CSE 326: Data Structures

Graph Algorithms
Graph Search

Lecture 23

James Fogarty

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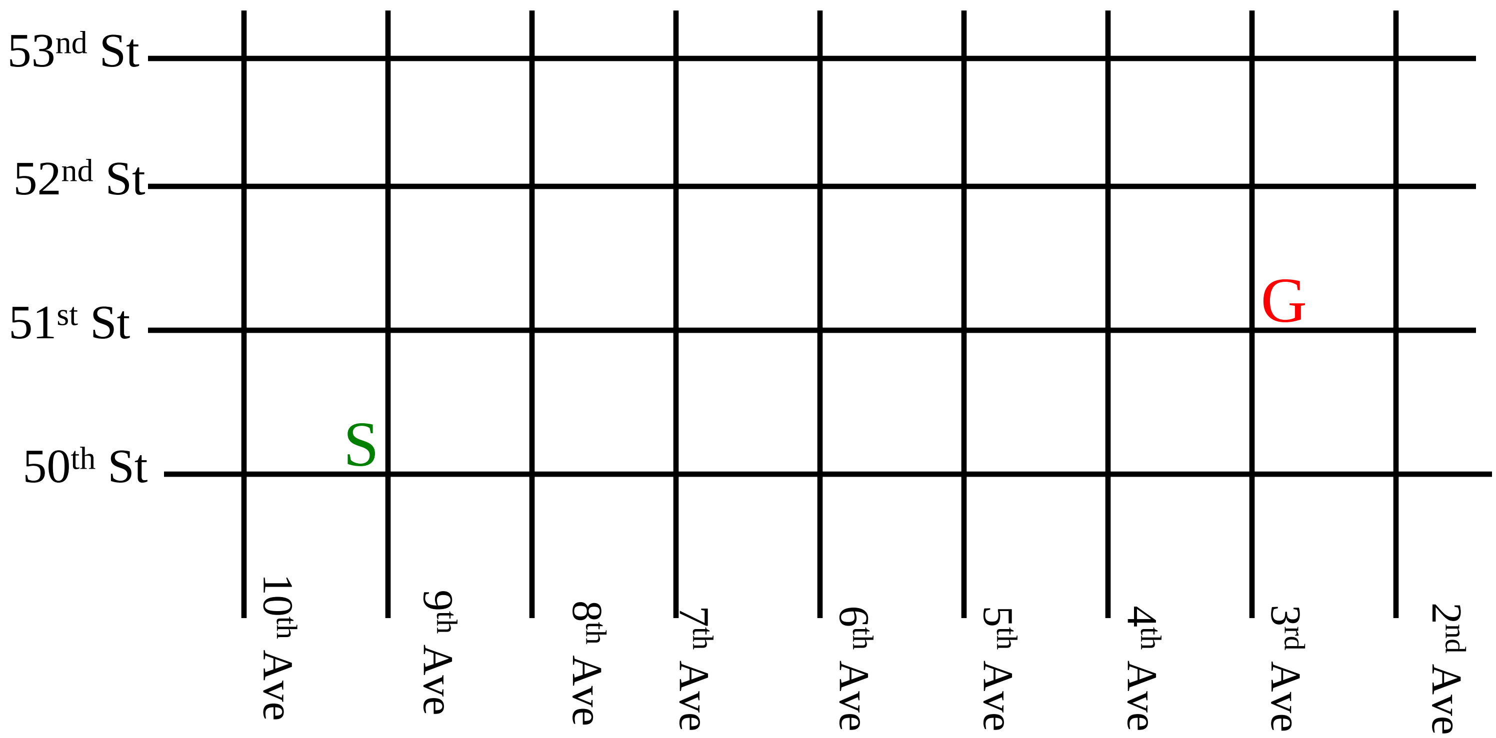
Problem: Large Graphs

