# CSE 322
# Autumn Quarter 1998
# Assignment 3
# Due Friday, October 23

All solutions should be neatly written or type set.

1. (10 points) A finite state transducer is a DFA with output (cf. page 60, problem 2.1.4). Formally, a finite state transducer is a 5-tuple $M = (K, \Sigma, \Gamma, \delta, s)$ where $K$ is the finite set of states, $\Sigma$ is the input alphabet, $\Gamma$ is the output alphabet, $s$ is the start state. The transition function $\delta$ maps $K \times \Sigma$ into $K \times \Gamma^*$. The semantics of the transition is that if $\delta(q, \sigma) = (p, w)$ then $M$ in state $q$ reading the symbol $\sigma$ goes to state $p$ outputting the string $w$. Given an input string $M$ produces the output which is the concatenation of all the strings outputted by $M$ during its moves. For example, examining the finite state transducer on the top of page 62, on input *abba* the transducer outputs *abb*.

   (a) Design a finite state transducer that implements binary addition. Assume the input alphabet has five symbols $\{(0,0),(0,1),(1,0),(1,1),\#\}$ and the output alphabet has just two symbols $\{0,1\}$. On input say $(1,1)(1,0)(1,1)\#$ your machine should output 0011. More generally, your machine considers the input $(a_0, b_0)(a_1, b_1)\cdots(a_n, b_n)\#$ to be the problem of adding the two numbers represented by the sequences $a$ and $b$ and obtaining the sum as a sequence $c$.

   $$\begin{array}{r} a_n a_{n-1} \cdots a_1 a_0 \\ +b_n b_{n-1} \cdots b_1 b_0 \\ \hline c_{n+1} c_n c_{n-1} \cdots c_1 c_0 \end{array}$$

   (Note that since our finite state transducer reads its input left-to-right, it is presented the lowest order bit first, which is on the right in the standard way of writing binary numbers.)

   (b) Design a finite state transducer that implements "short division" of decimal integers by 6. For example, on an input $3472\#$ your finite state transducer should produce the string $0578R4$. That is, the result of dividing 3472 by 6 is 578 with a remainder of 4. On input $42\#$ the output should be $07R0$.

2. (10 points) Let $M_1 = (K_1, \Gamma, \delta_1, s_1, F_1)$ be a DFA and let $M_2 = (K_2, \Sigma, \Gamma, \delta_2, s_2)$ be a finite state transducer. Assume for this problem that if $\delta_2(q, \sigma) = (p, w)$ then $w \in \Gamma$, that is $w$ is a string of length 1.

   (a) Construct a DFA $M$ that accepts the language

   $$L = \{w : w \in \Sigma^* \text{ and on input } w, M_2 \text{ outputs a member of } L(M_1)\}.$$

   (Hint: recognize that in some way $M_2$ will have to be run on the input $w$ at the same time that $M_1$ is run on the output produced. In order to run $M_1$ and $M_2$ in parallel you will need some sort of "cross product" construction.)

   (b) Write a "behavioral lemma" that relates the behavior of $M$ with that of $M_1$ and $M_2$ in way that is sufficient to prove that $M$ accepts the language $L$.

   (c) (2 points extra credit) Do parts (a) and (b) removing the restriction that $M_2$ outputs a string of length 1 per step.

3. (10 points) Consider the following DFA. Use the state elimination method to construct an equivalent regular expression. Show the steps of the state elimination method. Describe in words the language accepted by the DFA.