# CSE 326: Data Structures
# Lecture #7
# Binary Search Trees

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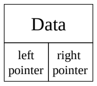
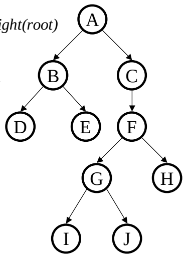
## Binary Trees

- Properties

  *Notation:*
  *depth(tree) = MAX {depth(leaf)} = height(root)*
  - max # of leaves = $2^{depth(tree)}$
  - max # of nodes = $2^{depth(tree)+1} - 1$
  - max depth = n-1
  - average depth for n nodes = $\sqrt{n}$ (over all possible binary trees)
- Representation:





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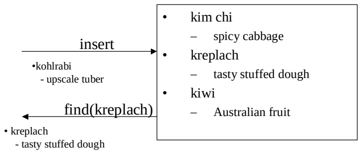
## Dictionary & Search ADTs

- Operations
  - create
  - destroy
  - insert
  - find
  - delete



- Dictionary: Stores *values* associated with user-specified *keys*
  - keys may be any (homogenous) comparable type
  - values may be any (homogenous) type
  - implementation: data field is a struct with two parts
- Search ADT: keys = values

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## Naïve Implementations

| | unsorted array | sorted array | linked list |
|---|---|---|---|
| insert (w/o duplicates) | (if no stretch) | | |
| find | | | |
| delete | (if no shrink) | | |

*Goal: fast find like sorted array, dynamic inserts/deletes like linked list*

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## Naïve Implementations

| | unsorted array | sorted array | linked list |
|---|---|---|---|
| insert (w/o duplicates) | find + O(1)<br>(if no stretch) | O(n) | find + O(1) |
| find | O(n) | O(log n) | O(n) |
| delete | find + O(1)<br>(if no shrink) | O(n) | find + O(1) |

*Goal: fast find like sorted array, dynamic inserts/deletes like linked list*

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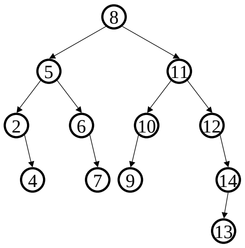
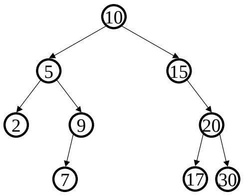
## Binary Search Tree Dictionary Data Structure

- Search tree property
  - all keys in left subtree smaller than root's key
  - all keys in right subtree larger than root's key
  - result:
    - easy to find any given key
    - inserts/deletes by changing links



## In Order Listing



visit left subtree
visit node
visit right subtree

In order listing:

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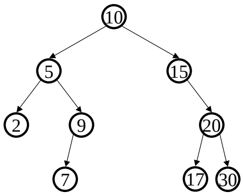
## In Order Listing



visit left subtree
visit node
visit right subtree

In order listing:
2→5→7→9→10→15→17→20→30

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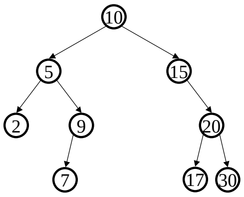
## Finding a Node



```
Node *& find(Comparable x,
             Node * root)
{
  if (root == NULL)
    return root;
  else if (x < root->key)
    return find(x,root->left);
  else if (x > root->key)
    return find(x, root->right);
  else
    return root;
}
```

runtime:

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## Insert

Concept: proceed down tree as in Find; if new key not found, then insert a new node at last spot traversed

```
void insert(Comparable x,  Node * root) {
  assert ( root != NULL );
  if (x < root->key){
      if (root->left == NULL)
              root->left = new Node(x);
      else insert( x, root->left ); }
  else if (x > root->key){
      if (root->right == NULL)
              root->right = new Node(x);
      else insert( x, root->right ); } }
```

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## Tricky Insert

C++ trick: use reference parameters

```
void insert(Comparable x,  Node * & root) {
  if ( root == NULL )
      root = new Node(x);
  else if (x < root->key)
      insert( x, root->left );
  else
      insert( x, root->right ); }
```

runtime:

Works even when called with empty tree –

```
  node * myTree = NULL;
  insert( something,  myTree );
```

sets the variable myTree to point to the newly created node

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## Digression: Value vs. Reference Parameters

- Value parameters (Object foo)
  - copies parameter
  - no side effects
- Reference parameters (Object & foo)
  - shares parameter
  - can affect actual value
  - use when the value needs to be changed
- Const reference parameters (const Object & foo)
  - shares parameter
  - cannot affect actual value
  - use when the value is too intricate for pass-by-value

# *Really* Tricky Insert

```
void insert(Comparable x,  Node * & root){

  Node * & target = find(x, root);
  if ( target == NULL )
     target = new Node(x); }
```

# BuildTree for BSTs

Suppose $a_1, a_2, \ldots, a_n$ are inserted into an initially empty BST:

1. $a_1, a_2, \ldots, a_n$ are in increasing order

2. $a_1, a_2, \ldots, a_n$ are in decreasing order

3. $a_1$ is the median of all, $a_2$ is the median of elements less than $a_1$, $a_3$ is the median of elements greater than $a_1$, etc.