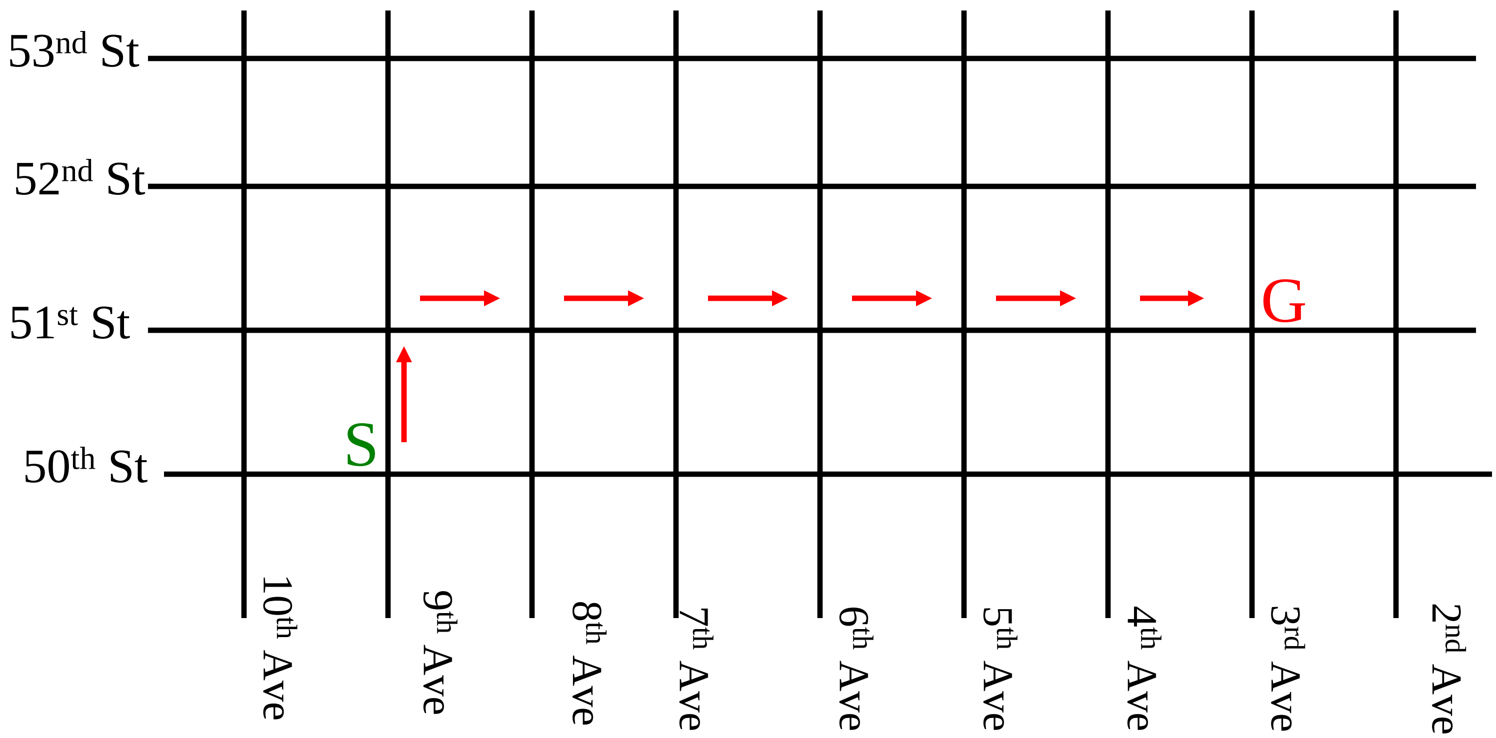
- It is expensive to find optimal paths in large graphs, using BFS or Dijkstra's algorithm (for weighted graphs)
- How can we search large graphs efficiently by using "commonsense" about which direction looks most promising?

