CSE 322
Autumn Quarter 1998
Assignment 4
Due Friday, October 30

All solutions should be neatly written or type set.

1. (10 points) Use the pumping lemma for regular languages to carefully prove that the language
$$L = \{a^n b^m c^{n+m}\}$$
is not regular.

2. (10 points) Consider the finite language $L_n = \{xx : x \in \{a,b\}^* \text{ and } |x| = n\}$.

   (a) Construct a DFA M_n that accepts L_n. Try to do it with as few states as possible. Can you do it in fewer than $(n+2)2^n$ states?

   (b) Prove that any NFA that accepts L_n must have at least 2^n states. (Be careful because you are proving this result for NFAs not DFAs.)

3. (10 points) Let $M = (K, \Sigma, \delta, s, F)$ be a DFA and assume that $K = \{1, 2, ..., n\}$. Define the $n \times n$ Boolean matrix T such that $T[i,j] = 1$ if and only if for some $a \in \Sigma$ $\delta(i,a) = j$.

   (a) Use Warshall's algorithm to compute the Boolean matrix T^* which has the property that $T^*[i,j] = 1$ if and only if for some $w \in \Sigma^*$, $(i,w) \vdash_M^* (j,e)$. That is, $T^*[i,j] = 1$ if and only if the state j is reachable from the state i.

   (b) Assuming T and T^* have been computed, use these matrices in an algorithm to determine if $L(M)$ is empty.

   (c) Assuming T and T^* have been computed, use these matrices in an algorithm to determine if $L(M)$ is finite.