CSE 322
Autumn Quarter 1998
Assignment 6
Due Friday, November 20, 1998

All solutions should be neatly written or type set.

1. (10 points) Consider the grammar $G = (V, \Sigma, R, S)$ where

$$\begin{aligned}
V &= \{+, *, (,), id, T, F, E\} \\
\Sigma &= \{+, *, (,), id\} \\
R &= \{E \rightarrow E + T, \\
&= E \rightarrow T, \\
&= T \rightarrow T * F, \\
&= T \rightarrow F, \\
&= F \rightarrow (E), \\
&= F \rightarrow id\}
\end{aligned}$$

   (a) Design a PDA M_T by the "top down" construction that accepts $L(G)$. Give a leftmost derivation of $(id + id) * id + id$. Beside it give the sequence of configurations from M_T that corresponds to the leftmost derivation.

   (b) Design a PDA M_B by the "bottom up" construction that accepts $L(G)$. Give a rightmost derivation of $(id + id) * id + id$. Beside it give the sequence of configurations from M_B that corresponds to the rightmost derivation.

2. (10 points) A context-free grammar $G = (V, \Sigma, R, S)$ is *linear* if every production has at most one nonterminal on the right hand side. That is, if $A \rightarrow \alpha$ is a production then $\alpha \in \Sigma^*((V - \Sigma) \cup \{e\})\Sigma^*$. For example, the grammar with productions $\{S \rightarrow aSb, S \rightarrow e\}$ is linear. A *single-turn* PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is one that reverses its stack at most once in any computation. That is, whenever

$$(s, w, e) \vdash_M^* (q_1, w_1, \gamma_1) \vdash_M (q_2, w_2, \gamma_2) \vdash_M^* (q_3, w_3, \gamma_3) \vdash_M (q_4, w_4, \gamma_4) \text{ and } |\gamma_1| > |\gamma_2|$$

then it must be the case that

$$|\gamma_3| \geq |\gamma_4|.$$

(a) Construct a single-turn PDA for the grammar with the two productions $\{S \rightarrow aSb, S \rightarrow e\}$ that accepts the same language.

(b) Give a general construction of a single-turn PDA M that accepts the language of a linear grammar G.

3. (10 points) Let $G = (V, \Sigma, R, S)$.

(a) A nonterminal A *productive* if if $A \Rightarrow_G^* w$ for some $w \in \Sigma^*$. That is, some terminal string can be generated from A. Give an algorithm for finding all the productive nonterminals in a grammar G.

(b) Use the algorithm in part (a) as part of an algorithm for deciding if the language generated by a context-free grammar is empty.

(c) Use the algorithm in part (a) to construct a context-free grammar G' such that $L(G) = L(G')$ and for all α, if $S \Rightarrow_{G'}^* \alpha$ then $\alpha \Rightarrow_{G'}^* w$ for some $w \in \Sigma^*$. That is, G' and G generate the same language and in G' every partial derivation can be eventually completed into the derivation of some terminal string

(d) (5 points extra credit) Show that the context-free languages are closed under the prefix operation. (Hint: you will need part (c) in your construction.)