Example



Plan a route from 9th & 50th to 3rd & 51st

Example



Plan a route from 9th & 50th to 3rd & 51st

Best-First Search

The *Manhattan distance* (Δ x+ Δ y) is an estimate of the distance to the goal

- It is a *search heuristic*

❑ Best-First Search

- Order nodes in priority to minimize estimated distance to the goal

❑ Compare: BFS / Dijkstra

- Order nodes in priority to minimize distance from the start

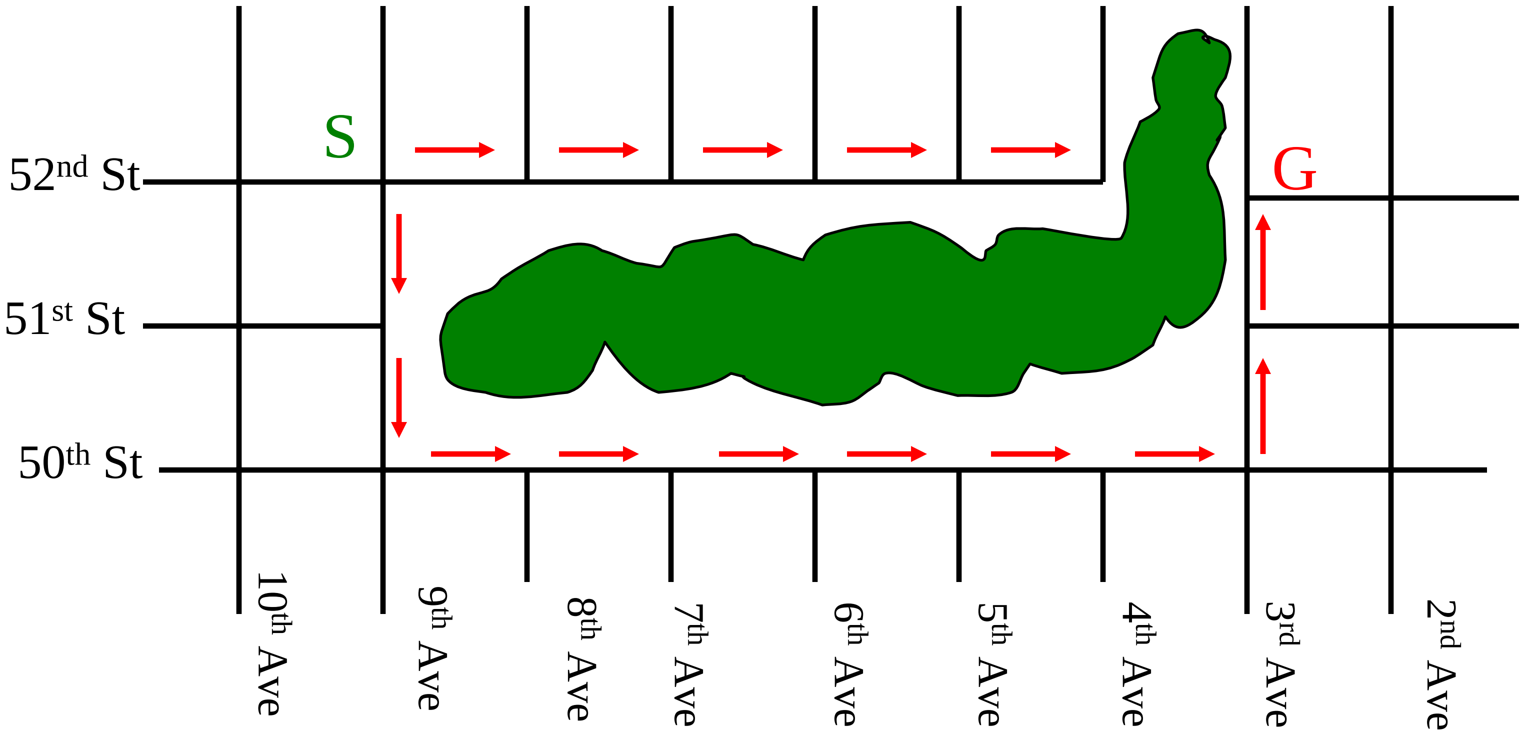
Best-First Search

Open – Heap (priority queue)
Criteria – Smallest key (highest priority)
h(n) – heuristic estimate of distance from n to closest goal

