Data Compression: Huffman Coding

10.1.2 in Weiss (p.389)

Why compress files?

- Resources are limited
  - Long-term storage (disk space)
  - Internet transfers (network bandwidth)
  - Fast memory access (cache)
- Because we can

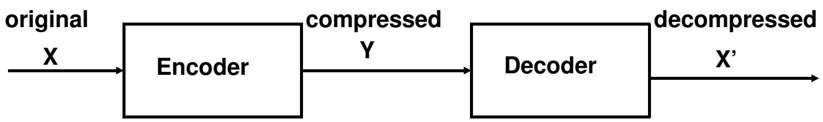
Is compression possible?

- Most data contains redundancies
  - E.g. Human-readable text
  - Not all combinations are equally likely.
  - In English, some letter pairs ("qu", "th", etc.) appear more frequently than others.
- The essential information content is much less
  - Information theory developed by Shannon in 1950s
  - If you have *n* equally likely symbols, how many bits do you need to represent them?

What can be compressed?

- Which of the following would we require in pristine shape? (lossless)
  - C++ source file
  - Binary executable
  - Photograph of your thumb
  - Video of a monkey eating a banana
  - MP3 ringtones
  - E-mail

Data Compression



- **Lossless** compression X = X'
- **Lossy** compression X != X'
- **Compression Ratio** |X|/|Y|
  - Where |X| is the # of bits in X.

Reversible or Entropy Coding

Irreversible Coding

Lossy Compression

- Ideal for signals with more data than humans can process (high-fidelity).
- Most audio and video information can be removed without being noticed.

Standards:

- JPEG (Joint Photographic Experts Group)
- MPEG (Motion Picture Experts Group)
- MP3 (MPEG-1, Layer 3)

Can get compression ratios of 10:1

Lossless Compression

Can get compression ratios of 4:1

- No data is lost.
- Information is low-fidelity to begin with.

Standards:

- Gzip, Unix compress, zip, GIF

Another technique is **run-length encoding (RLE)**, part of several compression techniques (BMP, PCX, TIFF, PDF)

A run of characters is replaced by the **number of characters** of that typeand a **single character**:

RTAAAAAADEEEE

RT*6AD*4E

Lossless Compression of text

Really only need 7 bits for 128 things

ASCII = fixed 8 bits per character

Example: "hello there"

- 11 characters * 8 bits = 88 bits

Can we encode this message using fewer bits?

We could look JUST at the message,
there are only 6 possible characters + one space. = 7 things
– needs 3 bits.

Encode: aabddcaa = could do as 16 bits (each character = 2 bits each)
Huffman can do as 14 bits

Huffman Coding 1951

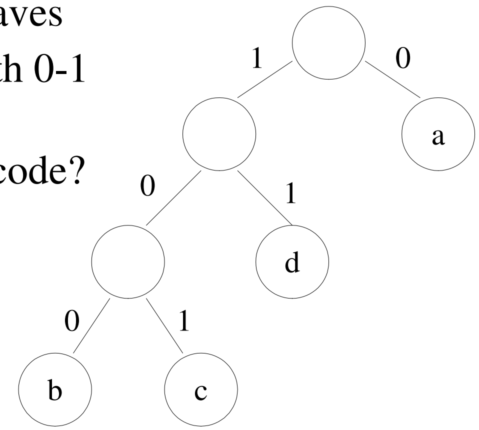
- Uses *frequencies* of symbols in a string to build a **prefix code**.
- **Prefix Code** – no code in our encoding is a prefix of another code.

Letter	code
a	0
b	100
c	101
d	11

Note: codes are **variable length** – (0 to 3 bits per character)

Huffman Tree

- All symbols at leaves
- Edges labeled with 0-1
- Why does this guarantee prefix code?



Decoding a Prefix Code

Loop
  start at root of tree
    loop
      if bit read = 1 then take 1-child
      else, take 0-child
    until node is a leaf
    Report character found!
Until end of the message

Decode: 11100010100110

Letter	code
a	0
b	100
c	101
d	11

8 characters:

- 8*8 bits = 64 bits in ASCII
- 8*2 bits = 16 bits (if used 2 bits each)
- 14 bits = Huffman (uses frequency

Why did we need the code to be a prefix code?

Cost of a Huffman Tree

Cost of a Huffman Tree containing n symbols is the expected length of a codeword.

$$C(T) = p_1*r_1+p_2*r_2+p_3*r_3+....+ p_n*r_n$$

For previous example = (.50 * 1) + (.125 * 3) + (.125 * 3) + (.25 * 2)

Where:

p_i = the probability that a symbol occurs

r_i = the length of the path from the root to the node

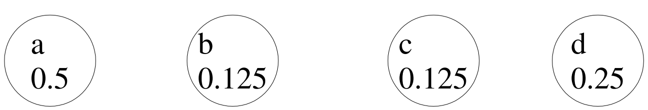
Constructing a Huffman Tree

Letter	Frequency	code
a	.50	0
b	.125	100
c	.125	101
d	.25	11

What is the cost of this tree?

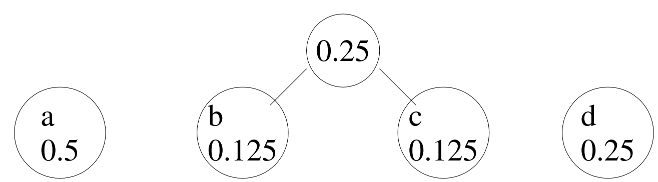
Huffman Tree Construction Part the First

- Given a symbol-frequency table:
  - Start with a forest of one-node trees
  - One for each symbol
  - Associate a frequency with each tree



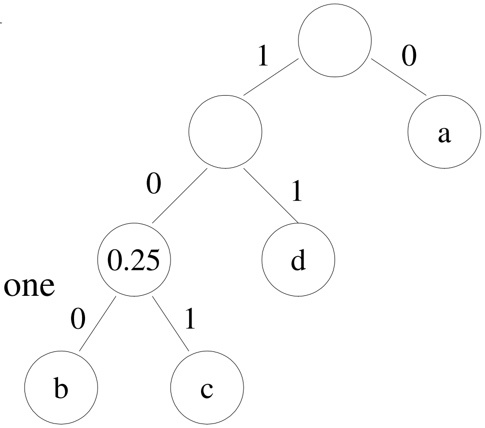
Huffman Tree Construction Part the Second

- While there is more than one tree
  - Pick the two trees with smallest frequency
  - Combine them into one tree
    - And add their frequencies



Huffman Tree Construction Part the Third

- Pick arbitrary 0-1 labellings for the edges
  - More than one Huffman tree is possible
  - How to get from one Huffman tree to another?



Digression: Why "anti-compress" files?

- Error-correcting codes
  - By adding redundancies into data instead of removing it, we can make it robust to noise.
  - Noise on our communication channel will corrupt this redundancy.
    - CD/DVD optical storage
    - Hard disk magnetic storage
    - WiFi
    - Ethernet / CDMA
  - Examples: checksums, phonetic alphabet