Welcome to

CSE 326

Data Structures

Staff

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See web-page for office hours.

Web Page

- All info is on the web page for CSE 326
  - › http://www.cs.washington.edu/326
  - › also known as
    - http://www.cs.washington.edu/education/courses/326/04wi

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CSE 326 E-mail List

- Subscribe by going to the class web page.
- E-mail list is used for posting announcements by instructor and TAs.
- It is your responsibility to subscribe. It might turn out to be very helpful for assignments hints, corrections etc.

Textbook

- *Data Structures and Algorithm Analysis in Java (or in C++)*, by Weiss

- See Web page for errata and source code

Grading

- Dry assignments 25% - submit in singles
- Wet assignments (programming projects) 25% - can submit in pairs.
- Midterm 20%
  - Friday, Feb 6, 2004
- Final 30%
  - Group I : 8:30-10:20 a.m. Thursday, Mar. 18, 2004
  - Group II: 2:30-4:20 p.m. Tuesday, Mar. 16, 2004

Class Overview

- Introduction to many of the basic data structures used in computer software
  - Understand the data structures
  - Analyze the algorithms that use them
  - Know when to apply them
- Practice design and analysis of data structures.
- Practice using these data structures by writing programs.

Goal

- You will understand
  - what the tools are for storing and processing common data types
  - which tools are appropriate for which need
- So that you will be able to
  - make good design choices as a developer, project manager, or system customer

Course Topics

- Introduction to Algorithm Analysis
- Lists, Stacks, Queues
- Search Algorithms and Trees
- Hashing and Heaps
- Sorting
- Disjoint Sets
- Graph Algorithms

Reading

- Chapters 1 and 2, *Data Structures and Algorithm Analysis in Java*, by Weiss
  - › Most of Chapter 2 will be seen in class next week.

Data Structures: What?

- Need to organize program data according to problem being solved
- Abstract Data Type (ADT) - A data object and a set of operations for manipulating it
  - › List ADT with operations **`insert`** and **`delete`**
  - › Stack ADT with operations **`push`** and **`pop`**
- Note similarity to Java classes
  - › private data structure and public methods

Data Structures: Why?

- Program design depends crucially on how data is structured for use by the program
  - › Implementation of some operations may become easier or harder
  - › Speed of program may dramatically decrease or increase
  - › Memory used may increase or decrease
  - › Debugging may be become easier or harder

Terminology

- Abstract Data Type (ADT)
  - Mathematical description of an object with set of operations on the object. Useful building block.
- Algorithm
  - A high level, language independent, description of a step-by-step process
- Data structure
  - A specific family of algorithms for implementing an abstract data type.
- Implementation of data structure
  - A specific implementation in a specific language

Algorithm Analysis: Why?

- Correctness:
  - Does the algorithm do what is intended.
- Performance:
  - What is the running time of the algorithm.
  - How much storage does it consume.
- Different algorithms may correctly solve a given task
  - Which should I use?

Evaluating an algorithm

Mike: My algorithm can sort 10^6 numbers in 3 seconds.

Bill: My algorithm can sort 10^6 numbers in 5 seconds.

Mike: I’ve just tested it on my new Pentium IV processor.

Bill: I remember my result from my undergraduate studies (1985).

Mike: My input is a random permutation of 1..10^6.

Bill: My input is the sorted output, so I only need to verify that it is sorted.

Program Evaluation / Complexity

- Processing time is surely a bad measure!!!
- We need a ‘stable’ measure, independent of the implementation.

* A complexity function is a function T: N à N.

T(n) is the number of operations the algorithm does on an input of size n.

* We can measure three different things.

- Worst-case complexity
- Best-case complexity
- Average-case complexity

The RAM Model of Computation

- Each simple operation takes 1 time step.
- Loops and subroutines are not simple operations.
- Each memory access takes one time step, and there is no shortage of memory.

For a given problem instance:

- Running time of an algorithm = # RAM steps.
- Space used by an algorithm = # RAM memory cells

useful abstraction ⇒ allows us to analyze algorithms in a machine independent fashion.

Big O Notation

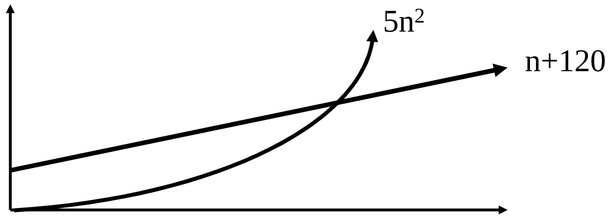
- Goal :
  - › A stable measurement independent of the machine.
- Way:
  - › ignore constant factors.
- $f(n) = O(g(n))$ if $c \cdot g(n)$ is upper bound on $f(n)$

⇔ There exist c, N, s.t. for any $n \geq N$, $f(n) \leq c \cdot g(n)$

Consider large inputs (asymptotic behavior)