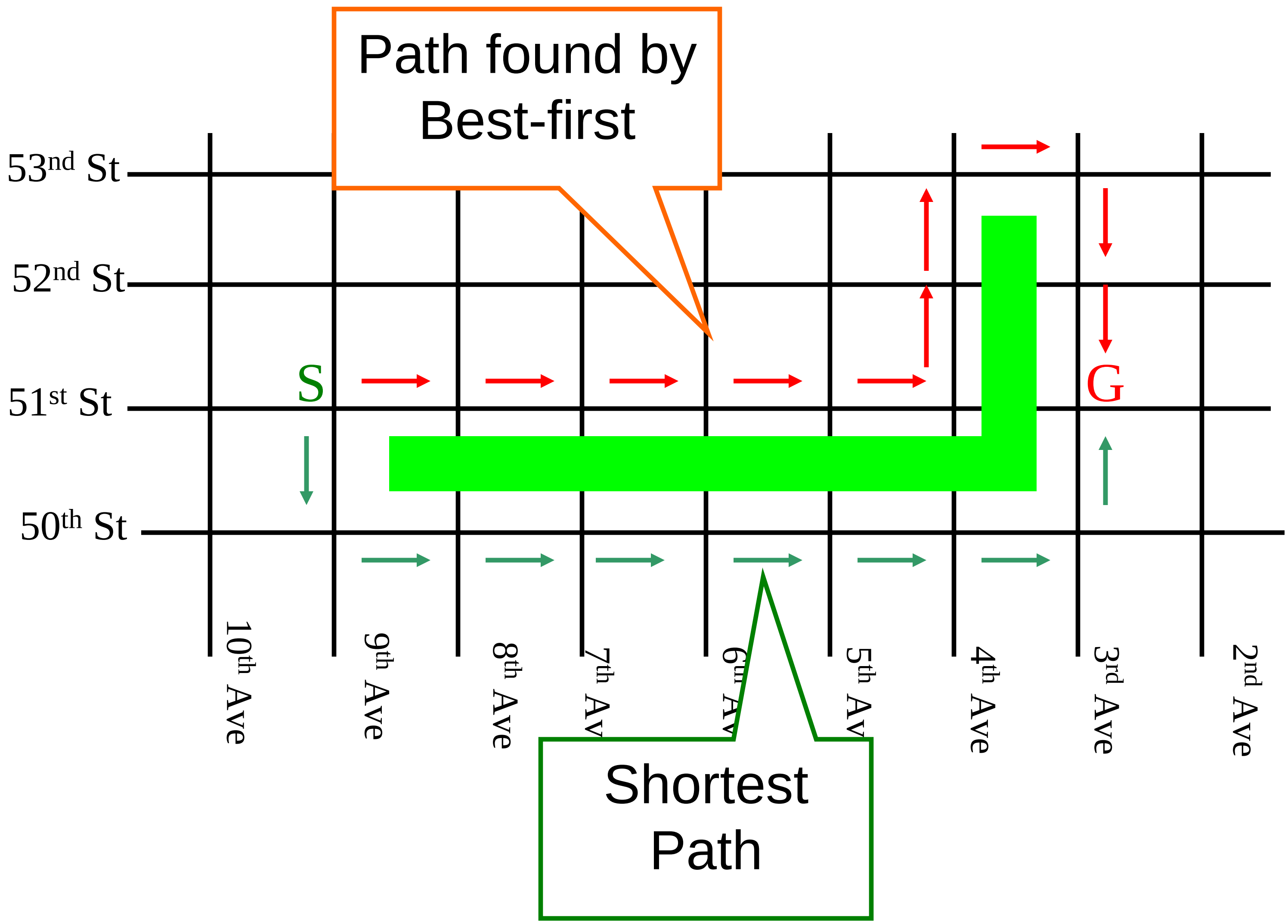
```
Best_First_Search( Start, Goal_test)
  insert(Start, h(Start), heap);
  repeat
      if (empty(heap)) then return fail;
      Node := deleteMin(heap);
      if (Goal_test(Node)) then return Node;
      for each Child of node do
          if (Child not already visited) then
              insert(Child, h(Child),heap);
      end
      Mark Node as visited;
  end
```

