CSE326 Homework #6

Due: Wednesday August 3.

1. (a) Give an example of a graph with negative edges but no negative cost cycles where Dijkstra's algorithm gives the wrong answer.

   (b) Suppose you are given a graph that has negative-cost edges but no negative-cost cycles. Why does the following strategy fail to find shortest paths: uniformly add a constant k to the cost of every edge so that all costs become non-negative, run Dijkstra's algorithm, return the result with edge costs reverted back to the original costs (i.e. with k subtracted). Give an argument as well as a small example where it fails.

2. Consider the following sequence of disjoint union / find operations: `union(1,2), union(2,3), union(3,4), union(4,5), union(5,6), union(6,7), union(7,8), union(9,10), union(11,12), union(13,14), union(15,16), union(1,10), union(1,12), union(14,15), union(1,16)`. In this problem we don't assume that the inputs to union are roots, so that two find operations are performed during the union to find the roots before pointing one root to another. Show the resulting up tree after these operations for each case below. In each case count the number of nodes visited in all the find operations. In the case of path compression some nodes are visited twice.

   (a) There is no path compression on the finds and the root of the first argument points to the root of the second argument.

   (b) There is path compression on the finds and the root of the first argument points to the root of the second argument.

   (c) There is no path compression on the finds and weighted union is used.

   (d) There is path compression on the finds and weighted union is used.