Ignore constants

Big O Notation



For all $n \geq 5$ (N=5)
$n+120 \leq 5n^2$

$\Rightarrow n+120 = O(n^2)$

Ω, Θ Notation

- $f(n) = \Omega(g(n))$ if $c \cdot g(n)$ is lower bound on $f(n)$

⇔ There exist c, N, s.t. for any $n \geq N$, $f(n) \geq c \cdot g(n)$

- $f(n) = \Theta(g(n))$ if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$

⇔ There exist c_1, c_2, N, s.t. for $n \geq N$,

$$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$$

Ω, Θ Notation

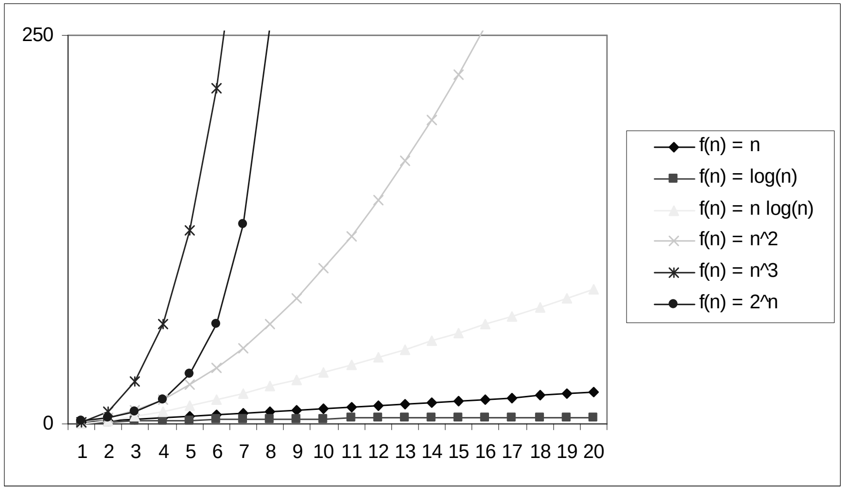
Examples:

$4x^2+100 = O(x^2)$	$4x^2+100 \neq \Theta(x^3)$
$4x^2+100 = \Omega(x^2)$	$4x^2+100 = O(x^3)$
$4x^2+100 = \Theta(x^2)$	$4x^2+100 = \Omega(x)$
$4x^2 -100 = O(x^2)$	$4x^2 + x\log x = O(x^2)$
$123400 = O(1)$	

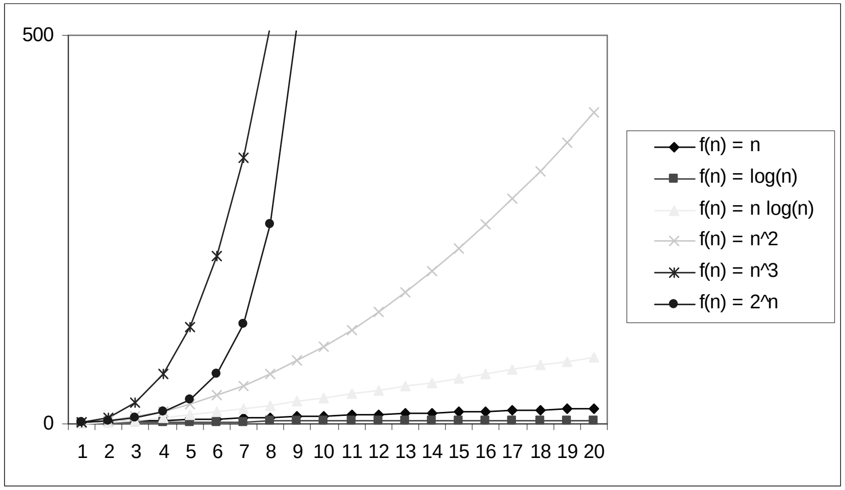
Growth Rates

- Even by ignoring constant factors, we can get an excellent idea of whether a given algorithm will be able to run in a reasonable amount of time on a problem of a given size.
- The “big O” notation and worst-case analysis are tools that greatly simplify our ability to compare the efficiency of algorithms.