4. data is randomly ordered

# Analysis of BuildTree

- Worst case is $O(n^2)$

$1 + 2 + 3 + \ldots + n = O(n^2)$

- Average case assuming all input sequences are equally likely is $O(n \log n)$
  - equivalently: average depth of a node is $\log n$
  - proof: see *Introduction to Algorithms*, Cormen, Leiserson, & Rivest

## Proof that Average Depth of a Node in a BST constructed from random data is O(log n)

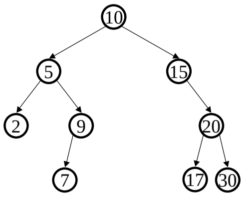
- Calculate sum of all depths, divide by number of nodes
- D(n) = sum of depths of all nodes in a random BST containing n nodes
- D(n) = D(left subtree)+D(right subtree) +1*(number of nodes in left and right subtrees)
- D(n) = D(L)+D(n-L-1)+(n-1)
- For random data, all subtree sizes equally likely

$$D(n) = \left(\frac{1}{n}\sum_{L=0}^{n-1}(D(L)+D(n-L-1))\right)+(n-1)$$

$$D(n) = \left(\frac{2}{n}\sum_{L=0}^{n-1}D(L)\right)+(n-1)$$

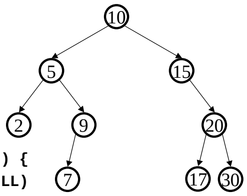
$$D(n) = O(n\log n)$$

# Deletion



Why might deletion be harder than insertion?

# FindMin/FindMax



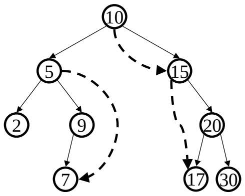
```
Node * min(Node * root) {
  if (root->left == NULL)
    return root;
  else
    return min(root->left); }
```

*How many children can the min of a node have?*

## Successor

Find the next larger node
in this node's subtree.

- *not next larger in entire tree*

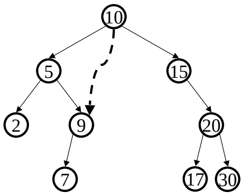


```
Node * succ(Node * root) {
  if (root->right == NULL)
    return NULL;
  else
    return min(root->right);
}
```

*How many children can the successor of a node have?*

## Predecessor

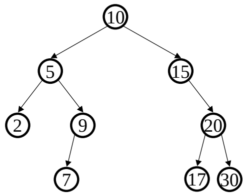
Find the next smaller node
in this node's subtree.



```
Node * pred(Node * root) {
  if (root->left == NULL)
    return NULL;
  else
    return max(root->left);
}
```

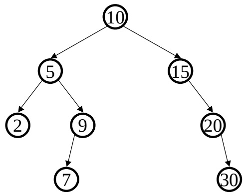
## Deletion - Leaf Case

Delete(17)



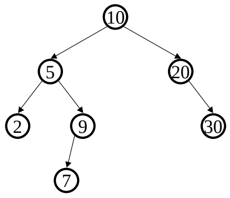
## Deletion - One Child Case

Delete(15)



## Deletion - Two Child Case

Delete(5)

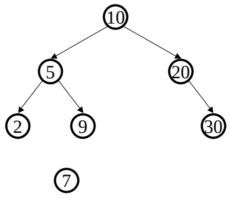


replace node with value guaranteed to be between the left and right subtrees: the successor

*Could we have used the predecessor instead?*

## Deletion - Two Child Case

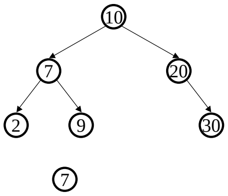
Delete(5)



always easy to delete the successor – always has either 0 or 1 children!

# Deletion - Two Child Case

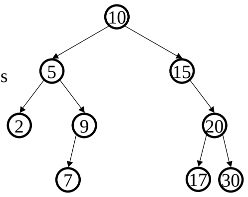
Delete(5)



Finally copy data value from deleted successor into original node

# Lazy Deletion

- Instead of physically deleting nodes, just mark them as deleted
  - \+ simpler
  - \+ physical deletions done in batches
  - \+ some adds just flip deleted flag
  - – extra memory for deleted flag
  - – many lazy deletions slow finds
  - – some operations may have to be modified (e.g., min and max)



# Dictionary Implementations

| | unsorted array | sorted array | linked list | BST |
|---|---|---|---|---|
| insert | find + O(1) | O(n) | find + O(1) | O(Depth) |
| find | O(n) | O(log n) | O(n) | O(Depth) |
| delete | find + O(1) | O(n) | find + O(1) | O(Depth) |

BST’s looking good for shallow trees, *i.e.* the depth D is small (log n), otherwise as bad as a linked list!