CSE 326: Data Structures
Topic #7: Don't Sweat It - Splay It

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AVL Trees: Are They Worth It?

Advantages
- Rotations are cool!

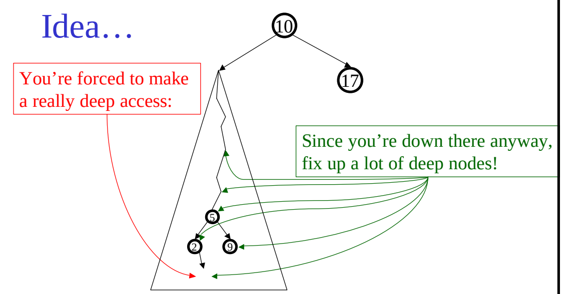
Disadvantages
- Wouldn't want to meet one in a dark alley at night

Splay What?

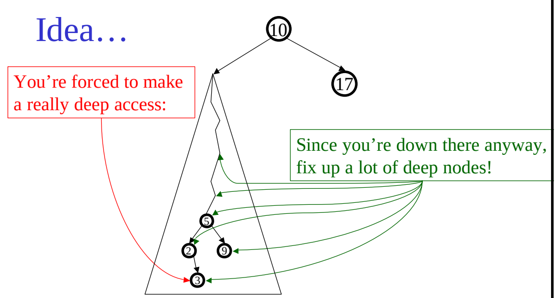
- Blind adjusting version of AVL trees
  - Why worry about balances? Just rotate anyway!
- *Amortized* time for all operations is O(log n)
- Worst case time is O(n)
  - But guaranteed to happen rarely
- Insert/Find always rotates node *to the root*!

Analogy: AVL is to Splay trees as….

Idea…



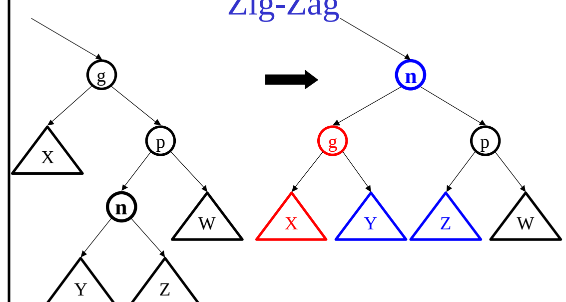
Idea…



Details…

1. Find or insert a node **n**
2. *Splay* **n** to the root using:
   zig-zag, **zig-zig**, or plain ol' **zig**
3. Helps the new root (**n**) *and* many others!

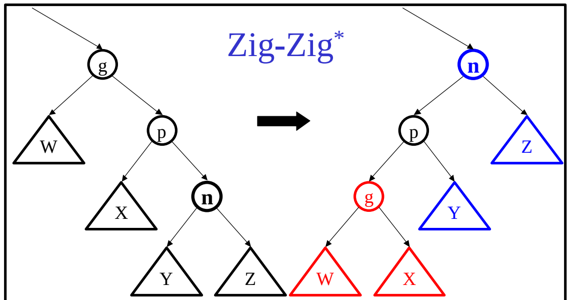
Zig-Zag*



*Just like an…

Which nodes improve depth?

Zig-Zig*

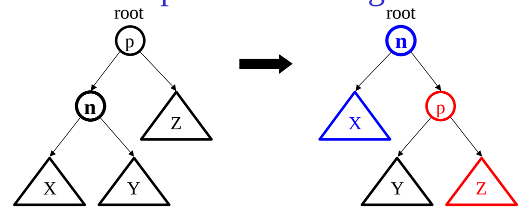


*Is this an AVL zig-zig?

How to implement?

Why does this help?

Special Case: Zig



Final result from splaying to root:

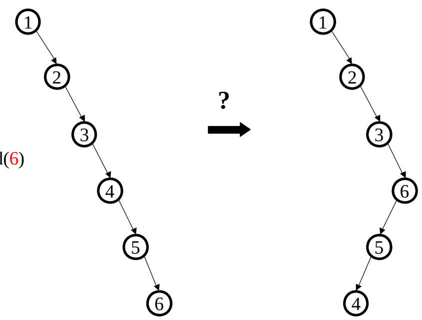
Relative depth of p, Y, Z?

Relative depth of everyone else?

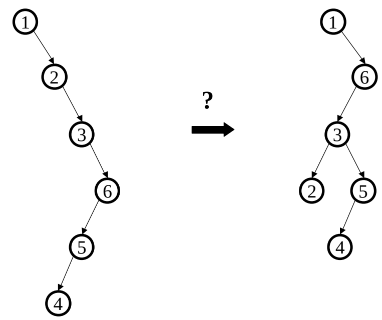
Why not drop zig-zig and just zig all the way?