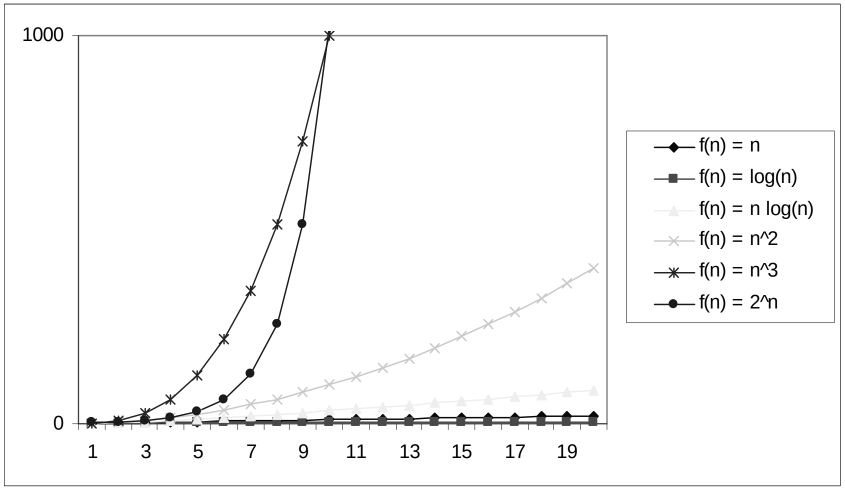
Practical Complexity



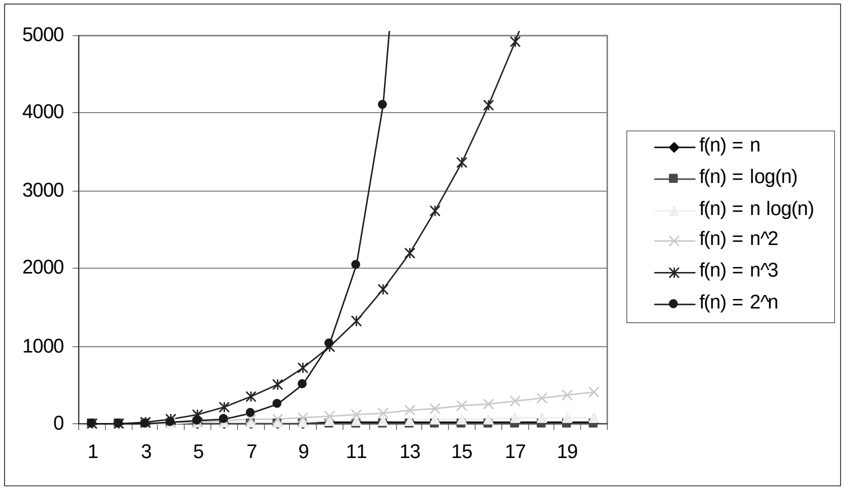
Practical Complexity



Practical Complexity



Practical Complexity



Big O Fact

- A polynomial of degree k is $O(n^k)$
- Proof:
  - › Suppose $f(n) = b_k n^k + b_{k-1} n^{k-1} + \ldots + b_1 n + b_0$
    - Let $a = \max_i \{b_i\}$
  - › $f(n) \le an^k + an^{k-1} + \ldots + an + a$

    $\le kan^k \le cn^k$ (for c=ka).

Iterative Algorithm for Sum

- Find the sum of the first **`num`** integers stored in an array **`v`**.

```
sum(v[ ]: integer array, num: integer): integer{
    temp_sum: integer ;
    temp_sum := 0;
    for i := 0 to num – 1 do
        temp_sum := v[i] + temp_sum;
    return temp_sum;
}
```

Note the use of pseudocode

Programming via Recursion

- Write a *recursive* function to find the sum of the first **`num`** integers stored in array **`v`**.

```
sum (v[ ]: integer array, num: integer): integer {
    if (num = 0) then
        return 0
    else
        return (v[num-1] + sum(v,num-1));
}
```

Pseudocode

- In the lectures algorithms will be presented in pseudocode.
  - This is very common in the computer science literature
  - Pseudocode is usually easily translated to real code.
  - This is programming language independent
- Pseudocode should also be used for homework (dry ones)

Review: Induction

- Suppose
  - S(k) is true for fixed constant k
    - Often k = 0
  - S(n) implies S(n+1) for all n >= k
- Then S(n) is true for all n >= k

Proof By Induction

- Claim:S(n) is true for all n >= k
- Base:
  - Show S(n) is true for n = k
- Inductive hypothesis:
  - Assume S(n) is true for an arbitrary n
- Step:
  - Show that S(n) is then true for n+1

Induction Example: Geometric Closed Form

- Prove $a^0 + a^1 + \ldots + a^n = (a^{n+1} - 1)/(a - 1)$ for all $a \neq 1$
  - Basis: 1. show that $a^0 = (a^{0+1} - 1)/(a - 1)$:
    $a^0 = 1 = (a^1 - 1)/(a - 1)$. 2. Show true for n=2.
  - Inductive hypothesis:
    - Assume $a^0 + a^1 + \ldots + a^n = (a^{n+1} - 1)/(a - 1)$
  - Step (show true for n+1):
    $a^0 + a^1 + \ldots + a^{n+1} = a^0 + a^1 + \ldots + a^n + a^{n+1}$
    $= (a^{n+1} - 1)/(a - 1) + a^{n+1} = (a^{n+1+1} - 1)/(a - 1)$

Program Correctness by Induction

- **Basis Step:** sum(v,0) = 0. ü
- **Inductive Hypothesis (n=k):** Assume sum(v,k) correctly returns sum of first k elements of v, i.e. **`v[0]+v[1]+…+v[k-1]`**
- **Inductive Step (n=k+1):** sum(v,n) returns **`v[k]+sum(v,k)`** which is the sum of first k+1 elements of v. ü

Algorithms vs Programs

- Proving correctness of an algorithm is very important
  - a well designed algorithm is guaranteed to work correctly and its performance can be estimated
- Proving correctness of a program (an implementation) is fraught with weird bugs
  - Abstract Data Types are a way to bridge the gap between mathematical algorithms and programs