  2. Throw it away and start a new dictionary.
  3. Double the dictionary, adding one more bit to indices.
  4. Throw out the least recently visited entry to make room for the new entry.

Notes on LZW

- Extremely effective when there are repeated patterns in the data that are widely spread.
- Negative: Creates entries in the dictionary that may never be used.
- Applications:
  - Unix compress, GIF, V.42 bis modem standard

LZ77

- Ziv and Lempel, 1977
- Dictionary is implicit
- Use the string coded so far as a dictionary.
- Given that $x_1x_2...x_n$ has been coded we want to code $x_{n+1}x_{n+2}...x_{n+k}$ for the largest k possible.

Solution A

- If $x_{n+1}x_{n+2}...x_{n+k}$ is a substring of $x_1x_2...x_n$ then $x_{n+1}x_{n+2}...x_{n+k}$ can be coded by <j,k> where j is the beginning of the match.
- Example

ababababa babababababababab....

coded

ababababa babababa bababab....

<2,8>

Solution A Problem

- What if there is no match at all in the dictionary?

  ababababa cabababababababab....

  coded

- Solution B. Send tuples <j,k,x> where
  - If k = 0 then x is the unmatched symbol
  - If k > 0 then the match starts at j and is k long and the unmatched symbol is x.

Solution B

- If $x_{n+1}x_{n+2}...x_{n+k}$ is a substring of $x_1x_2...x_n$ and $x_{n+1}x_{n+2}...\ x_{n+k}x_{n+k+1}$ is not then $x_{n+1}x_{n+2}...x_{n+k}\ x_{n+k+1}$ can be coded by

  $$<j,k,\ x_{n+k+1}>$$

  where j is the beginning of the match.
- Examples

ababababa cabababababababab....

ababababa c ababababab ababab....

<0,0,c> <1,9,b>

Solution B Example

a bababababababababababab.....
<0,0,a>

a b ababababababababababab.....
<0,0,b>

a b aba bababababababababab.....
<1,2,a>

a b aba babab ababababababab.....
<2,4,b>

a b aba babab abababababa bab.....
<1,10,a>

Surprise Code!

a bababababababababababab$

<0,0,a>

a b ababababababababababab$

<0,0,b>

a b ababababababababababab$

<1,22,$>

Surprise Decoding

<0,0,a><0,0,b><1,22,$>

<0,0,a>	a
<0,0,b>	b
<1,22,$>	a
<2,21,$>	b
<3,20,$>	a
<4,19,$>	b
...	
<22,1,$>	b
<23,0,$>	$

Surprise Decoding

<0,0,a><0,0,b><1,22,$>

<0,0,a>	a
<0,0,b>	b
<1,22,$>	a
<2,21,$>	b
<3,20,$>	a
<4,19,$>	b
...	
<22,1,$>	b
<23,0,$>	$

Solution C

- The matching string can include part of itself!
- If $x_{n+1}x_{n+2}\ldots x_{n+k}$ is a substring of

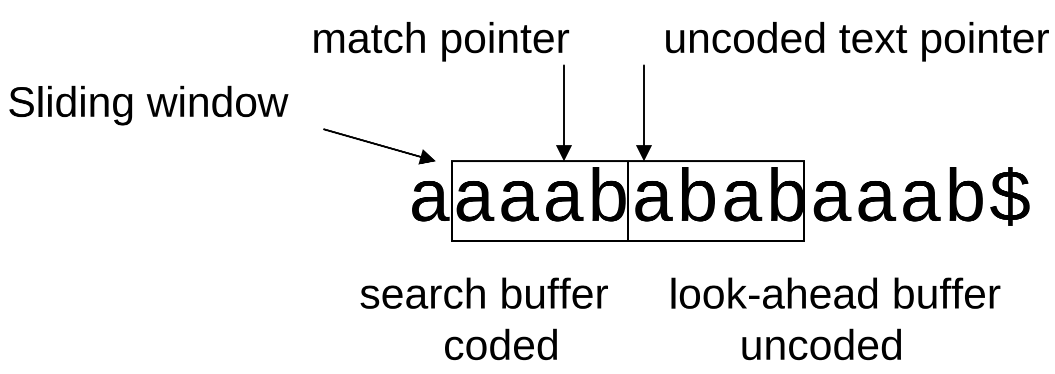
$$x_1x_2\ldots x_n\, x_{n+1}x_{n+2}\ldots x_{n+k}$$

that begins at $j \leq n$ and $x_{n+1}x_{n+2}\ldots\, x_{n+k}x_{n+k+1}$ is not then $x_{n+1}x_{n+2}\ldots x_{n+k}\, x_{n+k+1}$ can be coded by

$$<j,k,\, x_{n+k+1}>$$

Bounded Buffer – Sliding Window

- We want the triples <j,k,x> to be of bounded size. To achieve this we use bounded buffers.
  - Search buffer of size s is the symbols $x_{n-s+1}...x_n$ j is then the offset into the buffer.
  - Look-ahead buffer of size t is the symbols $x_{n+1}...x_{n+t}$
- Match pointer can start in search buffer and go into the look-ahead buffer but no farther.



tuple
<2,5,a>