

Linear Probing

The keys are: 89, 18, 49, 58, 69

Table size = 10

$\text{hash}_i(x) = (x + i) \bmod 10$. Try $\text{hash}_0(x)$, $\text{hash}_1(x)$, ...

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Linear Probing

The keys are: 89, 18, 49, 58, 69

Table size = 10

$\text{hash}_i(x) = (x + i) \bmod 10$. Try $\text{hash}_0(x)$, $\text{hash}_1(x)$, ...

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	89

$$h_0(89) = 89 \bmod 10 = 9$$

Linear Probing

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Table size = 10

$\text{hash}_i(x) = (x + i) \bmod 10$. Try $\text{hash}_0(x)$, $\text{hash}_1(x)$, ...

0	
1	
2	
3	
4	
5	
6	
7	
8	18
9	89

$$h_0(18) = 18 \bmod 10 = 8$$

Linear Probing

The keys are: 89, 18, 49, 58, 69

Table size = 10

$\text{hash}_i(x) = (x + i) \bmod 10$. Try $\text{hash}_0(x)$, $\text{hash}_1(x)$, ...

0	49
1	
2	
3	
4	
5	
6	
7	
8	18
9	89

$$h_0(49) = 49 \bmod 10 = 9 \text{ (occupied)}$$

$$h_1(49) = (49+1) \bmod 10 = 0$$

Linear Probing

The keys are: 89, 18, 49, 58, 69

Table size = 10

$\text{hash}_i(x) = (x + i) \bmod 10$. Try $\text{hash}_0(x)$, $\text{hash}_1(x)$, ...

0	49
1	58
2	
3	
4	
5	
6	
7	
8	18
9	89

$$h_0(58) = 58 \bmod 10 = 8 \text{ (occupied)}$$

$$h_1(58) = (58+1) \bmod 10 = 9 \text{ (occupied)}$$

$$h_2(58) = (58+2) \bmod 10 = 0 \text{ (occupied)}$$

$$h_3(58) = (58+3) \bmod 10 = 1$$

Linear Probing

The keys are: 89, 18, 49, 58, 69

Table size = 10

$\text{hash}_i(x) = (x + i) \bmod 10$. Try $\text{hash}_0(x)$, $\text{hash}_1(x)$, ...

0	49
1	58
2	69
3	
4	
5	
6	
7	
8	18
9	89

$$h_0(69) = 69 \bmod 10 = 9 \text{ (occupied)}$$

$$h_1(69) = (69+1) \bmod 10 = 0 \text{ (occupied)}$$

$$h_2(69) = (69+2) \bmod 10 = 1 \text{ (occupied)}$$

$$h_3(69) = (69+3) \bmod 10 = 2$$

Quadratic Probing

The keys are: 89, 18, 49, 58, 69

Table size = 10

$\text{hash}_i(x) = (x + i^2) \bmod 10$. Try $\text{hash}_0(x)$, $\text{hash}_1(x)$, ...

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Quadratic Probing

The keys are: 89, 18, 49, 58, 69

Table size = 10

$\text{hash}_i(x) = (x + i^2) \bmod 10$. Try $\text{hash}_0(x)$, $\text{hash}_1(x)$, ...

0	
1	
2	
3	
4	
5	
6	
7	
8	18
9	89

$$h_0(89) = 89 \bmod 10 = 9$$

$$h_0(18) = 18 \bmod 10 = 8$$

Quadratic Probing

The keys are: 89, 18, 49, 58, 69

Table size = 10

$\text{hash}_i(x) = (x + i^2) \bmod 10$. Try $\text{hash}_0(x)$, $\text{hash}_1(x)$, ...

0	49
1	
2	
3	
4	
5	
6	
7	
8	18
9	89

$$h_0(49) = 49 \bmod 10 = 9 \text{ (occupied)}$$

$$h_1(49) = (49+1) \bmod 10 = 0$$

Quadratic Probing

The keys are: 89, 18, 49, 58, 69

Table size = 10

$\text{hash}_i(x) = (x + i^2) \bmod 10$. Try $\text{hash}_0(x)$, $\text{hash}_1(x)$, ...

0	49
1	
2	58
3	
4	
5	
6	
7	
8	18
9	89

$$h_0(58) = 58 \bmod 10 = 8 \text{ (occupied)}$$

$$h_1(58) = (58+1) \bmod 10 = 9 \text{ (occupied)}$$

$$h_2(58) = (58+4) \bmod 10 = 2$$

Quadratic Probing

The keys are: 89, 18, 49, 58, 69

Table size = 10

$\text{hash}_i(x) = (x + i^2) \bmod 10$. Try $\text{hash}_0(x)$, $\text{hash}_1(x)$, ...

0	49
1	
2	58
3	69
4	
5	
6	
7	
8	18
9	89

$$h_0(69) = 69 \bmod 10 = 9 \text{ (occupied)}$$

$$h_1(69) = (69+1) \bmod 10 = 0 \text{ (occupied)}$$

$$h_2(69) = (69+4) \bmod 10 = 3$$

Double Hashing

Define two hash function: $\text{hash}(x)$ and $\text{step}(x)$

$\text{hash}_i(x) = (\text{hash}(x) + i * \text{step}(x)) \bmod 10$.

