B-Trees

CSE 326
Data Structures
Lecture 10

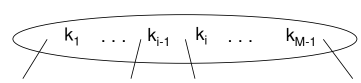
Disk vs. Memory

- Disks many times slower than memory:
  - › Processor measured in GH = 10^9 cycles per second
  - › Main memory measured in microsec. = 10^6 per second
  - › Disk seek measured in miliseconds = 10^3 per second
- i.e. ~ 1 million instructions per disk lookup
- Measuring runtime by pointer lookups meaningless if data can't fit in main memory

Trees on disk

- Each pointer lookup means seeking the disk
- Want as shallow a tree as possible
- Balanced binary tree with N nodes has height ________?
- Balanced M-ary tree with N nodes has height ________?

M-ary trees



N-ary trees

- In the limit, branching factor of N gives height 1
  - › Just gives linear array, obviously not good
- What should we use in practice?
- Just as caches read in a block at a time, disks read in a block at a time.

Example

- 1k byte page
- Key 8 bytes, pointer 4 bytes
- (M-1)8 + 4M = 1024

  12 M = 1032

  $M = \lfloor 1032/12 \rfloor = 86$

B-Trees

B-Trees are multi-way search trees commonly used in database systems or other applications where data is stored externally on disks and keeping the tree shallow is important.

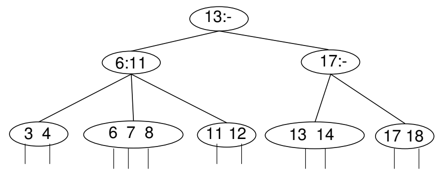
A B-Tree of order M has the following properties:

1. The root is either a leaf or has between 2 and M children.
2. All nonleaf nodes (except the root) have between $\lceil M/2 \rceil$ and M children.
3. All leaves are at the same depth.

All data records are stored at the leaves.
Leaves store between $\lceil M/2 \rceil$ and M data records.
Internal nodes only used for searching.

Example

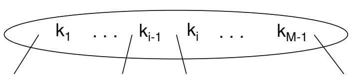
- B-tree of order 3 has 2 or 3 children per node



B-Tree Details

Each (non-leaf) internal node of a B-tree has:

- Between $\lceil M/2 \rceil$ and M children.
- up to M-1 keys $k_1 < k_2 < ... < k_{M-1}$

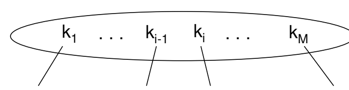


Keys are ordered so that:
$k_1 < k_2 < ... < k_{M-1}$

B-Tree Details

Each leaf node of a B-tree has:

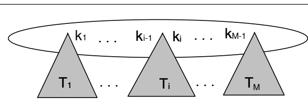
- Between $\lceil M/2 \rceil$ and M keys and pointers.



Keys are ordered so that:
$k_1 < k_2 < ... < k_{M-1}$

Keys point to data on other pages.

Properties of B-Trees



Children of each internal node are "between" the items in that node.
Suppose subtree T_i is the i-th child of the node:

all keys in T_i must be between keys k_{i-1} and k_i

i.e. $k_{i-1} \leq T_i < k_i$

k_{i-1} is the smallest key in T_i

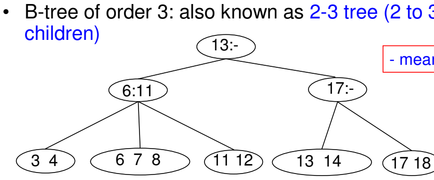
All keys in first subtree $T_1 < k_1$

All keys in last subtree $T_M \geq k_{M-1}$

Example: Searching in B-trees

- B-tree of order 3: also known as 2-3 tree (2 to 3 children)

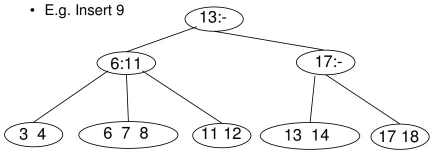
- means empty slot



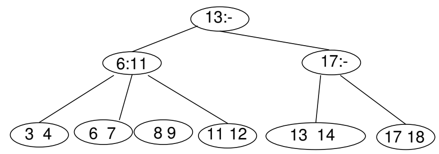
- Examples: Search for 9, 14, 12
- Note: If leaf nodes are connected as a Linked List, B-tree is called a B+ tree – Allows sorted list to be accessed easily

Inserting into B-Trees

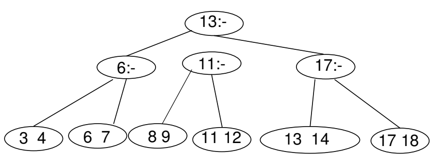
- Insert X: Do a Find on X and find appropriate leaf node
  - If leaf node is not full, fill in empty slot with X
    - E.g. Insert 5
  - If leaf node is full, split leaf node and adjust parents up to root node
    - E.g. Insert 9



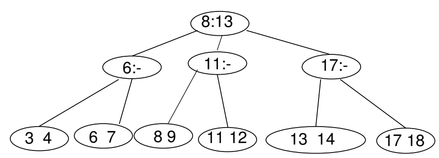
Insert Example



Insert Example

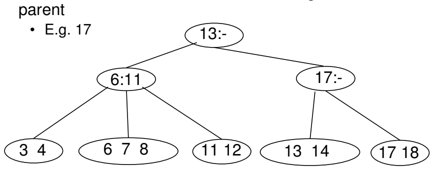


Insert Example

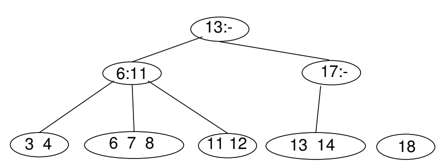


Deleting From B-Trees

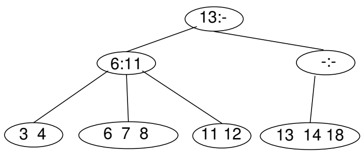
- Delete X : Do a find and remove from leaf
  - Leaf underflows – borrow from a neighbor
    - E.g. 11
  - Leaf underflows and can't borrow – merge nodes, delete parent
    - E.g. 17



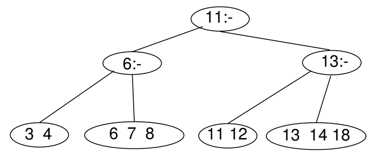
Delete Example



Delete Example



Delete Example



Run Time Analysis of B-Tree Operations

- For a B-Tree of order M
  - Each internal node has up to M-1 keys to search
  - Each internal node has between $\lceil M/2 \rceil$ and M children
  - Depth of B-Tree storing N items is $O(\log_{\lceil M/2 \rceil} N)$
- Example: M = 86
  - $\log_{43} N = \log_2 N / \log_2 43 = .184 \log_2 N$
  - $\log_{43} 1{,}000{,}000{,}000 = 5.51$

Summary of Search Trees

- Problem with Search Trees: Must keep tree balanced to allow
  - fast access to stored items
- AVL trees: Insert/Delete operations keep tree balanced
- Splay trees: Repeated Find operations produce balanced trees on average
- Multi-way search trees (e.g. B-Trees): More than two children
  - per node allows shallow trees; all leaves are at the same depth
  - keeping tree balanced at all times