CSE 322
Autumn Quarter 1998
Assignment 1
Due Friday, October 9

All solutions should be neatly written or type set. All major steps in proofs must be justified.

1. (10 points) The problem is designed to strengthen your ability to prove facts by induction. You will need to review the definition w^i and w^R on page 43. You may also use the fact that $(xy)^R = y^R x^R$ for strings x and y and other basic facts about concatenation of strings.
   (a) Prove that $w^{i+1} = ww^i$ for all $i \geq 0$ and all strings w.
   (b) Prove that $(w^R)^i = (w^i)^R$ for all $i \geq 0$ and all strings w.
2. (10 points) In this problem you will need to use strong induction. Carefully prove that if L is a regular language then so is $L^R = \{w^R : w \in L\}$. You can do this by proving by induction on the length of the regular expression γ that there is a regular expression γ' with the property that $\mathcal{L}(\gamma') = \mathcal{L}(\gamma)^R$, that is, the language described by γ' is the reversal of the language described by γ.
3. (10 points) Let $\Sigma = \{a, b\}$. Write regular expressions that describe each of the following languages. You do not have to prove that your regular expression is correct.
   (a) The set of strings in Σ^* with a number of a's equal to 1 mod 3.
   (b) The set of strings in Σ^* with at most three b's.
   (c) The set of strings in Σ^* with exactly one occurrence of bbb.