Splaying Example

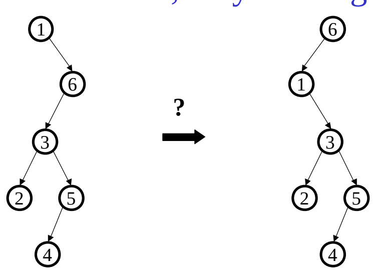
Find(6)



Still Splaying 6

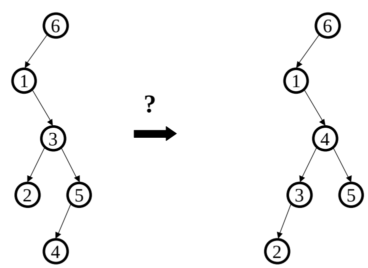


Almost There, Stay on Target

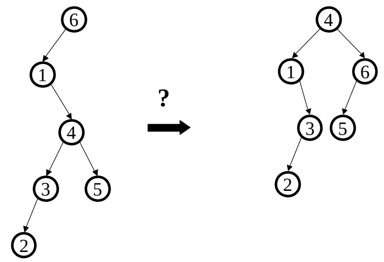


Splay Again

Find(4)



Example Splayed Out



Why Splaying Helps

- If a node *n* on the access path is at depth *d* before the splay, it's at about depth *d/2* after the splay
  - Exceptions are the root, the child of the root (and descendants), and the node splayed
- Overall, nodes which are below nodes on the access path tend to move closer to the root
- Splaying gets amortized O(log n) performance.

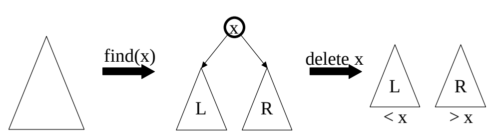
Splay Operations: Find

- Find the node in normal BST manner
- Splay the node to the root

Splay Operations: Insert

- Insert the node in normal BST manner
- Splay the node to the root

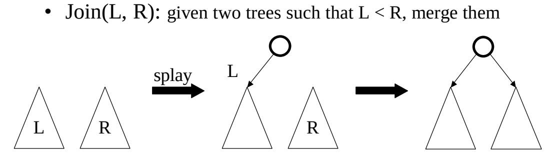
Splay Operations: Remove



Now what?

Join

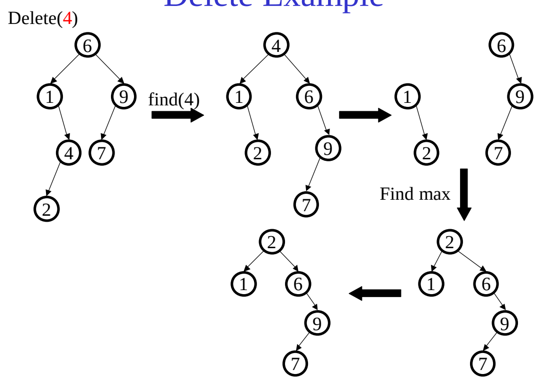
- Join(L, R): given two trees such that L < R, merge them



- Splay on the maximum element in L then attach R

Does this work to join any two trees?

Delete Example

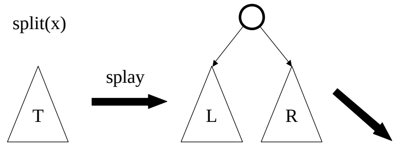


Nifty Splay Operation: Splitting

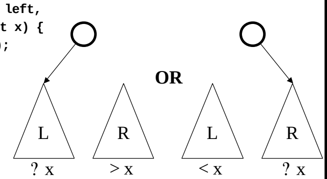
- Split(T, x) creates two BSTs L and R:
  - all elements of T are in either L or R (`T = L ? R`)
  - all elements in L are ? x
  - all elements in R are ? x
  - L and R share no elements (`L ? R = ?`)

How do we split a splay tree?

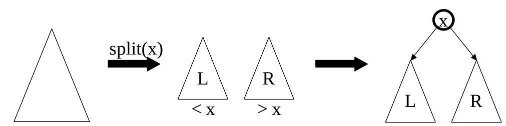
Splitting Splays




```
void split(Node * root, Node *& left,
           Node *& right, Object x) {
  Node * target = root ->find(x);
  splay(target);
  if (target < x) {
    left = target ->left;
    target->left = NULL;
    right = target;
  }
  ...
}
```



Pssstt: Another Way to Insert




```
void insert(Node *& root, Object x) {
  Node * left, * right;
  split(root, left, right, x);
  root = new Node(x, left, right);
}
```

Interesting note: split-and-insert was the original algorithm. But insert-and-splay has better constants

Splay Tree Summary

- All operations are in amortized O(log n) time
- Splaying can be done top-down; better because:
  - only one pass
  - no recursion or parent pointers necessary
- Splay trees are *very* effective search trees
  - Relatively simple
  - No extra fields required
  - Excellent *locality* properties: frequently accessed keys are cheap to find

Coming Up

- *Really* big search trees
- Hashing
- More on HW #3's *mysterious* benefactor…

To Do

- Finish Chapter 4, start Chapter 5
- Continue HW 3 – part B due Tuesday, 11 p.m.