# CSE 322
# Autumn Quarter 1998
# Assignment 8
# Due Friday, December 4, 1998

All solutions should be neatly written or type set.

1. (10 points) Use a Turing machine diagram to design a machine that when it starts with initial tape and head in configuration $\underline{\triangleright} a^n$ eventually halts with tape and head in configuration $\triangleright a^n \underline{\sqcup} a^{n^2}$. Hint: your machine will likely need tape symbols beyond $\triangleright, \sqcup$, and $a$.

2. (10 points) Use a Turing machine diagram to design a machine that decides the language $L = \{wcw : w \in \{a,b\}^*\}$. That is, your machine must always halt, go to state $y$ if the input is in the language $L$ and go to the state $n$ if the input is not in the language $L$. The initial configuration of the tape and head is $\underline{\triangleright} wcw$.

3. (10 points) Let $M = (K, \Sigma, \delta, s, H)$ be a Turing machine. Define $\Gamma = \Sigma - \{\triangleright\}$. Define a *string configuration* to be a string in $\triangleright \Gamma^* K \Gamma^*$. That is, a string configuration is a string of the form $\triangleright wqv$ where $w, v \in \Gamma^*$ and $q \in K$. The string configuration $\triangleright wqv$ represents the configuration $(q, \triangleright w, v)$. We say that $\triangleright wqv \vdash_M \triangleright w'q'v'$ if $(q, \triangleright w, v) \vdash_M (q', \triangleright w', v')$, that is if the corresponding configurations also fall in the $\vdash_M$ relation. Design a context-free grammar $G = (V, \Sigma \cup K \cup \{\#\}, R, S)$ with the property that

$$L(G) = \{x\#y^R : x, y \in \triangleright \Gamma^* K \Gamma^*,\ |x| = |y|,\ \text{and } x \vdash_M y\}.$$

Note that $\#$ is a symbol that is not in $\Sigma \cup K$ and $\Sigma$ and $K$ are disjoint.