CSE 322
Autumn Quarter 1998
Assignment 5
Due Friday, November 13, 1998

All solutions should be neatly written or type set.

1. (10 points) Consider the language

$$L = \{a^i b^j c^k : i, j, k \geq 0 \text{ and } i = j \text{ or } j = k\}$$

   (a) Design a context-free grammar that generates L and explain what each of your non-terminals is generating.

   (b) Demonstrate that your grammar is ambiguous.

2. (10 points) A context-free grammar $G = (V, \Sigma, R, S)$ is *right-linear* if every production in R has one of the following forms (i) $A \to wB$ where $A, B \in V - \Sigma$ and $w \in \Sigma^*$ or (ii) $A \to w$ where $A \in V - \Sigma$ and $w \in \Sigma^*$.

   (a) Consider the right-linear grammar $G = (V, \Sigma, R, S)$ where

$$\begin{aligned} V &= \{a, b, S, A, B\} \\ \Sigma &= \{a, b\} \\ R &= \{S \to aaS, S \to abA, S \to e, A \to bA, A \to baB, \\ &\quad\; B \to aB, B \to bbS\} \end{aligned}$$

   Design an NFA that accepts $L(G)$.

   (b) Prove that the language generated by a right-linear grammar is always regular.

3. (10 points) Consider the definition of regular expressions given on page 48 of the text.

   (a) Design an unambiguous grammar with exactly one nonterminal that generates the set of regular expressions over an alphabet Σ.

(b) Suppose we decide that full parentheses are not really ideal for regular expressions because they make them hard to read. Instead, we define *informal regular expressions* that make parentheses optional. However, in the informal regular expressions we interpret Kleene star with the strongest binding power, concatenation with the next strongest binding power, and union the least binding power. For example, the informal expression $ab^* \cup c^*$ means $((ab^*) \cup c^*)$ not $a(b^* \cup c)^*$ or anything else. Naturally, we still allow parentheses in our informal regular expressions. For example, we may want to write the informal regular expression $(a \cup b)^* c$. Design an unambiguous grammar for informal regular expressions over Σ that correctly parses expressions with respect to the binding powers of the operation of Kleene star, concatenation, and union. (Hint: you will probably want to model your grammar after a grammar like that in example 3.1.3 on page 117 of the text.)