CSE 322
Autumn Quarter 1998
Assignment 7
Due Wednesday, November 25, 1998

All solutions should be neatly written or type set.

1. (10 points) The strong pumping lemma for context-free languages is stated:

   For each context-free language L there is a number $N > 0$ such that for all $w \in L$ with $|w| > N$, there are u, v, x, y, and z such that

   1. $w = uvxyz$,
   2. $|v| + |y| > 0$,
   3. $|vxy| \leq N$,
   4. $uv^ixy^iz \in L$ for all $i \geq 0$.

   Use the strong pumping lemma to show that the language

   $$L = \{a^ib^jc^k : 0 \leq i \leq j \leq k\}$$

   is not context-free.

   Hint: In your proof by contradiction (i) the N will be given to you, (ii) then you can choose the w, (iii) then the u, v, x, y, and z will be given to you, (iv) then there will be a number of cases each one leading to the contradiction. The main thing to remember is that you don't have control over the choice of the five parts u, v, x, y, and z.

2. (10 points) Recall the definition of the finite state transducer from Assignment 3. A finite state transducer is a 5-tuple $M = (K, \Sigma, \Gamma, \delta, s)$ where K is the finite set of states, Σ is the input alphabet, Γ is the output alphabet, s is the start state. The transition function δ maps $K \times \Sigma$ into $K \times \Gamma^*$. The semantics of the transition is that if $\delta(q, \sigma) = (p, w)$ then M in state q reading the symbol σ goes to state p outputting the string w. Given an input string M produces the output which is the concatenation of all the strings outputted by M during its moves.

Given a string x in the alphabet Σ we define

$$F_M(x) = w \text{ where } M \text{ ouputs } w \text{ on input } x.$$

Note that $w \in \Gamma^*$. If L is a language in the alphabet Σ then define

$$F_M(L) = \{F_M(x) : x \in L\}.$$

For example if $L = \{a^n b^n : n \geq 0\}$ and M is a finite state transducer with just one state that when reading an a outputs a a and on reading b outputs bb, then $F_M(L) = \{a^n b^{2n} : n \geq 0\}$

(a) Let $L = \{a^m b a^n c a^m b a^n : m, n \geq 0\}$. Design a finite state transducer M such that

$$F_M(L) = \{a^m b^n a^m b^n : m, n \geq 0\}.$$

(b) It is a fact that if L is context-free and M is a finite state transducer then $F_M(L)$ is also context-free. Use this fact and other known closure properties of context-free languages to show that the language $L = \{ww : w \in \{a, b\}^*\}$ is not context-free. You may use the fact that the language $\{a^m b^n a^m b^n : m, n \geq 0\}$ is not context-free.