Obstacles

Best-FS eventually will expand vertex to get back on the right track



Non-Optimality of Best-First



Improving Best-First

- Best-first is often tremendously faster than BFS/Dijkstra, but might stop with a non-optimal solution
- How can it be modified to be (almost) as fast, but guaranteed to find optimal solutions?
- A* - Hart, Nilsson, Raphael 1968
    - One of the first significant algorithms developed in AI
    - Widely used in many applications

A*

Exactly like Best-first search, but using a different criteria for the priority queue:

minimize (distance from start) +
(estimated distance to goal)

priority f(n) = g(n) + h(n)

f(n) = priority of a node

g(n) = true distance from start

h(n) = heuristic distance to goal

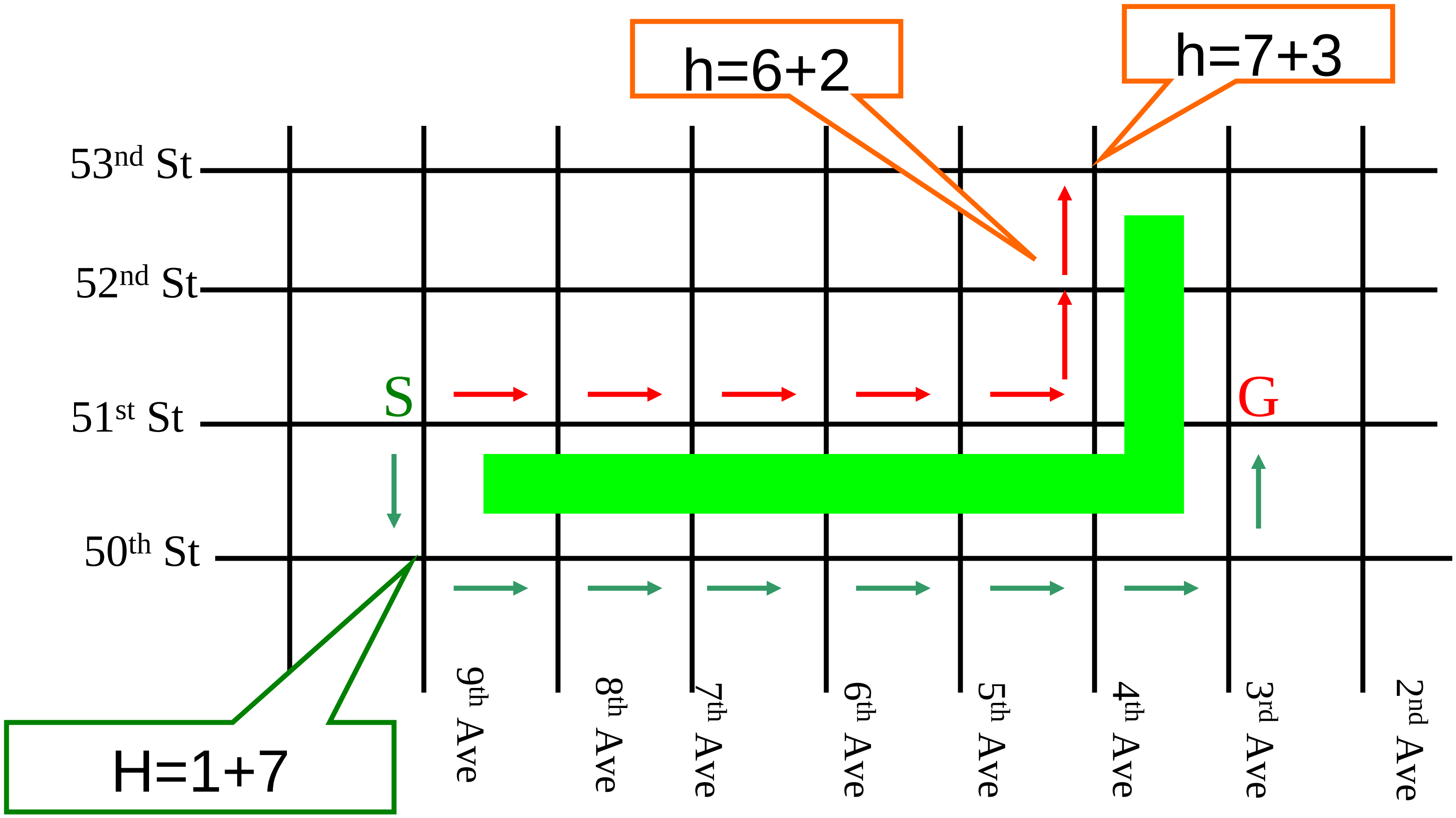
Optimality of A*

Suppose the estimated distance is *always* less than or equal to the true distance to the goal

- heuristic is a lower bound

Then: when the goal is removed from the priority queue, we are guaranteed to have found a shortest path!

A* in Action


h=6+2
h=7+3
53nd St
52nd St
51st St
50th St
S
G
H=1+7
9th Ave
8th Ave
7th Ave
6th Ave
5th Ave
4th Ave
3rd Ave
2nd Ave

Application of A*: Speech Recognition

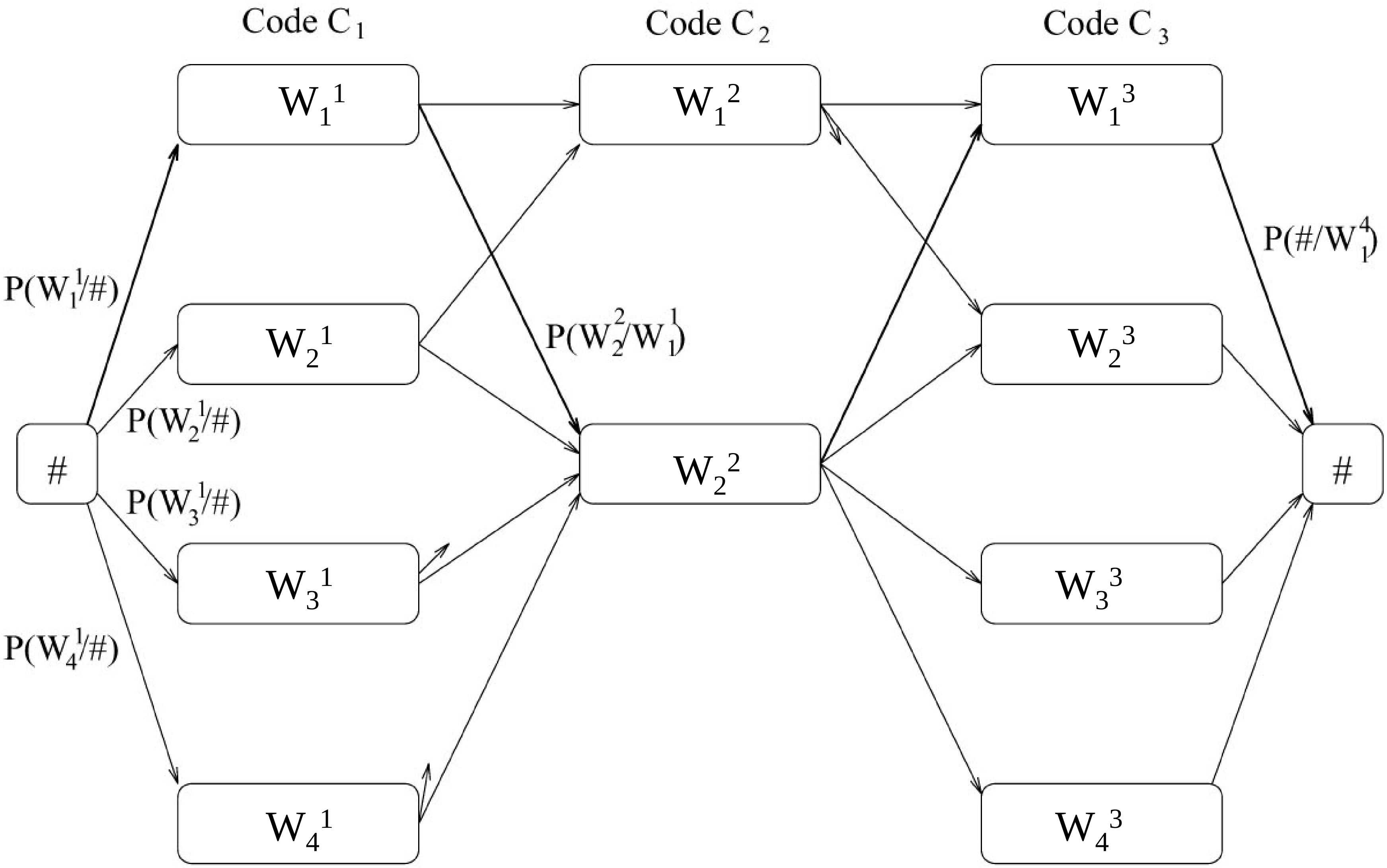
(Simplified) Problem:

