CSE 326 Lecture 14: Sorting

- Today's Topics:
  - Elementary Sorting Algorithms:
    - Bubble Sort
    - Selection Sort
    - Insertion Sort
  - Shellsort
- Covered in Chapter 7 of the textbook

Sorting: Definitions

- Input: You are given an array A of data records, each with a key (which could be an integer, character, string, etc.).
  - There is an *ordering* on the set of possible keys
  - You can compare any two keys using <, >, ==
- For simplicity, we will assume that A[i] contains only one element – the key
- Sorting Problem: Given an array A, output A such that:
  For any i and j, if $i < j$ then $A[i] \leq A[j]$
- *Internal sorting*: all data in main memory
- *External sorting:* data on disk

Why Sort?

- Sorting algorithms are among the most frequently used algorithms in computer science
  - Crucial for efficient retrieval and processing of large volumes of data. E.g. Database systems
- Allows binary search of an N-element array in O(log N) time
- Allows O(1) time access to *k*th largest element in the array for any *k*
- Allows easy detection of any duplicates

Sorting: Things to Think about…

- Space: Does the sorting algorithm require extra memory to sort the collection of items?
  - Do you need to copy and temporarily store some subset of the keys/data records?
  - An algorithm which requires O(1) extra space is known as an **in place** sorting algorithm

Sorting: More Things to Think about…

- Stability: Does it rearrange the order of input data records which have the same key value (duplicates)?
  - E.g. Given: Phone book sorted by name. Now sort by county – is the list still sorted by name within each county?
  - Extremely important property for databases
  - A **stable sorting algorithm** is one which does not rearrange the order of duplicate keys

Sorting 101: Bubble Sort

- Idea: "Bubble" larger elements to end of array by comparing elements i and i+1, and swapping if A[i] > A[i+1]
  - Repeat from first to end of unsorted part
- Example: Sort the following input sequence:
  - 21, 33, 7, 25

Sorting 101: Bubblesort

```
/* Bubble sort pseudocode for integers
*  A is an array containing N integers */

for(int i=0;i<N;i++) {
    /* From start to the end of unsorted part */
    for(int j=1;j<(N-i);j++) {
      /* If adjacent items out of order, swap */
      if( A[j-1] > A[j] ) SWAP(A[j-1],A[j]);
    }
}
```

- Stable? In place? Running time = ?

Sorting 102: Selection Sort

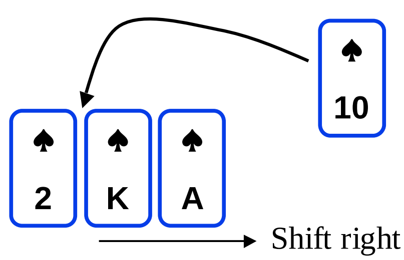
- Bubblesort is stable and in place, but $O(N^2)$ – can we do better by moving items more than 1 slot per step?
- Idea: Scan array and select smallest key, swap with A[1]; scan remaining keys, select smallest and swap with A[2]; repeat until last element is reached.
- Example: Sort the following input sequence:
  - 21, 33, 7, 25
- Is selection sort stable (suppose you had another 33 instead of 7)? In place?
- Running time = ?

Sorting 102: Selection Sort

- Bubblesort is $O(N^2)$ – can we do better by moving items more than 1 slot per step?
- Idea: Scan array and select smallest key, swap with A[1]; scan remaining keys, select smallest and swap with A[2]; repeat until last element is reached.
- Example: Sort the following input sequence:
  - 21, 33, 7, 25
- NOT STABLE. In place (extra space = 1 temp variable).
- Running time = N steps with N-1, …, 1 comparisons
  = N-1 + … + 1 = $O(N^2)$

Sorting 103: Insertion Sort

- What if first *k* elements of array are already sorted?
  - E.g. 4, 7, 12, 5, 19, 16
- Idea: Can *insert* next element into proper position and get k+1 sorted elements, *insert* next and get k+2 sorted etc.
  - 4, 5, 7, 12, 19, 16
  - 4, 5, 7, 12, 19, 16
  - 4, 5, 7, 12, 16, 19 Done!
  - Overall, N-1 passes needed
  - Similar to card sorting…
  - Start with empty hand
  - Keep inserting…



Sorting 103: Insertion Sort

```
/* Insertion sort pseudocode for integers
*  A is an array containing N integers */

  int j, P, Tmp;
  for(P = 1; P < N; P++ ) {
    Tmp = A[ P ];
    for(j = P; j > 0 && A[ j - 1 ] > Tmp; j-- )
       A[ j ] = A[ j - 1 ]; //Shift A[j-1] to right
    A[ j ] = Tmp; // Found a spot for A[P] (= Tmp)
   }
```

- Is Insertion sort in place? Stable?
- Running time = ?

Sorting 103: Insertion Sort

```
  int j, P, Tmp;
  for(P = 1; P < N; P++ ) {
    Tmp = A[ P ];
    for(j = P; j > 0 && A[ j - 1 ] > Tmp; j-- )
       A[ j ] = A[ j - 1 ]; //Shift A[j-1] to right
    A[ j ] = Tmp; // Found a spot for A[P] (= Tmp)
   }
```

- Insertion sort: In place (O(1) space for Tmp) and stable
- Running time: Worst case is reverse order input = $\Theta(N^2)$
  - Best case is input already sorted = O(N).

