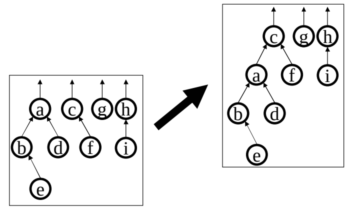
CSE 326: Data Structures

Topic #11: Disjoint Set ADT (2)

Ashish Sabharwal
Autumn, 2003

Improving Union



Could we do a better job on this union?

Union-by-size: Code

```
int Union(int x, int y) {
  // If up[x] and up[y] aren't both
  // -1, this algorithm is in trouble

  if (size[x] > size[y]) {
    up[y] = x;
    size[x] += size[y];
  }
  else {
    up[x] = y;
    size[y] += size[x];
  }
}
```

new runtime for Union():

new runtime for Find():

Union-by-size: Find Analysis

- Complexity of Find: Θ(max node depth)
- All nodes start at depth 0
- Node depth increases
  - Only when it is part of smaller tree in a union
  - Only by one level at a time

  Result: **tree size doubles when node depth increases by 1**

Find runtime = Θ(node depth) =

runtime for m finds and n-1 unions =

Nifty Storage Trick

- Use the same array representation as before
- Instead of storing **`–1`** for the root, simply store **`–size`**

[Read section 8.4, page 276]

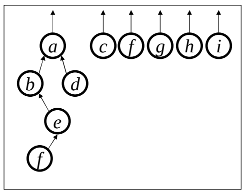
How about Union-by-height?

- Can still guarantee $\Theta(\log n)$ worst case depth

  Left as an exercise!
  (will probably appear in Homework #3)

- Problem: Union-by-height doesn't combine very well with the new find optimization technique we'll see next

Improving Find

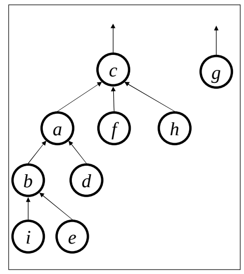


While we're finding *f*, could we do anything else?

Hint: think splay trees…

Path Compression!

find(*e*)



➡

Recall: it need *not* be a binary tree!

Path Compression: Code

```
int Find(Object x) {
  // x had better be in
  // the set!
  int xID = hTable[x];
  int i = xID;

  // Get the root for
  // this set
  while(up[xID] != -1) {
    xID = up[xID];
  }
```

```
  // Change the parent for
  // all nodes along the path
  while(up[i] != -1) {
      temp = up[i];
      up[i] = xID;
      i = temp;
  }
  return xID;
}
```

(New?) runtime for Find:

Interlude: A Really Slow Function

Ackermann's function is a <u>really</u> big function A(*x*, *y*) with inverse $\alpha(x, y)$ which is <u>really</u> small

How fast does $\alpha(x, y)$ grow?

$\alpha(x, y) = 4$ for *x* **far** larger than the number of atoms in the universe (2^{300})

α shows up in:

- Computation Geometry (surface complexity)
- Combinatorics of sequences

A More Comprehensible Slow Function

log* *x* = number of times you need to compute log to bring value down to at most 1

E.g. log* 2 = 1

log* 4 = log* 2^2 = 2

log* 16 = log* 2^{2^2} = 3 (log log log 16 = 1)

log* 65536 = log* $2^{2^{2^2}}$ = 4 (log log log log 65536 = 1)

log* 2^{65536} = …………… = 5

Take this: $\alpha(m,n)$ grows even slower than log* *n* !!

Complex Complexity of Union-by-Size + Path Compression

Tarjan proved that, with these optimizations, *p* union and find operations on a set of *n* elements have worst case complexity of $O(p \cdot \alpha(p, n))$

For *all practical purposes* this is amortized constant time: $O(p \cdot 4)$ for *p* operations!

- Very complex analysis – worse than splay tree analysis etc. that we skipped!
- Tarjan is also the (very smart) splay tree guy