- System hears a sequence of 3 words
- It is unsure about what it heard
  - For each word, it has a set of possible "guesses"
  - E.g.: Word 1 is one of { "hi", "high", "I" }
- What is the most likely sentence it heard?

Speech Recognition as Shortest Path

Convert to a shortest-path problem:

- Utterance is a "layered" DAG
- Begins with a special dummy "start" node
- Next: A layer of nodes for each word position, one node for each word choice
- Edges between every node in layer i to every node in layer i+1
  - Cost of an edge is smaller if the pair of words frequently occur together in real speech
    - + Technically: - log probability of co-occurrence
- Finally: a dummy "end" node
- Find shortest path from start to end node



Code C 1
Code C 2
Code C 3
W 1 1
W 1 2
W 1 3
P(W 1 1/#)
P(#/W 1 4)
W 2 1
P(W 2 2/W 1 1)
W 2 3
P(W 2 1/#)
#
W 2 2
#
P(W 3 1/#)
W 3 1
W 3 3
P(W 4 1/#)
W 4 1
W 4 3

Summary: Graph Search

Depth First

- Little memory required
- Might find non-optimal path

Breadth First

- Much memory required
- Always finds optimal path

Iterative Depth-First Search

- Repeated depth-first searches, little memory required

Dijskstra's Short Path Algorithm

- Like BFS for weighted graphs

Best First

- Can visit fewer nodes
- Might find non-optimal path

A*

- Can visit fewer nodes than BFS or Dijkstra
- Optimal if heuristic estimate is a lower-bound

Dynamic Programming

Algorithmic technique that systematically records the answers to sub-problems in a table and re-uses those recorded results (rather than re-computing them).

Simple Example: Calculating the Nth Fibonacci number.