Lower Bound on Simple Sorting Algorithms

- An *inversion* is a pair of elements in wrong order
  - ➪ $i < j$ but $A[i] > A[j]$
- Our simple sorting algorithms so far swap adjacent elements (explicitly or implicitly): swapping removes 1 inversion
  - ➪ Running time proportional to no. of inversions in array
- Given N distinct keys, total of N(N-1)/2 possible inversions. Average list contains: N(N-1)/4 inversions
  - ➪ Average running time of Insertion sort is $\Theta(N^2)$
- Any sorting algorithm that swaps adjacent elements requires $\Omega(N^2)$ time: Each swap removes only one inversion

Shellsort: Breaking the Quadratic Barrier

- Main Insight: Insertion sort runs fast on nearly sorted sequences do *several passes of Insertion sort* on different subsequences of elements
- Example: Sort 19, 5, 2, 1
  1. Do Insertion sort on subsequences of elements spaced apart by **2**: 1st and 3rd, 2nd and 4th
     - ➪ 19, *5*, 2, *1* 2, *1*, 19, *5*
  2. Do Insertion sort on subsequence of elements spaced apart by **1**:
     - ➪ 2, 1, 19, 5 1, 2, 19, 5 1, 2, 19, 5 1, 2, 5, 19
- Note: Fewer number of shifts than plain Insertion sort
  - ➪ 4 versus 6 for this example

Shellsort: Overview

- Named after Donald Shell – first algorithm to achieve $o(N^2)$
  - Running time is $O(N^x)$ where x = 3/2, 5/4, 4/3, …, or 2 depending on "increment sequence"
- In our example, we used the increment sequence: N/2, N/4, …, 1 = 2, 1 (for N = 4 elements)
  - This is Shell's original increment sequence
- Shellsort: Pick an *increment sequence* $h_t > h_{t-1} > \ldots > h_1$
  - Start with k = t
  - Insertion sort all subsequences of elements that are h_k apart so that $\mathbf{A[i] \le A[i+h_k]}$ for all i (known as an *h_k-sort*)
  - Go to next smaller increment h_{k-1} and repeat until k = 1 (note: $h_1 = 1$)

Shellsort: An Example (a pathetic one)

- Example: Shell's original sequence: $h_t = N/2$ and $h_k = h_{k+1}/2$
  - Sort 21, 33, 7, 25
  - Try it! (What is the increment sequence?)

Shellsort: An Example

- Example: Shell's original sequence: $h_t = N/2$ and $h_k = h_{k+1}/2$
  - Sort 21, *33*, 7, *25* (N = 4, increment sequence = 2, 1)
  - 7, *25*, 21, *33* (after 2-sort)
  - 7, 21, 25, 33 (after 1-sort)

Shellsort: The Nuts and Bolts

```
/* Shell sort pseudocode for integers
*  A is an array containing N integers */

   int i, j, Increment, Tmp;
   for( Increment = N/2; Increment > 0; Increment /= 2 )
     for( i = Increment; i < N; i++ ) {
       Tmp = A[ i ];

       for( j = i; j >= Increment &&
                A[ j - Increment ] > Tmp ; j -= Increment )
          A[ j ] = A[ j - Increment ];

       A[ j ] = Tmp;
     }
```

- Note: The two inner `for` loops correspond almost exactly to the code for Insertion sort!
- Running time = ? (What is the worst case?)

Shellsort: Run Time Analysis

- Simple to code but hard to analyze
  - Run time depends on increment sequence
- What about the increment sequence h_k = N/2, N/4, …, 2, 1?
  - Upper bound
    - Shellsort does h_k insertion sorts with N/h_k elements for k = 1 to t
    - Running time = $O(\Sigma_{k=1\ldots t}\, h_k\, (N/h_k)^2)$
      $= O(N^2\, \Sigma_{k=1\ldots t}\, 1/h_k) =$ **$O(N^2)$**
  - Lower bound
    - What is the worst case?

Shellsort: Run Time Analysis

- What about the increment sequence N/2, N/4, …, 2, 1?
  - Lower bound
    - What is the worst case?
    - Suppose smallest elements in odd positions, largest in even positions in sorted order:
      2, 11, 4, 12, 6, 13, 8, 14
    - None of the passes N/2, N/4, …, 2 do anything!
    - Last pass (h_1 = 1) must shift N/2 smallest elements to first half and N/2 largest elements to second half
    - 4 shifts 1 slot, 6 shifts 2, 8 shifts 3, … = 1 + 2 + 3 + … (N/2 terms)
    - Run time = At least N^2 steps = **$\Omega(N^2)$**

Shellsort: Can we do better?

- The reason we got $\Omega(N^2)$ was because of increment sequence
  - Adjacent increments have common factors (e.g. 8, 4, 2, 1)
  - We keep comparing same elements over and over again
  - Need to increment so that different elements are compared in different passes
- Is there a better increment sequence than N/2, N/4, …, 2, 1?

Shellsort: How to Break the $O(N^2)$ Barrier

- Hibbard's increment sequence: $2^k - 1, 2^{k-1} - 1, \ldots, 7, 3, 1$
  - Adjacent increments have no common factors
  - Worst case running time of Shellsort with Hibbard's increments = $\Theta(N^{1.5})$ (Theorem 7.4 in text)
  - Average case running time for Hibbard's = $O(N^{1.25})$ in simulations but *nobody has been able to prove it!* (next homework assignment?)
- Final thoughts on the "Simple Sorts" discussed today:
  - Insertion sort good for small input sizes (~20)
  - Shellsort better for moderately large inputs (~10,000)

After Midterm: The crème de la crème of Sorts:

Heapsort, Mergesort, and Quicksort

Next Class: Midterm Review

To Do:

Midterm on Wed Feb 12: Read Chapters 1 through 6

HW #3 due: Thu Feb 13