Try $\text{hash}(x)$, $\text{hash}(x) + \text{step}(x)$, $\text{hash}(x) + 2\text{step}(x)$, ...

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

The keys are: 89, 18, 49, 58, 69

Table size = 10

$\text{hash}(x) = x \bmod 10$

$\text{step}(x) = 7 - (x \bmod 7)$

Double Hashing

The keys are: 89, 18, 49, 58, 69

$\text{hash}(x) = x \bmod 10$, $\text{step}(x) = 7 - (x \bmod 7)$

$\text{hash}_i(x) = (\text{hash}(x) + i * \text{step}(x)) \bmod 10$.

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	89

$$h_0(89) = 89 \bmod 10 = 9$$

Double Hashing

The keys are: 89, 18, 49, 58, 69

$\text{hash}(x) = x \bmod 10$, $\text{step}(x) = 7 - (x \bmod 7)$

$\text{hash}_i(x) = (\text{hash}(x) + i * \text{step}(x)) \bmod 10$.

0	
1	
2	
3	
4	
5	
6	
7	
8	18
9	89

$$h_0(18) = 18 \bmod 10 = 8$$

Double Hashing

The keys are: 89, 18, 49, 58, 69

$\text{hash}(x) = x \bmod 10$, $\text{step}(x) = 7 - (x \bmod 7)$

$\text{hash}_i(x) = (\text{hash}(x) + i * \text{step}(x)) \bmod 10$.

0	
1	
2	
3	
4	
5	
6	49
7	
8	18
9	89

$$h_0(49) = 49 \bmod 10 = 9 \text{ (occupied)}$$

$$\text{step}(49) = 7 - (49 \bmod 7) = 7 - 0 = 7.$$

$$h_1(49) = (9 + 7) \bmod 10 = 6$$

Double Hashing

The keys are: 89, 18, 49, 58, 69

$\text{hash}(x) = x \bmod 10$, $\text{step}(x) = 7 - (x \bmod 7)$

$\text{hash}_i(x) = (\text{hash}(x) + i * \text{step}(x)) \bmod 10$.

0	
1	
2	
3	58
4	
5	
6	49
7	
8	18
9	89

$$h_0(58) = 58 \bmod 10 = 8 \text{ (occupied)}$$

$$\text{step}(58) = 7 - (58 \bmod 7) = 7 - 2 = 5.$$

$$h_1(58) = (8 + 5) \bmod 10 = 3$$

Double Hashing

The keys are: 89, 18, 49, 58, 69

$\text{hash}(x) = x \bmod 10$, $\text{step}(x) = 7 - (x \bmod 7)$

$\text{hash}_i(x) = (\text{hash}(x) + i * \text{step}(x)) \bmod 10$.

0	69
1	
2	
3	58
4	
5	
6	49
7	
8	18
9	89

$h_0(69) = 69 \bmod 10 = 9$ (occupied)

$\text{step}(69) = 7 - (69 \bmod 7) = 7 - 6 = 1$.

$h_1(69) = (9+1) \bmod 10 = 0$

Double Hashing

The keys are: 89, 18, 49, 58, 69, 60, 23

$\text{hash}(x) = x \bmod 10$, $\text{step}(x) = 7 - (x \bmod 7)$

$\text{hash}_i(x) = (\text{hash}(x) + i * \text{step}(x)) \bmod 10$.

0	69
1	
2	60
3	58
4	
5	
6	49
7	
8	18
9	89

$h_0(60) = 60 \bmod 10 = 0$ (occupied)

$\text{step}(60) = 7 - (60 \bmod 7) = 7 - 4 = 3$.

$h_1(60) = (0+3) \bmod 10 = 3$ (occupied)

$h_2(60) = (0+2*3) \bmod 10 = 6$ (occupied)

$h_3(60) = (0+3*3) \bmod 10 = 9$ (occupied)

$h_4(60) = (0+4*3) \bmod 10 = 2$

Double Hashing

The keys are: 89, 18, 49, 58, 69, 60, 23

$\text{hash}(x) = x \bmod 10$, $\text{step}(x) = 7 - (x \bmod 7)$

$\text{hash}_i(x) = (\text{hash}(x) + i * \text{step}(x)) \bmod 10$.

0	69
1	
2	60
3	58
4	
5	
6	49
7	
8	18
9	89

$h_0(23) = 23 \bmod 10 = 3$ (occupied)

$\text{step}(23) = 7 - (23 \bmod 7) = 7 - 2 = 5$.

$h_1(23) = (3+5) \bmod 10 = 8$ (occupied)

$h_2(23) = (3+2*5) \bmod 10 = 3$ (occupied)

$h_3(23) = (3+3*5) \bmod 10 = 8$ (occupied)

$h_4(23) = (3+4*5) \bmod 10 = 3$ (occupied)

Double Hashing

Table size = 10

$\text{step}(x) = 5$

Result: Infinite loop!!!!

Why does it happen?

Can it be avoided?