CSE 322: Introduction to Formal Models in Computer Science
Assignment #2
April 8, 1998
due: Wednesday, April 15

There are two pages to this assignment.

1. Exercise 1.8.3. Make your regular expressions as short and simple as you can. Give a very brief explanation of why each one is correct.

2. Exercise 2.1.2. Instead of describing informally, give an equivalent regular expression. Make your regular expressions as short and simple as you can. Give a very brief explanation of why each one is correct.

3. Exercise 2.1.3 (a), (b), and (c). Just give the state diagram for each automaton. Make your state diagrams as simple as you can. Give a very brief explanation of why each one is correct.

4. (a) Give the state diagram for a deterministic finite automaton that accepts the language

   $$L = \{w \in \{0,1\}^* \mid w \text{ is the binary representation of a multiple of } 5\}.$$

   For the purposes of this problem, assume that e represents the integer 0, and that leading 0's are o.k. For instance, e, 11001, and 00101 are all in L, but 110 and 00001 are not.
   Hint: Let the state set be $\{q_0, q_1, q_2, q_3, q_4\}$, and maintain the property that $(q_0, w) \vdash_M^* (q_i, e)$ if and only if $w' \bmod 5 = i$, where w' is the integer with binary representation w. Now think, for example, about what the remainder mod 5 of (the integer with binary representation) $w1$ would be, if you know that the remainder mod 5 of (the integer with binary representation) w is 3.

   (b) Suppose you wanted to solve part 4a now for multiples of k instead of multiples of 5, and you took the state set to be $\{q_0, q_1, \ldots, q_{k-1}\}$, generalizing the hint above. For $\sigma \in \{0,1\}$, what would $\delta(q_i, \sigma)$ be? What is the initial state s and the set F of final states?

(c) By induction on $|w|$, prove that your construction in part 4b satisfies $(q_0, w) \vdash_M^* (q_i, e)$ if and only if $w' \bmod k = i$, where w' is the integer with binary representation w. Problem 2(a) on Assignment #1 may be useful.

(d) Conclude that your automaton in part 4b accepts the (binary representations of) multiples of k.