## CSE 326 - Data Structures <br> Winter 2004 <br> Dry assignment \#1.

Due date: $1 / 16 / 04$ (see submission instructions in course web-page).

1. Let $T_{\text {bar }}(n)$ define the time complexity, as a function of $n$, of executing $\operatorname{bar}(\mathrm{n})$, and let $T_{\text {foo }}(n)$ define the time complexity, as a function of $n$, of executing foo( $n$ ).
```
foo( n : integer): void { bar( k: integer ):void {
    m: integer;
    m:= n* n* n;
    bar(m );
}
```

```
    if (k<=1)
```

    if (k<=1)
        return;
        return;
    print( "X" );
    print( "X" );
    bar( k / 2 );
    bar( k / 2 );
    }

```
}
```

Complete: $\mathrm{T}_{\mathrm{bar}}(\mathrm{n})=\Theta($ $\qquad$ )

$$
\mathrm{T}_{\mathrm{foo}}(\mathrm{n})=\Theta(\ldots)
$$

Explain your answers.
2. For each of the following questions, briefly explain your answer.
a. If I prove that an algorithm takes $O\left(n^{2}\right)$ worst-case time, is it possible that it takes $O(n)$ on some inputs?
b. If I prove that an algorithm takes $O\left(n^{2}\right)$ worst-case time, is it possible that it takes $O(n)$ on all inputs?
c. If I prove that an algorithm takes $\Theta\left(\mathrm{n}^{2}\right)$ worst-case time, is it possible that it takes $O(n)$ on some inputs?
d. If I prove that an algorithm takes $\Theta\left(\mathrm{n}^{2}\right)$ worst-case time, is it possible that it takes $O(n)$ on all inputs?
3. Write (in pseodocode) a recursive function 'MaxPair' that gets an array a[] of integers and its size n (it is known that $\mathrm{n}>1$ ), and returns the maximal sum of two consecutive
elements in a[] (that is $\operatorname{Max}(\mathrm{a}[\mathrm{j}-1]+\mathrm{a}[\mathrm{j}]: 1 \leq \mathrm{j} \leq \mathrm{n}-1)$. You are not allowed to use loops in your solution.
What is the time and space complexity?
4. A 'frame matrix' is an $\mathrm{N}^{*} \mathrm{~N}$ matrix in which all the values along the same frame are identical. For example, the matrix below is a $5 * 5$ frame matrix.

| 7 | 7 | 7 | 7 | 7 |
| :--- | :--- | :--- | :--- | :--- |
| 7 | 14 | 14 | 14 | 7 |
| 7 | 14 | -9 | 14 | 7 |
| 7 | 14 | 14 | 14 | 7 |
| 7 | 7 | 7 | 7 | 7 |

Suggest a data structure for storing a frame matrix, whose space complexity is $\mathrm{O}(\mathrm{N})$ (for an $\mathrm{N} * \mathrm{~N}$ frame matrix with $\mathrm{N}^{2}$ elements). Using your suggested data structure, implement (write in pseudo-code) the following operations; each should have time complexity $\mathrm{O}(1)$ :
get $(i, j)$ returns the value of the element whose location is ( $i, j$ ).
put $(i, j, x)$ - set the value $x$ at location ( $i, j$ ) AND in all the locations in the frame to which ( $\mathrm{i}, \mathrm{j}$ ) belongs, in a way that the resulting matrix is still a frame matrix.
5. $t$ is a linked list. What is the result of executing rec_func ( $t$ ) (defined below)? What is its time and space complexity? Explain briefly.

```
rec_funcl(t node_pointer, r node_pointer): node_pointer
{
    tail node_pointer;
    if (t = NULL) return r;
    tail := t.next;
    t.next := r;
    return rec_funcl(tail, t);
}
rec_func(t node_pointer): node_pointer
{
    return rec_func1(t, NULL);
}
```