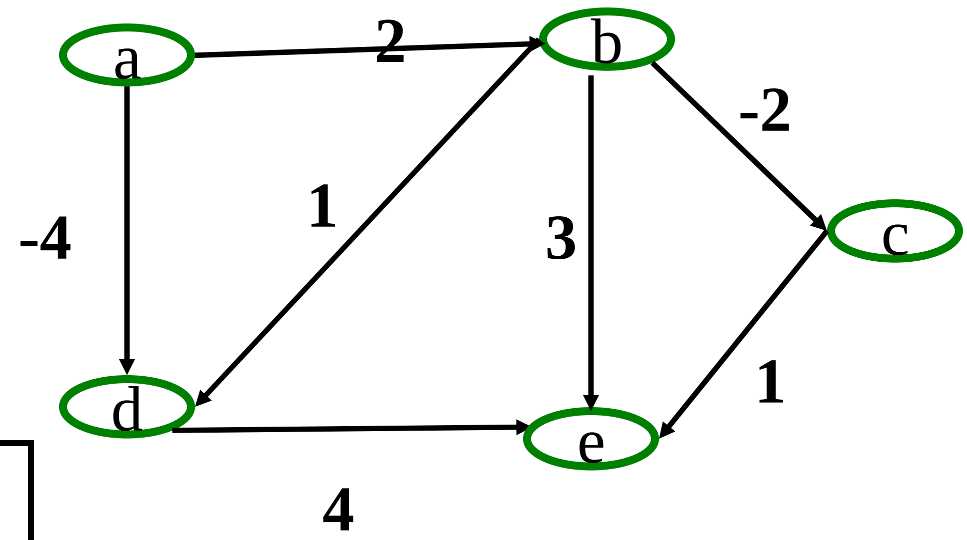
$$Fib(N) = Fib(N-1) + Fib(N-2)$$

Floyd-Warshall

```
for (int k = 1; k =< V; k++)
 for (int i = 1; i =< V; i++)
  for (int j = 1; j =< V; j++)
   if ( ( M[i][k]+ M[k][j] ) < M[i][j] )
      M[i][j] =  M[i][k]+ M[k][j]
```

Invariant: After the kth iteration, the matrix includes the shortest paths for all pairs of vertices (i,j) containing only vertices 1..k as intermediate vertices

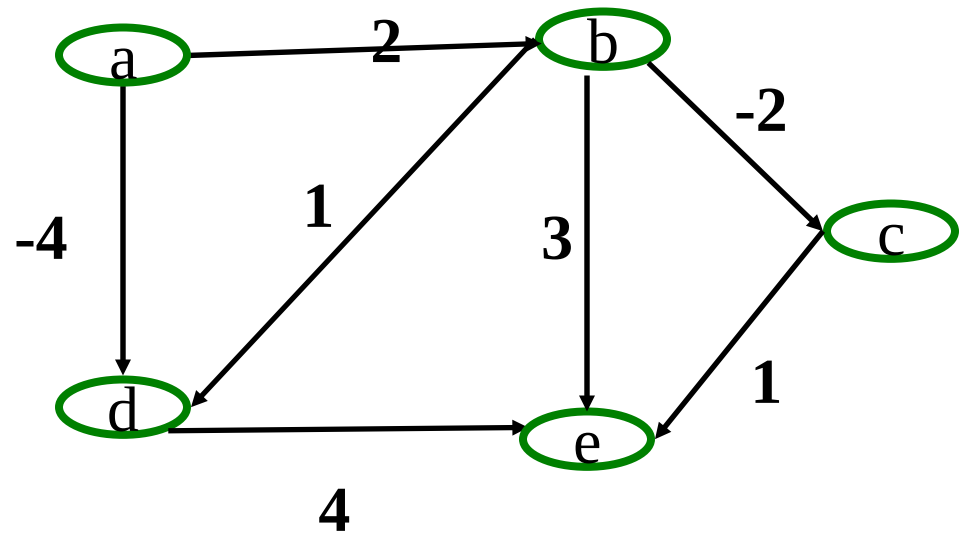
Initial state of the matrix:



	a	b	c	d	e
a	0	2	-	-4	-
b	-	0	-2	1	3
c	-	-	0	-	1
d	-	-	-	0	4
e	-	-	-	-	0

$$M[i][j] = \min(M[i][j], M[i][k] + M[k][j])$$

Floyd-Warshall - for All-pairs shortest path



	a	b	c	d	e
a	0	2	0	-4	0
b	-	0	-2	1	-1
c	-	-	0	-	1
d	-	-	-	0	4
e	-	-	-	-	0

Final Matrix Contents